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LINEAR ARRAY PATTERN SYNTHESIS FOR WIDE BAND SECTOR NULLING

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1. INTRODUCTION

The problem of wide band interference suppression from certain direction has received much attention during the last decade [1, 2]. Wide band interference nulling problems often arise when the interfering signal direction varies with time or is not known exactly. Interference suppression can be achieved by imposing broad null sectors in the interference direction while keeping the main beam directed towards the desired signal. One way of achieving approximate wide band pattern nulling is to impose a sufficient number of equispaced nulls over the prescribed pattern sector. A numerical study has shown that the sidelobe cancellation is independent of the actual pattern type and is determined by the number of equispaced nulls and the number of sidelobes to be canceled [3]. On the other hand, the power response of the array is integrated over a spatial region of interest and is forced to be less than or equal to a small quantity [2]. The array synthesis problem can be formulated as a least-square null constrained optimization problem where the optimum complex weight vector is obtained by minimizing the weight vector norm subject to linear and quadratic constraints. In fact, independent of the particular pattern synthesis technique wide sector nulling rapidly uses up the available degrees of freedom.

Clearly, it is beneficial to let the algorithm estimate the most efficient number of nulls to suppress the prescribed sector while using the rest of the pattern nulls to approximate the perturbed pattern in some sense. In this work a minimax approximation technique is applied to impose sector nulling using complex coefficients. This technique utilizes the necessary degrees of freedom to suppress the prescribed broad sectors below a certain level while the rest of the degrees of freedom are used to approximate the original pattern in the minimax sense. Furthermore, the sidelobe level (SLL) and the main beam characteristics can be controlled directly which is more meaningful in antenna arrays. Moreover, as the null synthesis methods based on linearized solutions of the amplitude-only or the position-only control methods are incapable of realizing asymmetric nulls about the main beam [1, 4–6], the complex coefficient method can impose any asymmetric nulls at arbitrary directions in the sidelobe region.

2. STATEMENT OF THE PROBLEM

The array pattern of a linear array with $2N$ equispaced isotropic elements can be expressed as

$$F_0(u) = \sum_{n=1}^{2N} \omega_n e^{j d_n k u} \tag{1}$$

$$d_n = d_0 \left(\frac{2N+1}{2} - n \right) \tag{2}$$

where ω_n, d_0, k, and u denote the n th initial complex element excitation, the interelement spacing, wave number and the sine function of the angle θ from broad side, respectively. Furthermore, if the initial current excitations satisfy the following relation

$$\omega_n = \omega^*_{2N-n+1} \tag{3}$$

where $(^*)$ denotes the complex conjugation. Then the initial array pattern can be written as

$$F_0(u) = \sum_{n=1}^{N} (\omega_n e^{jd_n ku} + \omega_n^* e^{-jd_n ku}) \tag{4}$$

Let the complex coefficients be expressed as

$$\omega_n = a_n + jb_n \tag{5}$$

then equation (4) can be manipulated to be re-expressed as

$$F_0(u) = 2 \sum_{n=1}^{N} [a_n \cos(d_n ku) - b_n \sin(d_n ku)] \tag{6}$$

An array with conventional feed network has a frequency dependent pattern such that an m th point source at a fixed direction u_m with relative frequency bandwidth B_m will appear to cover an angular pattern sector

$$\Delta u_m = B_m \cdot u_m \tag{7}$$

Conventionally, imposing a set of closely spaced single nulls over the desired sector will create a broad null in the pattern. When sufficient number of pattern nulls, say $N1$, have been allocated to suppress the wide band sector, Δu_m, below a certain level then the remaining $(N - N1)$ pattern nulls are used to approximate the original pattern in the least-square sense [1–3]. Because wide sector nulling rapidly uses the available degrees of freedom, we have to know exactly the most efficient number of equispaced nulls to suppress the prescribed interference.

3. OPTIMIZATION PROCEDURE

In this work, broad band sector nulling is achieved by calculating a set of new complex element excitations, $\{\alpha_n, \beta_n\}$, such that the perturbed array pattern has suppressed the required sector level while maintaining the main beam and SLL within certain tolerances. This particular scheme depends on using a sufficient number of the degrees of freedom to suppress the interference below the system noise level. The remaining degrees of freedom are used to approximate, in the minimax sense, the original pattern which the system maintains in an interference-free environment. The set of coefficients $\{\alpha_n, \beta_n\}$ are evaluated as the marginal cost in the simplex tableaux.

Let the perturbed pattern be expressed as :

$$F(u) = 2\sum_{n=1}^{N}[\alpha_n \cos(d_n k u) - \beta_n \sin(d_n k u)] \tag{8}$$

then to maintain the level of the perturbed pattern for the whole m th sector within a small quantity, δ_{2m}, and at the same time maintain the main beam characteristics as the initial pattern within certain tolerance, δ_1, the perturbed pattern should satisfy the following equation

$$F(u) = \begin{cases} F_0(u) \pm \delta_1 & 0 \le u \le u_0 \\ 0 \pm \delta_{2m} & u = u_m \pm 0.5\Delta u_m \text{ for } m = 1, 2, \ldots, M \\ 0 \pm \delta_3 & \text{elsewhere} \end{cases} \tag{9}$$

where u_0 is the location of the first null, δ_3 is the maximum SLL of the perturbed pattern, and M is the number of suppressed wide band sectors.

For a given continuous function $D(u)$ defined over the interval $[-1 \le u \le 1]$ and a specified tolerance δ_1, there exists a function $F(u)$ that is a linear combination of N-cosine and N-sine functions, according to Fourier theorem, of the form of equation (8), and satisfies the inequality

$$E(u) = W(u)|D(u) - F(u)| \le \delta_1 \tag{10}$$

where $D(u)$ and $W(u)$ are the desired and the weighting functions, respectively. For the problem under investigation $D(u)$ may represent the ideal characteristics

$$D(u) = \begin{cases} F_0(u) & 0 \le u \le u_0 \\ 0 & \text{elsewhere} \end{cases} \tag{11}$$

and the weighting function may be written as

$$W(u) = \begin{cases} 1 & 0 \leq u \leq u_0 \\ \dfrac{\delta_1}{\delta_{2m}} & u = u_m \pm 0.5\Delta u_m,\ m = 1, 2, \ldots, M \\ \dfrac{\delta_1}{\delta_3} & \text{elsewhere} \end{cases} \tag{12}$$

In practice, to minimize the deviation from the desired pattern at all points in the angular range, we discretize the interval $[-1 \leq u \leq 1]$ into a sufficient number of angular points L. Consequently, the problem under investigation can be reduced to an overdetermined system of linear equations in the coefficients $\{\alpha_n, \beta_n\}$. Mathematically, the problem can be stated as follows:

$$\begin{aligned} &\text{minimize } \delta_1 \text{ subject to} \\ &E(u_i) = W(u_i)|D(u_i) - F(u_i)| \leq \delta_1 \qquad \text{for } i = 1, 2, \ldots, L \end{aligned} \tag{13}$$

The perturbed function, $F(u)$, which satisfies Equation (13) in the minimax sense can be solved using the simplex method of linear programming [7]. Consequently, the following dual linear programming of a maximization problem is formulated as

$$\text{maximize} \sum_{i=1}^{L} W(u_i)D(u_i)(S_i - t_i) \tag{14}$$

subject to

$$\sum_{i=1}^{L}(S_i - t_i)W(u_i)\phi_n(u_i) = 0 \qquad \text{for } n = 1, 2, \ldots, 2N \tag{15}$$

where the basis functions of the constraints are given by,

$$\phi_n(u_i) = \begin{cases} \cos(d_n k u_i) & \text{for } n = 1, 2, \ldots, N \\ \sin(d_{n-N} k u_i) & \text{for } n = N+1, N+2, \ldots, 2N \end{cases}$$

$$\sum_{i=1}^{L}(S_i + t_i) \leq 1 \tag{16}$$

$$S_i > 0,\ t_i > 0 \tag{17}$$

S_i and t_i are slack variables.

A computer program has been developed to solve (14) subject to (15)–(17) as a minimax problem of an overdetermined system of linear equations at L sample points.

4. RESULTS AND DISCUSSION

This approach of antenna pattern synthesis to suppress multiple narrow and wide sectors in the sidelobe region is based on the minimax approximation. Therefore, the SLL can be controlled directly for arbitrary prescribed sector nulling and given main beam characteristics using equation 13. To validate the approach, several illustrative examples have been simulated to suppress wide sectors in the sidelobe region using complex current excitations. As the Chebyshev current distribution gives the optimum pattern in terms of the SLL and the main beam width, an initial Chebyshev array pattern is assumed for a 20 element equispaced array with $\lambda/2$ interelement spacing. The sample points, L, are chosen as 300 equally distributed points over the range $[-1 \leq u \leq 1]$.

Figure 1 shows the perturbed pattern (solid) with three prescribed wide sectors when their centers are imposed at $u_m = -0.35$, 0.35, and 0.75 and with relative bandwidths of 5%, 5%, and 10%, respectively. The corresponding suppressed sector levels are reduced to 56, 56, and 61 dB when the SLL of the perturbed pattern is maintained at 30 dB and the main beam width is almost unchanged compared with the initial pattern. The number of pattern nulls that are used to realize the suppressed sectors are 2, 2, and 3, respectively, as shown in Figure 1. So, this method estimates the required number of pattern nulls to realize the prescribed suppressed sectors and locates the positions of the remaining nulls to control the optimum SLL for the perturbed pattern in the minimax sense. Figure 2 shows five narrow band nulls imposed at $u_m = -0.55$, -0.25, 0.173, 0.35, and 0.45 using the minimax solution (solid) compared with the perturbed pattern using the least mean square error (LMSE) solution (dotted). From Figure 2, it is shown that the SLL of the minimax solution is maintained at 30 dB while the achieved SLL of the LMSE solution is reduced to 26 dB. The previous Figures show the capability of this method to suppress symmetric and/or asymmetric narrow and wide interferences.

Figure 3 shows the perturbed pattern of a 10% relative bandwidth suppressed sector with its center imposed at $u_m = 0.45$ using the

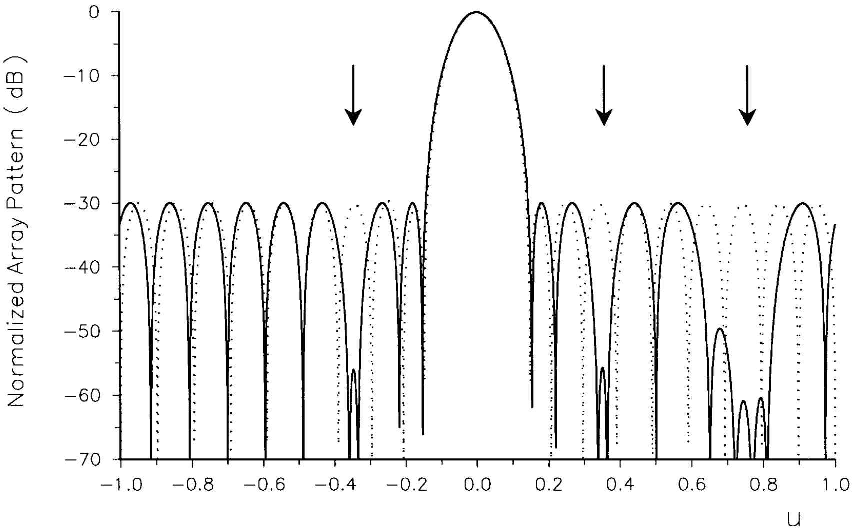

Figure 1. Simulated antenna array pattern: solid, perturbed pattern with three wide sectors imposed around $u_m = -0.35$, 0.35, and 0.75 for $B = 5\,\%$, 5%, and 10%, respectively; dotted, initial 30-dB Chebyshev pattern, $2N = 20$, $d_0 = \lambda/2$.

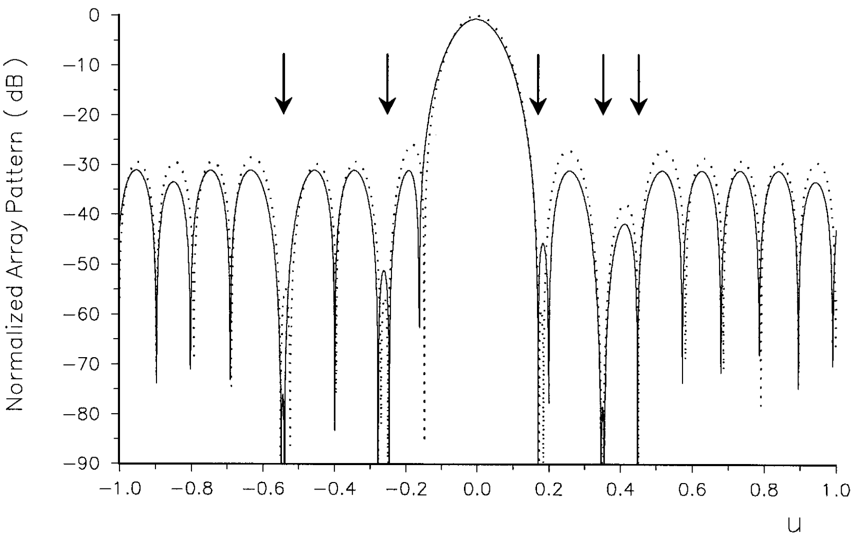

Figure 2. Simulated perturbed pattern with five narrow nulls imposed around $u_m = -0.55$, -0.25, 0.173, 0.35, and 0.45 for the minimax solution (solid) and the LMSE solution (dotted).

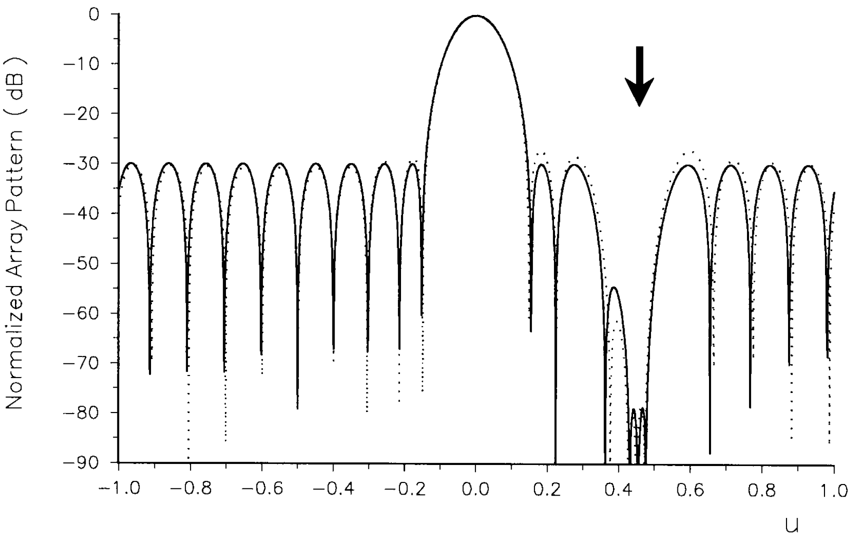

Figure 3. Simulated perturbed pattern with one wide sector of 10% relative bandwidth imposed around $u_m = 0.45$ for the minimax solution (solid) and the LMSE solution (dotted).

minimax solution (solid) compared with the perturbed pattern using the LMSE solution (dotted) when three equispaced nulls are imposed in the suppressed sector. The sector levels are reduced to 79 dB, and 78 dB for the minimax solution and the LMSE solution, respectively. However, the minimax solution maintains the SLL at 30 dB and also uses three nulls to suppress the prescribed sector while the achieved SLL of the LMSE solution is reduced to 27.2 dB. From the previous Figures, we conclude that the minimax approach allows the control of the SLL directly which is important in antenna pattern synthesis. Table 1 gives the computed complex current excitations of the first ten elements for Figures 1, 2, and 3. Note that the current ratios of the last ten elements are the complex conjugate of the first ten elements which means that the number of controllers is reduced to $2N$ only.

To get more insight into the performance of the proposed procedure for wide band jammer suppression, several examples are simulated numerically with a fixed SLL of 30 dB as given in Table 2. The sector depth and the number of pattern nulls used for sector suppression are listed for the given relative bandwidth. When the center of the sector is imposed at $u_m = 0.45$, the sector depth is reduced to 91, 79,

Table 1. Computed element complex current ratios, $\{\alpha_n + j\beta_n\}$, for Figs. 1, 2, and 3.

ELEM. NO	Fig. 1 $\{\alpha_n + j\beta_n\}$	Fig. 2 $\{\alpha_n + j\beta_n\}$	Fig. 3 $\{\alpha_n + j\beta_n\}$
1	3.40424 + j 0.39420	3.59854 + j 0.74711	3.54646 + j 1.261930
2	4.13682 - j 0.86832	2.89646 + j 0.39106	3.50861 - j 0.99829
3	5.45423 + j 0.84459	5.18549 + j 0.28918	6.21289 - j 0.45121
4	7.42635 - j 0.37693	7.26713 + j 0.47055	7.74625 + j 0.25507
5	10.16115 - j 0.04960	8.62844 + j 1.11936	9.19386 + j 0.09069
6	11.20823 + j 0.04615	10.86502 + j 1.00578	11.14639 - j 0.11199
7	12.41265 + j 0.08288	12.81833 + j 0.18403	12.83088 + j 0.06128
8	13.48050 - j 0.09057	14.08005 - j 0.53291	13.94186 + j 0.01518
9	15.35371 - j 0.10943	15.01270 + j 0.07166	14.99143 - j 0.29911
10	15.96517 + j 0.28282	15.90058 + j 0	15.88897 - j 0.22029

Table 2. Sector depth and number of nulls used for sector suppression, SLL $= 30$ dB, half power main beam width HPBW $= 6.46°$.

Angular Location u_m	Relative Bandwidth (%)	Sector Depth (dB)	No. of used nulls
0.45	5	91	3
0.45	10	79	3
0.45	20	66	4
0.173	5	59	2
0.173	10	52	2
-0.35 0.35 0.75	5 5 10	56 56 61	2 2 3

and 66 dB for 5%, 10%, and 20% relative bandwidth, respectively, while the main beam width is almost unchanged. A worst case of imposing a suppressed sector with a center at the peak of the first sidelobe ($u_m = 0.173$) and different relative bandwidths is given in Table 2. The corresponding sector levels are reduced to 59 and 52 dB, respectively. Furthermore, three wide band sectors are imposed in the sidelobe region as shown in Figure 1 and given in Table 2. The results show the validity and the efficiency of multiple wide band interference suppression by controlling the complex current excitations using the minimax approximation.

5. CONCLUSION

This paper presents a new technique for multiple narrow and wide band null synthesis by controlling the complex current excitations in linear arrays. The complex current excitations are determined using the minimax approximation approach to suppress multiple wide band sectors while maintaining the side lobe level and the main beam width almost unchanged. The approximation procedure is based on a linear programming approach to solve an overdetermined system of linear equations in the Chebyshev norm [7]. Expressing the error function as given in equation 13 gives the advantage of controlling the main beam width, SLL, and sector depth which is not obtainable by LMSE techniques.

The results show the formation of single and multiple wide band nulls at arbitrary prescribed directions. In fact, this method can impose arbitrary symmetric and asymmetric nulls in the sidelobe region and hence, overcome the limitations of the linearized amplitude-only or phase-only solutions. Also, unlike the LMSE techniques, we conclude that the minimax approach allows the control of the SLL directly while suppressing the prescribed sectors which is important in antenna pattern synthesis. Furthermore, this method estimates the required number of pattern nulls to realize the prescribed suppressed sector and locates the positions of the remaining nulls to control the optimum SLL for the perturbed pattern in the minimax sense.

REFERENCES

1. Steyskal, H., R. A. Shore, and R. L. Haupt, "Methods for null control and their effects on radiation pattern," *IEEE Trans. Antennas Propagat.*, Vol. AP-34, 404–409, 1986.
2. Er, M. H., "Linear antenna array pattern synthesis with prescribed broad nulls," *IEEE Trans. Antennas Propagt.*, Vol. AP-38, 1496–1498, 1990.
3. Steyskal, H., "Wide-band nulling performance versus number of pattern constraints for an array antenna," *IEEE Trans. Antennas Propagat.*, Vol. AP-31, 159–163, 1983.
4. Vu, T. B., "Method of null steering without using phase shifters," *IEE Proceedings*, Vol. 131, Pt. H, 242–246, 1984.
5. Ismail, T., and M. M. Dawoud, "Null steering in phased arrays by controlling the element positions," *IEEE Trans. Antennas Propagat.*, Vol. AP-39, 1561–1566, Nov. 1991.
6. Mismar, M. J., and T. Ismail, "Null steering using the minimax approximation by controlling only the current amplitudes," *Int. J. Electronics*, Vol. 78, 409–415, 1995.
7. Mismar, M. J., and I. H. Zabalawi, "Complex coefficient FIR Digital filters," *Circuits Systems Signal Process*, Vol. 13, 591–600, 1994.