RESEARCH ON CHIRAL AND BIANISOTROPIC MEDIA IN BYELORUSSIA AND RUSSIA IN THE LAST TEN YEARS

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1.   Introduction

The main goal of this review is to present the most interesting and recent results obtained by Byelorussian and Russian scientists in electromagnetics and optics of chiral, biisotropic and bianisotropic media to the readership of the *Progress in Electromagnetics Research Series.* For several reasons, very little is known about that research in the West. Political tension in the cold-war era created and supported firm boundaries between countries which, until recently, prevented the active exchange of ideas. At present, the economic crisis in the former Soviet Union creates novel obstacles in communication between the two worlds.

In the area of complex media electromagnetics, another communication problem exists–the communication between the specialists in optics, who have been doing research on optically active crystals for decades, and the microwave engineers and specialists in electromagnetic theory, who became interested in chiral and bianisotropic artificial materials only recently. Although the physics of the problems they try to solve is of course the same in the optics and in the microwave regime, the methods and even the terminology are often different.

In this paper, we make an attempt to partially cover both the gaps between the Eastern and the Western researchers, as well as between the specialists in optics and microwave theory and techniques. The first author of the present review is a physicist specializing in optics working in Byelorussia, and the second one is a specialist in electromagnetic theory and microwave engineering from Russia, working in the area of chiral, biisotropic and bianisotropic media electromagnetics. The third author is Head of the Optics Department of the Gomel State University, Byelorussia.

With the goal to review the current state-of-the-art of the research in our two countries, we unfortunately have no possibilities to give credits to other researchers in other countries and to give an extensive review of the research history with references to early papers. However, a few comments seem most appropriate there. In Byelorussia, the chiral research has a very long history with its roots in the famous works of Academician Fedor I. Fedorov*. In the early fifties, he developed the so called covariant methods which allowed one to study electromagnetic fields in anisotropic crystals independently of any coordinate system. In fact, he introduced and developed dyadic formalism in optics of crystals. Later on, F. Fedorov applied his method to optics of non-chiral crystals [1], acoustics of anisotropic media [2], boundary problems in crystallooptics [3], electromagnetics of chiral and bianisotropic media [4], and to elementary particles physics [5]. The covariant approach greatly simplifies the analysis, especially for anisotropic media. Presently, there are in Byelorussia several recognized research teams conducting fundamental and applied research in chiral and bianisotropic media electromagnetics, headed by

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* Fedor Ivanovich Fedorov passed away suddenly on 13th of October 1994 at his home in Minsk, Belarus. He was 83 years old.

F. I. Fedorov, B. V. Bokut* and A. N. Serdyukov. The first author belongs to that school and works in the group of Prof. A. N. Serdyukov.

A few remarks about the terminology and notations are needed. Historically, the electromagnetic chirality was studied in the optical region. Hence, the most common term for chiral media is *optically active* media. An alternative term in the Russian literature is *gyrotropic* media. In more recent Russian literature the name *chiral* has been adopted for novel artificial microwave materials, as it is also the case in the English language journals. However, the name *gyrotropic* is also in use for the microwave regime.

The vector notations vary in different Russian language books. It is probably most common to denote the scalar product by brackets as $(\overline{a}, \overline{b})$ or simply as $\overline{a}\,\overline{b}$. The vector product is denoted most often by rectangular brackets: $[\overline{a}, \overline{b}]$. A dyad formed of two vectors $\overline{a}$ and $\overline{b}$ is usually written as $\overline{a} \cdot \overline{b}$ or $\overline{a} \circ \overline{b}$. A dyadic which corresponds to the vector multiplication operation is often denoted as $\overline{a}^{\times}$, so that, for example, the product $\nabla^{\times}\overline{a}$ means $\nabla \times \overline{\overline{I}} \cdot \overline{a} = \nabla \times \overline{a}$, where $\overline{\overline{I}}$ is the unit dyadic. In the Russian language we have no way to distinguish between a dyad and a dyadic. As a consequence, dyadics are some times termed as "tensors", even in the dyadic notation. A matrix or tensor needs no special notation, when it is clear from the context whether it is a matrix or a scalar. Sometimes, calligraphic letters ($\mathcal{A}$) or square brackets ($[A]$) or double overline ($\overline{\overline{A}}$) or hat ($\hat{A}$) can be used. Also, a matrix proportional to the unit matrix is often not distinguished in notations from a scalar. However, especially in recent books and papers, notations similar to the ones usually seen in the modern Western literature are also in use.

Since such a variety of notations may be confusing, for this review we have translated the formulas from different sources into a single set of notations widely adopted in the Western literature. Here, $\overline{a} \cdot \overline{b}$ means the scalar product of two vectors, $\overline{a} \times \overline{b}$ stands for the vector product, $\overline{a}\overline{b}$ is a dyad, and $\overline{\overline{a}}$ is a dyadic or a 3×3-matrix. The unit dyadic is denoted by $\overline{\overline{I}}$.

Traditionally, in the fundamental research we use the Gauss units, not the SI. In this review we follow the original papers and keep the formulas in the Gauss units system.

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* Academician B. V. Bokut passed away on March 15, 1993, when this review was in preparation.

Most of the references here refer to the original Russian editions of Russian and Byelorussian scientific journals. Many of the leading journals are translated into English and reprinted by different publishing companies. Usually, the volume and the issue numbers in the Russian and the corresponding English editions coincide, but the page numbers may of course differ. Here we list the names of original Russian journals we refer to with their direct English translations. If we know that the title of the corresponding English language reprint differs from the previous, we give that English name as well, in italics.

- Doklady Akademii Nauk BSSR – Proceedings of the Academy of Sciences of the Byelorussian Soviet Socialist Republic

- Izvestiya Akademii Nauk BSSR, Seriya Fiziko-Matematicheskikh Nauk – Transactions of the Academy of Sciences of the Byelorussian Soviet Socialist Republic, Series of Physics and Mathematics

- Izvestiya Moskovskogo Universiteta – Moscow University Transactions

- Izvestiya Vys'shich Uchebnych Zavedenii, Fizika – Transactions of Higher Education Institutions, Physics

- Izvestiya Vys'shich Uchebnych Zavedenii, Radiofizika – Transactions of Higher Education Institutions, Radiophysics

- Kristallografiya – Crystallography – *Sov. Physics – Crystallography*

- Optika i Spektroskopiya – Optics and Spectroscopy

- Pis'ma v Zhurnal Tekhnicheskoi Fiziki – Letters to the Journal of Technical Physics

- Radiotekhnika i Elektronika – Radiotechnics and Electronics – *Sov. Journ. of Communication Technology and Electronics*

- Vestnik Moskovskogo Universiteta – News of the Moscow University

- Ukrainskii Fizicheskii Zhurnal – Ukrainian Physical Journal

- Zhurnal Experimentalnoi i Teoreticheskoi Fiziki – Journal of Experimental and Theoretical Physics – *Sov. Phys. – JETF*

- Zhurnal Prikladnoi Spektroskopii – Journal of Applied Spectroscopy

- Zhurnal Tekhnicheskoi Fiziki – Journal of Technical Physics – *Sov. Phys. – Technical Physics*

2. Material Equations for Chiral and Bianisotropic Media

The phenomenological theory of reciprocal bianisotropic media can be based on the following material equations with magneto-electric coupling [4], [6–14]:

$$\overline{D} = \overline{\overline{\epsilon}} \cdot \overline{E} + i\overline{\overline{\alpha}} \cdot \overline{H} \qquad \overline{B} = \overline{\overline{\mu}} \cdot \overline{H} - i\overline{\overline{\alpha}}^T \cdot \overline{E} \qquad (1)$$

valid in both the time and the frequency domain. Here, the tensor $\overline{\overline{\alpha}}$ measures the optical activity (or chirality or gyrotropy) of the medium, T denotes the transpose operation and i stands for the imaginary unit. Based on the energy conservation law, the Onsager–Casimir principle of kinetic coefficients symmetry and on the crystallographic symmetry, fundamental properties of the material tensors $\overline{\overline{\epsilon}}$, $\overline{\overline{\mu}}$ and $\overline{\overline{\alpha}}$ were determined [4], [6–14]. Possible alternative constitutive relations were analyzed, especially the model based on the concept of spatial dispersion [15–16]. As a result, it was established that different constitutive equations are actually equivalent (after appropriate redefinitions of the field vectors).

Table 1 shows some possible material relations suggested in the literature for reciprocal bianisotropic media with the corresponding boundary conditions on interfaces and the Poynting vector expressions. The calligraphic letters are used to distinguish the field vectors in different formalisms. The unit vector normal to an interface between two media (with the parameters marked by the indices 1 and 2) is denoted by $\overline{n}$. Trace of a dyadic $\overline{\overline{a}}$ is denoted as $\mathrm{Sp}\overline{\overline{a}}$ and $\overline{\overline{I}}$ is the unit dyadic.

The relations (1) were introduced and thoroughly analysed in [4], [6–14]. The equations (2) were used in [15, 16]. The constitutive equa-

tions (3) were introduced in [4], and (4) – in [17]. As is clearly seen, the relations (1) provide the most rational and convenient way of describing effects of chirality, especially in non-uniform media. The Byelorussian research has been based on using the constitutive equations (1), combined with the power of the covariant method of F. Fedorov.

No.	Material equations	Boundary conditions	Poynting vector
1.	$\overline{D}=\overline{\overline{\epsilon}}\cdot\overline{E}+i\overline{\overline{\alpha}}\cdot\overline{H}$ $\overline{B}=\overline{\overline{\mu}}\cdot\overline{H}-i\overline{\overline{\alpha}}^{T}\cdot\overline{E}$	$\overline{n}\times(\overline{E}_1-\overline{E}_2)=0$ $\overline{n}\times(\overline{H}_1-\overline{H}_2)=0$	$\overline{S}=\frac{c}{4\pi}\overline{E}\times\overline{H}$
2.	$\overline{\mathcal{D}}=\overline{\overline{\epsilon}}\cdot\overline{E}+(\overline{\overline{\gamma}}\cdot\nabla)\times\overline{E}$ $\overline{\mathcal{B}}=\overline{\overline{\mu}}\cdot\overline{\mathcal{H}}$	$\overline{n}\times(\overline{E}_1-\overline{E}_2)=0$ $-\overline{n}\times(\overline{\mathcal{H}}_1-\overline{\mathcal{H}}_2)$ $=\frac{1}{c}\left[\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\gamma}}_1\overline{\overline{I}}-\overline{\overline{\gamma}}_1\right)\cdot\frac{\partial\overline{E}_1}{\partial t}\right.$ $\left.-\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\gamma}}_2\overline{\overline{I}}-\overline{\overline{\gamma}}_2\right)\cdot\frac{\partial\overline{E}_2}{\partial t}\right]\times\overline{n}$	$\overline{S}=\frac{c}{4\pi}\overline{E}\times\overline{\mathcal{H}}$ $-\frac{1}{4\pi}\left[\overline{E}\times\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\gamma}}\overline{\overline{I}}-\overline{\overline{\gamma}}\right)\cdot\frac{\partial\overline{E}}{\partial t}\right]$
3.	$\overline{\mathcal{D}}=\overline{\overline{\epsilon}}\cdot\overline{\mathcal{E}}+(\overline{\overline{\gamma}}\cdot\nabla)\times\overline{\mathcal{E}}$ $\overline{\mathcal{B}}=\overline{\overline{\mu}}\cdot\overline{\mathcal{H}}+(\overline{\overline{\nu}}\cdot\nabla)\times\overline{\mathcal{H}}$	$\overline{n}\times(\overline{\mathcal{E}}_1-\overline{\mathcal{E}}_2)$ $=\frac{1}{c}\left[\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\nu}}_1\overline{\overline{I}}-\overline{\overline{\nu}}_1\right)\cdot\frac{\partial\overline{\mathcal{H}}_1}{\partial t}\right.$ $\left.-\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\nu}}_2\overline{\overline{I}}-\overline{\overline{\nu}}_2\right)\cdot\frac{\partial\overline{\mathcal{H}}_2}{\partial t}\right]\times\overline{n}$	$\overline{S}=\frac{c}{4\pi}\overline{\mathcal{E}}\times\overline{\mathcal{H}}$ $-\frac{1}{4\pi}\left[\overline{\mathcal{E}}\times\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\gamma}}\overline{\overline{I}}-\overline{\overline{\gamma}}\right)\cdot\frac{\partial\overline{\mathcal{E}}}{\partial t}\right.$ $\left.+\overline{\mathcal{H}}\times\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\nu}}\overline{\overline{I}}-\overline{\overline{\nu}}\right)\cdot\frac{\partial\overline{\mathcal{H}}}{\partial t}\right]$
4.	$\overline{\mathcal{D}}=\overline{\overline{\epsilon}}\cdot\overline{\mathcal{E}}+(\overline{\overline{\gamma}}\cdot\nabla)\times\overline{\mathcal{E}}$ $+i\overline{\overline{\alpha}}\cdot\overline{\mathcal{H}}$ $\overline{B}=\overline{\overline{\mu}}\cdot\overline{\mathcal{H}}+(\overline{\overline{\nu}}\cdot\nabla)\times\overline{\mathcal{H}}$ $-i\overline{\overline{\alpha}}^{T}\cdot\overline{\mathcal{E}}$	$\overline{n}\times(\overline{\mathcal{H}}_1-\overline{\mathcal{H}}_2)$ $=-\frac{1}{c}\left[\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\gamma}}_1\overline{\overline{I}}-\overline{\overline{\gamma}}_1\right)\cdot\frac{\partial\overline{\mathcal{E}}_1}{\partial t}\right.$ $\left.-\left(\frac{1}{2}\mathrm{Sp}\overline{\overline{\gamma}}_2\overline{\overline{I}}-\overline{\overline{\gamma}}_2\right)\cdot\frac{\partial\overline{\mathcal{E}}_2}{\partial t}\right]\times\overline{n}$	

Table 1.

3. The Green Function. Radiation and Scattering in Chiral Media

Historically, the propagation, reflection and refraction of light in source-free chiral regions were considered, starting from the work of Arago (1811), Fresnel (1823) and Pasteur (1848). To the best of our knowledge, the time-domain Green function for chiral media had been not found before the paper [19], where also some scattering problems were considered. In [19], isotropic reciprocal chiral media were studied.

Based on the Lorenz condition

$$\nabla \cdot \overline{A} + \frac{\epsilon\mu - \alpha^2}{c} \frac{\partial \phi}{\partial t} = 0 \qquad (2)$$

the following equations for the vector and scalar potentials $\overline{A}$ and ϕ were established:

$$\left[\left(\nabla \cdot \nabla - \frac{\epsilon\mu - \alpha^2}{c^2} \frac{\partial^2}{\partial t^2} \right) \overline{\overline{I}} + \frac{2i\alpha}{c} \frac{\partial}{\partial t} \nabla \times \overline{\overline{I}} \right] \cdot \overline{A} = -\frac{4\pi\mu}{c} \overline{j} \qquad (3)$$

$$\left(\nabla \cdot \nabla - \frac{\epsilon\mu - \alpha^2}{c^2} \frac{\partial^2}{\partial t^2} \right) \phi = -\frac{4\pi\mu}{\epsilon\mu - \alpha^2} \rho \qquad (4)$$

where $\overline{\overline{I}}$ is the unit dyadic, $\overline{j}$ stands for the electric current density and ρ is the electric charge density. The time domain dyadic Green function of the Eq. (3), valid for media with no frequency dispersion, was found in [19] in the form

$$\overline{\overline{G}}(\overline{R}, \tau) = \sqrt{\frac{\epsilon}{\mu}} \frac{1}{R} \left[\delta(R - v_+\tau)\overline{e}_+\overline{e}_+^* + \delta(R - v_-\tau)\overline{e}_-\overline{e}_-^* \right. \\ \left. + \sqrt{\frac{\epsilon\mu}{n_+n_-}} \delta(R - v_0\tau)\overline{e}_0\overline{e}_0 \right] \qquad (5)$$

where $\delta(x)$ is the Dirac delta function, $\overline{R}$ is the vector from the source to the field point, $\overline{e}_\pm$ stand for the unit right hand and left hand circularly polarized vectors, correspondingly, and $\overline{e}_0 = \overline{R}/R$ is the unit vector in the direction of $\overline{R}$. The vectors $\overline{e}_\pm$ are orthogonal to $\overline{e}_0$. $\tau = t - t'$ is the time elapsed since the radiation moment, $*$ denotes complex conjugate. As it is seen from (5), transverse circularly polarized electromagnetic waves propagate from the source point with the phase velocities $v_\pm = c/n_\pm$, whereas longitudinal wave components have the velocity $v_0 = c/\sqrt{n_+n_-}$. The two refraction indices read $n_\pm = \sqrt{\epsilon\mu} \pm \alpha$. Scalar solutions of (4) also have propagation velocity v_0. In [19], retarding potentials and dipole radiation were considered based on the Green function (5).

The Green function which takes into account frequency dispersion in chiral media under time-harmonic excitation was obtained in [20].

It was shown that the frequency dispersion effects are important, for example, near frequencies of high absorption, where the phase and group velocities may have the opposite directions.

In particular, for a spiral model of chiral molecules or artificial scatterers the following dispersion model holds [21]

$$\epsilon(\omega) = 1 + \frac{\beta_0^2}{\Omega_0^2 - \omega^2} \qquad \alpha(\omega) = A_0 \frac{\beta_0^2 \omega}{c \left(\Omega_0^2 - \omega^2\right)} \qquad (6)$$

where A_0 is a parameter determined by the size of the spirals, β_0^2 is proportional to the concentration of the spirals, and Ω_0 is the resonance absorption frequency. As can be shown [20], in the frequency range

$$\sqrt{\Omega_0^2 + \beta_0^2} < \omega < \sqrt{\Omega_0^2 + \beta_0^2} \left(1 + \frac{\beta_0^2 A_0^2}{2c^2}\right) \qquad (7)$$

the phase and group velocities of one of the circularly polarized eigenwaves have the opposite directions. For $A_0 \sim 10^{-7}$ cm and $\beta_0 \sim 10^{16}$ 1/s (in optics) the width (6) is about 5 Å in wavelengths. For the frequencies satisfying (7), the Green function gives waves travelling to the source. However, the group velocity vector is of course always directed from the source. In [20], the Green function and dipole radiation is studied for sources with arbitrary time variation.

The Green function [19] gives a possibility to consider electomagnetic wave scattering by fluctuations of the dielectric permittivity and the chirality parameter in isotropic optically active media. In [22], the extinction coefficients for the left and right hand circularly polarized waves were found:

$$h_\pm = \frac{2\omega^4 \epsilon V}{3\pi c^4} \overline{(\delta n_\pm)_V^2} \qquad (8)$$

where the index V refers to the averaging in space over a volume V:

$$\overline{(\delta n_\pm)_V^2} = \frac{1}{V^2} \overline{\left(\int \delta n_\pm dV\right)^2} \qquad (9)$$

$\delta n_\pm$ stand for the fluctuations of the two refractive indices, which depend on the fluctuations of the dielectric permittivity and chirality parameter as

$$\delta n_\pm = \delta \left(\sqrt{\epsilon} \pm \alpha\right) = \frac{\delta \epsilon}{2\sqrt{\epsilon}} \pm \delta \alpha$$

and the line over a term means the averaging over particles movements.

The difference in the extinction coefficients for the left and right hand circularly polarized waves leads to circular dichroism of scattering phenomena in lossless media. The dichroism effect at the length l can be measured by the parameter

$$\Gamma = \frac{1}{l} \tanh \left[\frac{l}{4}(h_- - h_+) \right] \tag{10}$$

or, for small scattering effects ($h_\pm l \ll 1$),

$$\Gamma = \frac{V \omega^4 \sqrt{\epsilon}}{3\pi c^4} \overline{(\delta\epsilon\delta\alpha)_V} \tag{11}$$

Energy dissipation due to the scattering as well as the circular dichroism effect can be described by introducing effective material parameters in (1):

$$\epsilon^{\text{eff}} = \epsilon + i\frac{\omega^2 \sqrt{\epsilon}}{6\pi c^3} V \overline{(\delta\epsilon)^2_V}$$

$$\alpha^{\text{eff}} = \alpha + i\frac{\omega^3 \sqrt{\epsilon}}{3\pi c^3} V \overline{(\delta\epsilon\delta\alpha)_V} \tag{12}$$

so that

$$n_\pm = \sqrt{\epsilon^{\text{eff}}} \pm \alpha^{\text{eff}} \tag{13}$$

In [22] the Rayleigh scattering in optically active fluids and gases was studied. The Einstein formula was generalized for gyrotropic fluids, and the Rayleigh formula was extended for gyrotropic gases:

$$h_\pm = \frac{(\epsilon - 1)\omega^4}{6\pi c^4 N} \left(\epsilon - 1 \pm \frac{4\alpha}{\sqrt{\epsilon}} \right) \tag{14}$$

where N is the chiral particles concentration.

Fluctuation-dissipation theorem for chiral media was considered in [18]. It was shown that the fluctuations of the electric and magnetic flux densities are correlated, in contrast to non-chiral media. As follows from the theorem established in [18], in chiral media with non-zero imaginary part of the chirality parameter α, the dielectric permittivity ϵ and the magnetic permeability μ must be complex, hence, in

lossy isotropic chiral media, all three material parameters are always complex.

At frequencies close to resonance frequencies of absorption, some specific waves with unusual properties can exist, as was found in [23]. In fact, within the frequency range (7), the inequality

$$0 \leq \epsilon(\omega) \leq \alpha^2(\omega) \tag{15}$$

can hold, which is required for the specific waves in question. For example, for a frequency ω_1, where $\epsilon(\omega_1) = 0$, only one transverse wave can propagate in chiral media. The wave is circularly polarized, and the sense of rotation depends on the sign of the chirality parameter α. This wave is an analog to the spiral electromagnetic wave in magnetized plasmas. However, in contrast to plasmas, where the spiral wave propagates along the direction of the external magnetic field only, the spiral wave in isotropic chiral media can propagate in any direction. At a frequency ω_2, where $\epsilon(\omega_2) = \alpha^2(\omega_2)$, in chiral media a transverse wave propagating with infinite phase velocity and non-zero Poynting vector can exist. At the same frequency ω_2 a longitudinal wave with zero Poynting vector can also propagate. In [23], reflection and transmission at an interface between isotropic half space and a chiral half space for frequencies (7), when (15) holds were studied. It was shown that the Cherenkov effect provides a possible way to generate specific waves in chiral media.

4.   Microscopic Theory of Optical Activity

Some general theorems for material parameters in chiral media, based on the Kramers-Kronig relations in chiral media [18] and the Onsager-Casimir symmetry relations were established in [24]. In particular, it was shown that the integral

$$\int_0^\infty \vartheta(\lambda)d\lambda = \int_0^\infty \frac{\vartheta(\omega)}{\omega^2}d\omega = 0 \tag{16}$$

where

$$\vartheta(\omega) = \omega \mathrm{Re}\alpha(\omega)/c \tag{17}$$

is the polarization rotation at a wavelength distance (rotational power). This means, for example, that a purely (at any frequency) right or left hand rotating medium can not exist. Consequently, the terms like *right (left) hand rotating* are not absolute, and one has to define the frequency range where the sense of rotation holds.

Using particular models for chiral particles, say, a spiral model (see (6)), one can have additional relations for the rotational power and the circular dichroism. For the spiral model [25],

$$\int_0^\infty \frac{D(\lambda)}{\lambda} d\lambda = 0 \ , \qquad \int_0^\infty \frac{\lambda D(\lambda)}{\lambda^2 - \lambda_0^2} d\lambda = 0$$

$$\int_0^\infty \lambda D(\lambda) d\lambda = -\pi^3 c^2 \frac{d^2 \vartheta(\omega)}{d\omega^2}\bigg|_{\omega=0} \tag{18}$$

$$\int_0^\infty \frac{\omega^2 + \Omega_0^2}{\left(\omega^2 - \Omega_0^2\right)^2} \vartheta(\omega) d\omega = 0$$

where $D(\omega) = \omega \mathrm{Im}\alpha(\omega)/c$ and $\lambda_0 = 2\pi c/\Omega_0$ is the wavelength which corresponds to the absorption resonance frequency. These results demonstrate that the sign of circular dichroism also varies with frequency. Eq. (18) gives a possibility to model the function $D(\lambda)$ for the frequencies where measurements are difficult or not possible.

The spiral model of chiral molecules can also be used in studies of non-linear properties of chiral media. In [26], some peculiarities of non-linear optical activity of crystals, known from earlier experiments, were explained based on the spiral model. The energy of a spiral molecule in an electromagnetic field was written in the form

$$U = -\overline{p} \cdot \overline{E} - \overline{m} \cdot \overline{H}$$

$$= \frac{e^2}{m_e(r^2 q^2 + 1)(\Omega_0^2 - \omega^2)} \left(\frac{1}{3} E_0^2 + \frac{r^4 q^2 \omega^2}{12c^2} H_0^2\right) \tag{19}$$

where $\overline{p}$ and $\overline{m}$ denote the electric and magnetic dipole moments of the molecule, e and m_e stand for the electron charge and its mass, r and q are the radius and the pitch of the spiral trajectory of electrons. Ω_0 is the electron resonant frequency (the resonant frequency of molecule absorption), E_0 and H_0 are the amplitude values of the external electric and magnetic fields. The variation in rotational power

of the medium caused by variations of the amplitude of the electro-magnetic field in the travelling wave is given by the formula

$$\Delta\vartheta = \frac{3m_{\mathrm{e}}(\Omega_0^2 - \omega^2)\vartheta}{2(E_0 rqe)^2}\left[\Delta U(q) + 2\Delta U(r)\right] \tag{20}$$

where ϑ is defined in (17), $\Delta U(q)$ and $\Delta U(r)$ denote the variations in the interaction energy (19) caused, correspondingly, by non-linear variations in the pitch and in the radius of the electron trajectory. Since the quantity in the square brackets in (20) is negative (the interaction energy tends to decrease), $\mathrm{sign}(\Delta\vartheta) = -\mathrm{sign}\vartheta$, provided that $\Omega_0 > \omega$. The last conclusion was supported by experiments [27] for non-linear crystals of SiO_2, $LiIO_3$, ZnP_2, CdP_2, and TeO_2.

Dependence of optical activity in crystals on the temperature was examined in [28] based on the spiral model described above. It was found that the resonance frequency Ω_0 and the parameter A_0 in (6) decrease with increasing temperature. This feature was observed ear-lier in experiments on quartz and some other chiral crystals [29]. De-pendence of the chirality parameter on both the temperature and the frequency was also considered in [28]. Some other microscopic aspects of the optical activity theory were discussed in [30, 31].

5.   Spherical and Cylindrical Waves. Biisotropic Waveguides

In optics of chiral media, the theoretical studies were traditionally re-stricted to plane electromagnetic wave propagation in uniform media, reflection and transmission at interfaces. For plane waves, the optical activity can be considered as an effect of first-order spatial dispersion, i.e., it can be described in terms of the dielectric permittivity dyadic as a function of the propagation vector [16]. It is one of the advantages of the material equations (1), that they can be effectively used for any spatial variation of the fields, not just for plane waves.

In more recent papers, spherical and cylindrical waves in chiral media were studied. In [32, 33], spherical waves in source free isotropic chiral media were considered. In [32], spherical wave solutions the wave equation for the electric field vector

$$\left[(\nabla \cdot \nabla + k_+ k_-)\overline{\overline{I}} + (k_+ - k_-)\nabla \times \overline{\overline{I}} \right] \cdot \overline{E} = 0 \qquad (21)$$

were given in terms of the scalar potential functions X and Ψ:

$$\overline{E} = \nabla \times (\overline{r}X) + \nabla \times (\nabla \times (\overline{r}\Psi)) \qquad (22)$$

where $k_\pm = n_\pm \omega/c$ stand for the wave numbers of the circularly polarized plane eigenwaves, $n_\pm = \sqrt{\epsilon} \pm \alpha$ are the corresponding refractive indices, $\overline{r}$ is the vector from the origin to the field point. Particular solutions to (21) can be expressed in terms of spherical special functions. There are two types of spherical wave solutions corresponding to the two wave numbers $k_\pm$, which are both hybrid with non-zero longitudinal electric field components. In [32], resonant frequencies and eigenmodes of spherical resonators with ideally conducting walls and filled with isotropic chiral media were found.

The wave equation for the vector potential $\overline{A}$ in media modelled by the material Eqs. (1) was obtained in [33] in the form

$$\left[\left(\nabla \cdot \nabla - \frac{n_+ n_-}{c^2} \frac{\partial^2}{\partial t^2} \right) \overline{\overline{I}} + \frac{1}{c}(n_+ - n_-)\frac{\partial}{\partial t}\nabla \times \overline{\overline{I}} \right] \cdot \overline{A} = 0 \qquad (23)$$

where the vector potential $\overline{A}$ is subject to the calibration condition $\nabla \cdot \overline{A} = 0$.

The quantum theory for electromagnetic fields in chiral media was built based on eigensolutions to (23) in the form

$$\overline{A}_{KJM}(\overline{r}, t) = \sum_{L=J,J+1} a_{KLM}^L(t) j_L(kr) \overline{Y}_{JM}^L(\theta, \phi) \qquad (24)$$

where $j_L(kr)$ are the spherical Bessel functions, and $\overline{Y}_{JM}^L(\theta, \phi)$ are the spherical vector eigenfunctions. It was found that the electromagnetic fields in chiral media can not possess a definite parity. Novel selection rules for impurity atoms radiation were established. In [38], using the spherical wave decomposition, frequency dispersive chiral media were treated by the quantum electrodynamics formalism. It was shown that the field energy, pulse, momentum and spin can be represented in a form similar to that in quantum electrodynamics. Photon wave functions were introduced.

The constitutive equations (1) (and other equivalent relations, see Table 1) are adequate for small spatial dispersion only, since only first-order spatial derivatives of the fields are taken into account. For larger chirality, more complex material equations with higher-order derivatives should be used. Spherical waves in media modelled by the second-order material equations

$$\overline{D} = \epsilon \overline{E} + i\alpha \overline{H} + (a\nabla \cdot \nabla \overline{\overline{I}} + b\nabla\nabla) \cdot \overline{E}$$

$$\overline{B} = \mu \overline{H} - i\alpha \overline{E} \tag{25}$$

were studied in [34]. Here, a and b are two extra complex material parameters. The corresponding wave equation for the electric field reads

$$\left[(\nabla \cdot \nabla + k_+ k_-)\overline{\overline{I}} + \left(\frac{k_+ k_-}{k_0^2} - 1 \right) \nabla\nabla + (k_+ - k_-)\nabla \times \overline{\overline{I}} \right] \cdot \overline{E} = 0 \tag{26}$$

where

$$k_\pm = \frac{\omega c}{a\mu\omega^2 + c^2} \left(\sqrt{\mu \left[\epsilon + a(\epsilon\mu - \alpha^2)\omega^2/c^2 \right]} \pm \alpha \right)$$

$$k_0 = \sqrt{\frac{\epsilon\mu - \alpha^2}{\mu(a + b)}}$$

The solutions of (26) were expressed as series of transverse and longitudinal electromagnetic waves. Some special cases were considered and the wave numbers which correspond to quasi-longitudinal waves were found.

The spherical wave solutions enabled researchers to consider scattering of plane circularly polarized electromagnetic waves by spherical scatterers in chiral media [35–37]. In particular, such scatterers as ideally conducting spheres, isotropic chiral spheres, dual-layer particles with metal cores and chiral coatings, and dual-layer isotropic chiral spherical particles were considered. The exact solutions were expressed in terms of series of vector spherical eigenfunctions. Scattered fields in the far-field zone were examined in detail for limiting cases of particles that are small or large compared to the wavelength. Scattering cross-sections, extinction and absorption coefficients were calculated. The

results can be used for studying scattering of ensembles of spherical particles in uniform chiral media, and in some other problems.

Cylindrical wave solutions in chiral media were considered in [39], based on the second-order material equations (25) and the wave equation (26). The solutions of (26) can be represented through scalar potentials Ψ, X, and Ψ_0:

$$\overline{E} = \nabla \times (\overline{a}X) + \nabla \times [\nabla \times (\overline{a}\Psi)] + \nabla\Psi_0 \qquad (27)$$

where $\overline{a}$ is a constant vector. The complete set of cylindrical eigenfunctions was found in [39]. The solutions correspond to solenoidal cylindrical waves and to potential fields. In the far-field zone, the solutions reduce to two transverse cylindrical waves with conical phase fronts. In that zone, the fields are locally circularly polarized. Employing the general solution [39], circular waveguides and coaxial guides with chiral filling can be studied.

An alternative way in studying biisotropic waveguides is to start from the decomposition of the electric and magnetic fields into the wavefield components. This allows solutions for more general cross-section geometry than circular and coaxial structures considered in [39]. The wave field approach to biisotropic waveguides was developed in co-operation with researchers from Helsinki University of Technology [40]. Plane chiral waveguides with anisotropic boundary impedance conditions were considered in [41, 42] using the vector circuit theory [43, 44].

Approximate analysis of rectangular waveguides filled with non-reciprocal biisotropic materials was given in [45]. The case when the waveguide height is small compared to the wavelength was considered. Under that assumption, locally quasi-static approximation for the field distribution along the vertical axis of the cross-section was employed. The analysis resulted in simple analytical solutions for the propagation factors and the field patterns.

6.   Bianisotropic Media Electromagnetics

Spherical waves in chiral bianisotropic media with scalar dielectric permittivity and dyadic chirality parameter were studied in [46]. In practice, that situation can be realized, for example, in uniaxial bian-

isotropic media at frequencies where two eigenvalues of the dyadic permittivity coincide. The equation for the magnetic flux density $\overline{B}$

$$\left[(\nabla \cdot \nabla + k^2)\overline{\overline{I}} - \frac{k_0}{k^2}(\nabla \cdot \overline{\overline{\gamma}} \cdot \nabla)\nabla \times \overline{\overline{I}}\right] \cdot \overline{B} = 0 \qquad (28)$$

where $k_0 = \omega/c$ and $k = \omega\sqrt{\epsilon}/c$ are the wave numbers and $\overline{\overline{\gamma}} = \mathrm{Sp}\overline{\overline{\alpha}}\,\overline{\overline{I}} - \overline{\overline{\alpha}}^{\mathrm{T}}$, was studied in [46]. The phase velocities of quasi-spherical electromagnetic waves in the far-field zone were found. Some special examples of real crystals were analysed and local phase velocities were calculated as functions of crystal orientation.

Electromagnetic waves in bianisotropic media with uniaxial symmetry were studied in [47–49]. Appropriate constitutive equations can be written as [49]

$$\overline{D} = \overline{\overline{\epsilon}} \cdot \overline{E} + i\,(-\alpha\overline{\overline{I}}_t + K\overline{\overline{J}}) \cdot \overline{H}$$
$$\overline{B} = \overline{\overline{\mu}} \cdot \overline{H} + i\,(\alpha\overline{\overline{I}}_t + K\overline{\overline{J}}) \cdot \overline{E} \qquad (29)$$

The dielectric permittivity $\overline{\overline{\epsilon}}$ and the magnetic permeability $\overline{\overline{\mu}}$ are uniaxial dyadics

$$\overline{\overline{\epsilon}} = \epsilon_t\overline{\overline{I}}_t + \epsilon_n\overline{z}_0\overline{z}_0$$
$$\overline{\overline{\mu}} = \mu_t\overline{\overline{I}}_t + \mu_n\overline{z}_0\overline{z}_0 \qquad (30)$$

where $\overline{z}_0$ stands for the unit vector along the geometrical axis, $\overline{\overline{I}}_t = \overline{x}_0\overline{x}_0 + \overline{y}_0\overline{y}_0$ is the transverse unit dyadic and $\overline{\overline{J}} = \overline{z}_0 \times \overline{\overline{I}}_t = \overline{y}_0\overline{x}_0 - \overline{x}_0\overline{y}_0$ is the 90 degree rotator in the transverse $(x-y)$ plane.

α is the same chirality parameter as in (1), and additional coupling of orthogonal transverse electric and magnetic fields is measured by the parameter K. For microwave applications, composite materials modelled by (29) can be fabricated by embedding small Ω-shaped metal elements in isotropic dielectric, together with chiral elements.

The non-chiral special case (with $\alpha = 0$ in (29)) was studied in [47, 48]. The eigenwaves in unbounded uniaxial omega media are linearly polarized plane TE- and TM-waves. The corresponding wave impedances are non-symmetric, i.e., they are different for the waves travelling in the opposite directions of the axis z. The theory of plane wave reflection and transmission in plane uniaxial omega slabs [48]

demonstrated novel possibilities for anti-reflection coatings design. By properly choosing the value of the coupling parameter K, the reflection coefficient from lossy slabs can be made very small for arbitrary complex permittivity and permeability of the material. Polarization patterns and propagation factors of eigenwaves in chiral uniaxial media subject to (29) were studied in [49].

Bianisotropic omega materials of more general symmetry modelled by the constitutive relations

$$\overline{D} = \overline{\overline{\epsilon}} \cdot \overline{E} + i\overline{\overline{K}}_{\mathrm{em}} \cdot \overline{H}$$
$$\overline{B} = \overline{\overline{\mu}} \cdot \overline{H} - i\overline{\overline{K}}_{\mathrm{me}} \cdot \overline{E} \tag{31}$$

were the subject of the paper [50]. The dielectric permittivity $\overline{\overline{\epsilon}}$ and the magnetic permeability $\overline{\overline{\mu}}$ in (31) are diagonal dyadics

$$\overline{\overline{\epsilon}} = \epsilon_0(\epsilon_{xx}\overline{x}_0\overline{x}_0 + \epsilon_{yy}\overline{y}_0\overline{y}_0 + \epsilon_{zz}\overline{z}_0\overline{z}_0)$$
$$\overline{\overline{\mu}} = \mu_0(\mu_{xx}\overline{x}_0\overline{x}_0 + \mu_{yy}\overline{y}_0\overline{y}_0 + \mu_{zz}\overline{z}_0\overline{z}_0) \tag{32}$$

For example, if the equivalent electric dipoles of molecules (or stems of the omega-particles in composite materials) are all aligned with the z-axis and the magnetic dipoles are in the y-direction (the loops lie in the ($x - z$) plane), then

$$\overline{\overline{K}}_{\mathrm{em}} = K\overline{z}_0\overline{y}_0$$
$$\overline{\overline{K}}_{\mathrm{me}} = K\overline{y}_0\overline{z}_0 \tag{33}$$

Waves in microstrip waveguiding structures with bianisotropic layers modelled by (31) and ferrite layers were considered in [50]. The combination of omega layers with biased ferrites gives novel possibilities in scanning antenna design and in other microwave applications.

In practice, chiral bianisotropic media, especially for applications in optics, can be manufactured as multilayered structures formed by anisotropic layers (so called *superlattices*). In the paper [51], effective macroscopic parameters for superlattices were obtained for the case of periodical structures with the period D smaller than the wavelength. The lattice is formed by dual layers of uniaxial dielectrics with the thicknesses d and d' ($d + d' = D$). The components of the material parameter tensors satisfy

$$\frac{\alpha_{3n}^{\text{eff}}}{\epsilon_{33}^{\text{eff}}} = x\frac{\alpha_{3n}}{\epsilon_{33}} + (1-x)\frac{\alpha'_{3n}}{\epsilon'_{33}}$$

$$\alpha_{kn}^{\text{eff}} - \alpha_{3n}^{\text{eff}}\frac{\epsilon_{k3}^{\text{eff}}}{\epsilon_{33}^{\text{eff}}} = x\left(\alpha_{kn} - \alpha_{3n}\frac{\epsilon_{k3}}{\epsilon_{33}}\right) + (1-x)\left(\alpha'_{kn} - \alpha'_{3n}\frac{\epsilon'_{k3}}{\epsilon'_{33}}\right) \quad (34)$$

Here, the non-primed and primed terms correspond to the first and the second layer of the period, respectively. The indices take the values $n = 1, 2, 3$ and $k = 1, 2$. The index 3 refers to the direction orthogonal to the interfaces.

The components of the effective permittivity tensor can be obtained by employing the simple rule

$$A^{\text{eff}} = xA + (1-x)A' \quad (35)$$

to the quantities

$$\frac{1}{\epsilon_{33}}, \qquad \frac{\epsilon_{n3}}{\epsilon_{33}}, \qquad \epsilon_{nk} - \frac{\epsilon_{n3}\epsilon_{k3}}{\epsilon_{33}}$$

By analysing several special cases, possibilities of synthesizing superlattices with desired gyrotropic properties were determined in [51]. For example, it is possible to have materials with isotropic equivalent dielectric permittivity at a given frequency (where the two eigenvalues of the permittivity tensor coincide). Such artificial media can be utilized, for example, as elements of frequency filters for applications in optics. The analysis was extended to superlattices with magnetic stuctures in [52].

For more complex cases when the layers are not necessary thin, effective parameters can be determined by the operator formalism [53–56].

The analysis of chiral properties becomes much more involved for biaxial crystals. It was shown in [57] that in biaxial chiral crystals of the symmetry class 222 eigenwaves in the directions of the optical axes are elliptically polarized, and the ellipticity depends on the anisotropy of the dielectric permittivity and on the angle between the optical axes. Circular eigenwaves may exist only at frequencies where the crystal becomes uniaxial or biisotropic.

Polarizations of eigenwaves in chiral crystals of the classes 2, m, $2mm$ in the directions of the optical axes were studied in [58].

It was proved that if the optical axes of planal crystals lie in a plane orthogonal to the symmetry plane, they behave as non-chiral crystals for the waves along an optical axis at a frequency where the crystal becomes uniaxial.

Electromagnetic pulses and Gaussian beams propagating in chiral dispersive uniaxial and biaxial media were studied in [59]. The solutions satisfy a generalized parabolic equation, obtained in the paper. Possible focusing of beams propagating in specific directions was considered.

Various aspects of parameter measurements of bianisotropic media were considered in [61–65]. In [62], a method for measuring chirality in transparent crystals based on measurements of ellipticity of transmitted waves was suggested. The technique discussed in [66–67] can retrieve the material parameters from the absolute values of the transmission coefficients measured at different angles of rotation of the sample with respect to the source and the analyser. For lossy media, a measurement technique based on the effect of ultrasound excitation by laser pulses was considered in [72–76].

Chiral media with intrinsic magnetic structures are of special interest. In such materials, both the optical activity and the Faraday effect cause polarization rotation. Eigenwaves in these media (the name "chiroferrites" was adopted in the Western literature) were studied in [68–69] Some other specific wave solutions were found in [70]. Experimental studies of chiral magnetic crystals in external magnetic field were reported in [71].

7. Non-Linear Bianisotropic Media

Because of chirality, non-linear crystals can exhibit novel interesting properties. Such crystals combine practical advantages of liquid crystals possessing spiral structures with that of non-linear media. Due to chirality, frequency selective and polarization sensitive reflection together with polarization rotation can be achieved. Due to non-linearity, parametric interaction and frequency transformation effects become possible. In [77], material parameters of non-linear crystals pumped by two circularly polarized waves with opposite senses of rotation travelling in isotropic chiral non-linear media were studied. The two waves were supposed to have different (but close) frequencies Ω_1 and Ω_2 (the corresponding wave numbers were denoted as $k_1(\Omega_1)$ and $k_2(\Omega_2)$) and

to have equal amplitudes E_0.

Superposition of the two waves results in a rotating electric field with the cartesian components

$$
\begin{aligned}
E_x &= E_0 \cos \phi \\
E_y &= E_0 \sin \phi
\end{aligned}
\tag{36}
$$

where the angle ϕ depends on the co-ordinate along the propagation direction z and the time t as

$$
\phi(z,t) = \Delta k\, z - \Delta \Omega\, t
\tag{37}
$$

with

$$
\Delta k = (k_1 - k_2)/2 = (\alpha \overline{\Omega} + \Delta \Omega \sqrt{\epsilon_0})/c
$$
$$
\overline{\Omega} = (\Omega_1 + \Omega_2)/2 , \qquad \Delta \Omega = (\Omega_1 - \Omega_2)/2
$$

where ϵ_0 denotes the dielectric permittivity in the absense of the pumping field.

The second order non-linear effect of the wave (36) on the dielectric permittivity dyadic $\overline{\overline{\epsilon}}$ results in

$$
\overline{\overline{\epsilon}}(z,t) = \overline{\overline{U}}(z,t) \cdot \overline{\overline{\epsilon}} \cdot \overline{\overline{U}}^{-1}(z,t)
\tag{38}
$$

where $\overline{\overline{U}}(z,t)$ stands for the operator of rotation on the angle ϕ (37) around the propagation direction z. The permittivity can be expressed as $\overline{\overline{\epsilon}} = \overline{\overline{\epsilon}}_0 - 2\Delta\epsilon \overline{a}\overline{a}$, where $\Delta\epsilon = -2\theta E_0^2$, θ is the coefficient of the electrooptical interaction, and $\overline{a}$ is a unit vector orthogonal to the axis z.

Propagation of a weak electromagnetic wave in a crystal with the permittivity (38) is governed by the wave equation

$$
\left(\nabla \times (\nabla \times \overline{\overline{I}}) + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \overline{\overline{\epsilon}}(z,t) \right) \cdot \overline{E} = 0
\tag{39}
$$

Exact solutions of the last equation can be written as sums of two coupled circularly polarized waves, proportional to unit right and left hand circularly polarized vectors $\overline{e}_+$ and $\overline{e}_-$:

$$
\begin{aligned}
\overline{E} = A \Big\{ &\overline{e}_- \exp[i(k(\omega) + \Delta k)z - i(\omega + \Delta\Omega)t] \\
&+ \zeta(\omega)\overline{e}_+ \exp[i(k(\omega) - \Delta k)z - i(\omega - \Delta\Omega)t] \Big\}
\end{aligned}
\tag{40}
$$

where ζ is the ellipticity of the wave and A its amplitude. It seems of interest to note that this is a rare case where it is possible to find exact solutions of non-uniform and non-stationary Maxwell equations. This is achieved by transforming the equations into a moving and rotating co-ordinate system, where the permittivity (38) becomes uniform in space and constant in time. The frequency ω, the wave number $k(\omega)$ and the ellipticity $\zeta(\omega)$ were found in [77] as solutions to appropriate dispersion equation. It was proved that when the Bragg condition at a frequency ω_0 of the small amplitude wave

$$\omega_0 \approx \frac{\Delta k c}{\sqrt{\bar{\epsilon}}} + \Delta\Omega \tag{41}$$

($\bar{\epsilon} = \epsilon_0 - \Delta\epsilon$) is satisfied, the wave reflection is the most sensitive to the polarization of the incident field. The incident wave can exchange its energy with the pumping waves. There exists an effect of parametric amplification of the transmitted or the scattered waves (depending on the propagation direction).

In the paper [78] it was established that by properly choosing the pumping frequencies Ω_1 and Ω_2 it is possible to have equal wave numbers of the pumping waves $k_1(\Omega_1)$ and $k_2(\Omega_2)$. In that case,

$$(\Omega_2 - \Omega_1)\sqrt{\epsilon_0} = \alpha(\Omega_1 + \Omega_2) \tag{42}$$

This means that the influence of the crystal chirality on the wave numbers of two interacting waves can be compensated by that of non-linearity. In this special case the resulting electric field of the pumping waves is rotating in time but is uniform in space.

Electromagnetic wave propagation in crystals with rotating but uniform structure was studied in [78] in detail. Effects of wavefront reversal and amplification of circularly polarized electromagnetic waves were demonstrated in microwave regime and for infrared light waves. Crystals with rotating structures demonstrate optical (or microwave) activity, and the polarization rotation angle at a wavelength distance reads

$$\vartheta = \frac{\omega_0^4 \Delta\epsilon^2}{8\bar{\epsilon}^{3/2} \Delta\Omega(\omega_0^2 - \Delta\Omega^2)} \tag{43}$$

As is seen, after a formal redefinition $\Delta\Omega\sqrt{\bar{\epsilon}}/c \to q$, (43) coincides with the corresponding formula for the cholesteric liquid crystals, with $q = 2\pi/P$, where P is the pitch of the cholesteric spiral.

Electromagnetic wave absorption in crystals with induced spiral rotating structures was considered in [79–80]. It was shown that in such crystals an effect similar to that of the Borrmann effect for X-rays and to the effect of absorption suppression in cholesteric liquid crystals can be observed. The maximum of the effect is at a frequency where the field is polarized orthogonally to the absorbing oscillators.

Similar non-linear effects are possible in anisotropic crystals of special symmetry classes, determined in [81].

Since recently, spectroscopy of non-linear chiral media has been developing very actively and quickly. A novel method of spectroscopy which allows to separate influences of different physical mechanisms for non-linear and chiral polarization rotation was advanced in [82]. Analytical expressions for non-linear rotation and distortion of the polarization ellipse after electromagnetic wave reflection from non-linear chiral crystals in the presence of a light wave of a different frequency were given in [83]. It was shown that by using two testing waves of different elliptical polarizations it is possible to extract more information about crystals compared to the conventional testing by a single linearly polarized electromagnetic wave.

In [84–85], different possible non-linear electron processes which lead to induced non-linear optical activity were considered. Quasi-linear plane waves in chiral non-linear media were studied in [86]. It was demonstrated that there may exist right and left hand circularly polarized waves which propagate as in linear chiral media, generating no harmonics. Their velocities depend on the corresponding wave amplitudes. However, linear combinations of these eigenwaves do not satisfy the non-linear field equations. For example, linearly polarized waves always generate harmonics.

8.   Diffraction of Light by Ultrasonic Waves in Chiral Media

Chirality of media affects considerably the processes of light diffraction by acoustic waves. The theory of acousto-optical interaction can be based on the material equations (1) with appropriate boundary conditions.

In [87] it was found that in chiral media there may exist low frequency collinear interaction which can give a novel way to built

high-resolution optical filters. Based on the theory of coupled modes, collinear diffraction in chiral absorbing media was considered. When the wave vectors of the incident and the diffracted waves point into the same direction, the Bragg condition reads

$$f = 2\frac{v}{\lambda_0}\mathrm{Re}\alpha \qquad (44)$$

where α is the chirality parameter in (1), f is the ultrasonic frequency, v is the ultrasonic phase velocity, and λ_0 is the light wavelength in vacuum. For the back scattering,

$$f = 2\frac{v}{\lambda_0}n \qquad (45)$$

where $n = \sqrt{\epsilon}$ is the refractive index in the corresponding non-chiral medium. In the case (44) only waves of different polarizations interact, whereas in the backscattering (45) the polarization pattern does not changes after scattering. In [87], the acousto-optical analog of the Borrmann effect in absorbing chiral crystals was considered. The polarization patterns of scattered waves were determined for linearly polarized incident fields.

The paper [88] deals with non-linear diffraction of high-power light waves by ultrasound in non-linear chiral media. Non-linearity of chiral materials considerably influences the diffraction processes. In particular, it was found that, for small Bragg angles, the interaction between the circularly polarized waves with the opposite senses of rotation is less sensitive to non-linearity than the isotropic diffraction, where two waves of the same polarization interact.

The coupled modes theory was applied in [89] to non-collinear Bragg diffraction in isotropic chiral media and in chiral cubic crystals. There exist four different types of interaction of circularly polarized waves and four corresponding Bragg angles. For the isotropic diffraction, the Bragg angle is

$$\phi_{\mathrm{B}} = \arcsin\frac{1}{2n}\frac{\lambda_0}{\Lambda} \qquad (46)$$

where λ_0 and Λ stand for the wavelengths of the light and the acoustic waves, respectively, and $n = \sqrt{\epsilon}$. For two interacting waves of different polarizations (anisotropic diffraction), there exist two Bragg angles, which differ from ϕ_{B} in (46) by

$$\Delta\phi = \pm\frac{2\text{Re}\alpha}{n\sin 2\phi_{\text{B}}} \tag{47}$$

As a results of diffraction of a linearly polarized wave coming at the Bragg angle (46), a complex structure of the diffracted field with three maxima can be observed for large thicknesses of the interaction area $l \geq \dfrac{\lambda_0}{\text{Re}\alpha}$. Similar processes were considered in the book [90]. In [89], the system of four differential equations for the amplitudes of interacting waves was solved for all four types of possible interactions. It was found that the diffracted field is always elliptically polarized, and the ellipticity depends on the anisotropy of the acousto-optical interaction and on the ultrasound power.

Raman-Nath scattering in chiral media is also of interest. The equation for the electric field of an electromagnetic wave interacting with ultrasound was written in [91] in the form

$$\left(\nabla\cdot\nabla\overline{\overline{I}} - 2\alpha\nabla\times\overline{\overline{I}}\frac{\partial}{\partial t} - \frac{\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)\cdot\overline{E} = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\overline{P} \tag{48}$$

where $\overline{P}$ is the dielectric polarization vector. Direction of the main axis and the ellipticity of the diffracted field were found for lossy chiral media. In [92], anisotropy of chiral crystals was taken into account and a novel method for measuring acousto-optical parameters of cubic chiral crystals was suggested. Electro-optical and acousto-optical interactions in uniaxial chiral media were considered in [93–94].

In the recent years, there is growing interest to acousto-optical interaction in media in external biasing electric and magnetic fields. This offers novel possibilities in improving characteristics of some optical devices and to create novel devices. Chiral media are of special interest, thanks to their interesting acousto-optical properties. In [95–96], based on the coupled mode theory, some cases of acousto-optical interactions in chiral media for special orientations of the external fields were considered. The results show that novel devices combining the functions of an acousto-optical modulator and a polarization switch can be built based on acousto-optical interaction in chiral media.

Papers [97–98] are devoted to material parameters measurement techniques for cubic chiral crystals. There exists a possibility to retrieve the complete set of parameters characterizing acousto-optical interactions from results of four polarization pattern measurements of

diffracted waves. Experiments made in the optical region for crystals $Bi_{12}GeO_{20}$ and $Bi_{12}SiO_{20}$ demonstrated effectiveness of the method.

Acousto-optical interactions in bianisotropic crystals of TeO_2 were studied in [99–101]. A possibility of polarization-independent acousto-optical modulation was theoretically established and con–firmed by an experiment. Some other aspects of the problem, including higher-order Bragg diffraction, were covered by the papers [102–107].

9. Holography in Chiral Crystals

Some chiral crystals are promising for use in systems of optical signal processing and information recording. A theoretical study of volume Denisyuk holography in chiral crystals was conducted in [108]. It was found that chirality of crystals affects significantly the mutual transformation of electromagnetic waves. The conditions which maximize the effect of power exchange were found. In the process of reading the information, chirality effects can be compensated by the Faraday effect in an external magnetic field. In [109], formation of holograms in optically isotropic chiral layers at oblique incidence of elliptically polarized time-harmonic waves was studied.

In many experiments, chirality is essential for both the processes of recording and reading of information. Coupled equations for wave interaction in isotropic chiral media in the Bragg regime were derived in [110]. The equations were solved for one of the simplest examples of possible geometrical configurations.

In [111–112], a phenomenological model of light diffraction by holographic grids, which takes into account piezoelectric, acousto-electric and chiral properties of crystals was advanced. The model is in good agreement with experiments and gives a possibility to determine optimum crystal orientations which maximize the diffraction effects. Diffraction of light by holographic grids in cubic chiral photo-refractive crystals was studied in [113–114]. The optimum orientations of the crystals with respect to the wave vectors directions were determined. In [115–116], diffraction of two coherent light waves by phase holographic grids formed in cubic photo-refractive crystals was considered. The conditions for maximum energy exchange depend on the wave polarizations and on the orientation of the crystal.

10. Conclusion

In the paper, studies covering various issues from basic electromagnetic properties of bianisotropic materials and appropriate material equations to the most recent research on applications of complex materials in microwave engineering and in optical signal processing, were briefly reviewed. With such a broad scope of review, it was not possible to give more details of the results and to credit other researchers. Because of scarcity of communication, there are many parallel independent studies, conducted by Western and Eastern scientists. For instance, in [117] and in [118], comparable studies on WKB approximation for waves in non-uniform chiral media were published. In both papers, references to some earlier studies on the topic can be found. The results of the first paper were used to improve accuracy of some optical devices by taking into account non-uniform structure of optically active crystals, whereas the results of the second paper are supposed to have applications in microwave engineering. Here, the exchange of ideas can be most stimulating.

The selection of the material definitely reflects the scientific interests of the authors. We tried to present the material so that it could be helpful for specialists in both optics and microwave engineering. The list of references could not be made complete, and it covers only most interesting and recent references to Byelorussian and Russian publications.

References

1. Fedorov, F. I., *Optics of Anisotropic Media*, Byelorussian Academy of Sciences, Minsk, 1958.
2. Fedorov, F. I., *Theory of Elastic Waves in Crystals*, Nauka, Moscow, 1965. The English translation: Plenum Press, New York, 1968.
3. Fedorov, F. I., and V. V. Filippov, *Reflection and Refraction of Light by Transparent Crystals*, Nauka i Tekhnika, Minsk, 1976.
4. Fedorov, F. I., *Theory of gyrotropy*, Nauka i Tekhnika, Minsk, 1976.
5. Fedorov, F. I., *Lorentz Group*, Nauka, Moscow, 1979.

6. Bokut, B. V., and A. N. Serdyukov, "On the phenomenological theory of natural optical activity", *Zhurnal Experimentalnoi i Teoreticheskoi Fiziki*, Vol. 61, No. 5, 1808–1813, 1971.

7. Bokut, B. V., A. N. Serdyukov and F. I. Fedorov, "On the form of constitutive equations in optically active crystals", *Optika i Spektroskopiya*, Vol. 37, No. 2, 288–293, 1974.

8. Bokut, B. V., A. N. Serdyukov and V. V. Shepelevich, "On the phenomenological theory of absorbing optically active media", *Optika i Spektroskopiya*, Vol. 37, No. 1, 120–124, 1974.

9. Bokut, B. V., A. N. Serduykov, F. I. Fedorov and N. A. Hilo, "On the boundary conditions in electrodynamics of optically active media", *Kristallografiya*, Vol. 18, No. 2, 227–233, 1973.

10. Bokut, B. V., and A. N. Serdyukov, "Frequency transformation of light waves in optically active media", *Zhurnal Prikladnoi Spektroskopii*, Vol. 12, No. 1, 65–71, 1970.

11. Bokut, B. V., and A. N. Serdyukov, "On conservation of momentum of electromagnetic field in optically active media", *Zhurnal Prikladnoi Spektroskopii*, Vol. 12, No. 1, 139–141, 1970.

12. Bokut, B. V., A. N. Serdyukov, and F. I. Fedorov, "Comment on the paper by V.M. Agranovich and V. L. Ginzburg "On the phenomenological electrodynamics of gyrotropic media", *Zhurnal Prikladnoi Spektroskopii*, Vol. 19, No. 2, 377–380, 1973.

13. Bokut, B. V., A. N. Serdyukov, "On the theory of optical activity of non-uniform media", *Zhurnal Prikladnoi Spektroskopii* , Vol. 20, No. 4, 677–682, 1974.

14. Bokut, B. V., A. N. Serdyukov, and F. I. Fedorov, "On the phenomenological theory of optically active crystals", *Kristallografiya*, Vol. 15, No. 5, 1002–1006, 1970.

15. Agranovich, V. M., and V. L. Ginzburg, *Kristallooptics with spatial dispersion and the theory of exitons*, Nauka, Moscow, 1965.

16. Landau, L. D., and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon Press, Oxford, Second Edition, 1984.

17. Chandrasekhar, P., and T. P. Srinivasan, "On optical properties of crystals", *J. Phys. C: Solid State Phys.*, Vol. 6, No. 4, 1085–1090, 1973.

18. Serdyukov, A. N., *Wave phenomena in gyrotropic media*, Doctoral Thesis, Minsk, 1985.

19. Gvozdev, V. V., and A. N. Serdyukov, "The Green function and fields radiated by moving charges in gyrotropic medium", *Optika i Spektroskopiya*, Vol. 47, No. 3, 544–548, 1979.

20. Gvozdev, V. V., and A. N. Serdyukov, "Electromagnetic waves in dispersive gyrotropic media", *Optika i Spektroskopiya*, Vol. 50, No. 2, 349–353, 1981.

21. Condon, E. U., "Theories of optical rotatory power", *Reviews of Modern Physics*, Vol. 9, 432–457, 1937.

22. Gvozdev, V. V., V. A. Penyaz, and A. N. Serdyukov, "On the scattering of electromagnetic waves in gyrotropic crystals", *Optika i Spektroskopiya*, Vol. 49, No. 6, 1168–1171, 1980.

23. Bokut, B. V., V. V. Gvozdev, and A. N. Serdyukov, "Specific waves in naturally gyrotropic media", *Zhurnal Prikladnoi Spektroskopii* , Vol. 34, No. 4, 701–706, 1981.

24. Bokut, B. V., V. A. Penyaz, and A. N. Serdyukov, "Dispersion summation rules in the optics of naturally gyrotropic media", *Optika i Spektroskopiya*, Vol. 50, No. 5, 929–933, 1981.

25. Semchenko, I. V., "Dispersion summation rules in the optics of naturally gyrotropic media: Spiral model of the molecules", *Doklady Akademii Nauk BSSR*, 1989, Vol. 33, No. 2, 114–116.

26. Semchenko, I. V., "On the microtheory of non-linear optical activity of quartz crystals: Spiral model of the molecules", *Zhurnal Prikladnoi Spektroskopii* , Vol. 46, No. 5, 855–858, 1987.

27. Vorivonchik, N. N., S. G. Garasevich, Yu. A. Myagchenko, S. I. Osipov, and A. B. Slobodyanyuk, "Non-linear optical activity of paratellurit crystals", *Ukrainskii Fizicheskii Zhurnal*, Vol. 32, No. 3, 399–401, 1987.

28. Semchenko, I. V., "Mechanisms of termo-induced non-linear optical activity in naturally gyrotropic crystals", *Doklady Akademii Nauk BSSR*, Vol. 33, No. 10, 888–890, 1989.

29. Baturina, O. A., B. Brszezina, M. Gavrankova, and Z. B. Perekalina, "Temperature dependence of optical rotation in the cubic phase of langbeinit crystals," *Kristallografiya*, Vol. 34, No. 2, 389–392, 1989.

30. Suslikov, L. M., Z. P. Gad'mashi, and Yu. A. Hazitarkhanov, "Gyrotropy of $AgGaS_2$ crystals", *Optika i Spektroskopiya*, Vol. 62, No. 4, 823–827, 1987.

31. Pashkevich, Yu. G., and S. A. Fedorov, "Microscopic theory of natural optical activity of molecular crystals", *Optika i Spektro-skopiya*, Vol. 71, No. 6, 979–984, 1991.

32. Godlevskaya, A. N., V. A. Karpenko, and A. N. Serdyukov, "Spherical electromagnetic waves in naturally gyrotropic crystals", *Optika i Spektroskopiya*, Vol. 59, No. 6, 1262–1265, 1985.

33. Afonin, A. A., A. N. Godlevskaya, V. N. Kapshai, and A. N. Serdyukov, "Spherical electromagnetic waves and quantum theory of electromagnetic fields in naturally gyrotropic media", *Zhurnal Prikladnoi Spektroskopii* , Vol. 45, No. 2, 307–312, 1986.

34. Godlevskaya, A. N., "Some peculiarities of spherical electromagnetic wave propagation in naturally gyrotropic media", *Doklady Akademii Nauk BSSR*, Vol. 31, No. 7, 616–618, 1987.

35. Godlevskaya, A. N., and V. N. Kapshai, "Electromagnetic waves scattering by a metal sphere in naturally gyrotropic medium", *Doklady Akademii Nauk BSSR*, Vol. 33, No. 4, 332–335, 1989.

36. Godlevskaya, A. N., and V. N. Kapshai, "Electromagnetic waves scattering by spherical particles in naturally gyrotropic media", *Optika i Spektroskopiya*, Vol. 68, No. 1, 122–127, 1990.

37. Afonin, A. A., A. N. Godlevkaya, V. N. Kapshai, S. P. Kurlovich, and A. N. Serdyukov, "Electromagnetic wave scattering by a dual-layer spherical particle in naturally gyrotropic medium", *Optika i Spektrosopiya*, Vol. 69, No. 2, 406–411, 1990.

38. Godlevskaya, A. N., and V. N. Kapshai, "Photon wave function in naturally gyrotropic media", *Optika i Spektroskopiya*, Vol. 66, No. 4, 830–834, 1989.

39. Kondratova, N. L., and A. N. Serdyukov, "Cylindrical electromagnetic waves in media with spatial dispersion", *Zhurnal Prikladnoi Spektroskopii*, Vol. 47, No. 2, 326–329, 1987.

40. Koivisto, P. K., S. A. Tretyakov, and M. I. Oksanen, "Waveguides filled with general biisotropic media," *Radio Science*, Vol. 28, No. 5, 675–688, 1993.

41. Oksanen, M. I., P. K. Koivisto, and S. A. Tretyakov, "Plane chiral waveguides with boundary impedance conditions,", *Microwave and Optic. Technol. Lett.*, Vol. 5, No. 2, 68–72, 1992.

42. Oksanen, M. I., P. K. Koivisto, and S. A. Tretyakov, "Vector circuit method applied to chiral slab waveguides," *J. Lightwave Technol.*, Vol. 10, No. 2, 150–155, 1992.

43. Oksanen, M. I., S. A. Tretyakov, and I. V. Lindell, "Vector circuit theory for isotropic and chiral slabs," *J. Electromagn. Waves Applic.*, Vol. 4, No. 7, 613–643, 1990.

44. Tretyakov, S. A., and M. I. Oksanen, "Electromagnetic waves in layered general biisotropic structures," *J. Electromagn. Waves Applic.*, Vol. 6, No. 10, 1393–1411, 1992.

45. Tretyakov, S. A., "Electromagnetic waves in rectangular wavegiudes of small height filled with biisotropic (non-reciprocal chiral) media," *Radiotekhnika i Elektronika*, Vol. 36, No. 11, 2090–2095, 1991.

46. Godlevskaya, A. N., and A. N. Serdyukov, "Quasi-spherical electromagnetic waves in gyrotropic crystals with scalar dielectric permittivity,"*Optika i Spektroskopiya*, Vol. 69, No. 4, 851–855, 1990.

47. Tretyakov, S. A., and A. A. Sochava, "Proposed composite material for non-reflecting shields and antenna radomes", *Electron. Lett.*, Vol. 29, No. 12, 1048–1049, 1993.

48. Tretyakov, S. A., and A. A. Sochava, "Novel uniaxial bianisotropic materials: reflection and transmission in planar structure," *Pro–gress in Electromagenetic Research*, PIER 9, A. Priov [Ed.], 157–179, EMW Publishing, Cambridge, MA, 1994.

49. Tretyakov, S. A., and A. A. Sochava, "Eigenwaves in uniaxial chiral omega media", *Microw. and Optic. Technol. Lett.*, Vol. 6, No. 12, 701–705, 1993.

50. Tretyakov, S. A., "Thin pseudochiral layers: Approximate boundary conditions and potential applications", *Microw. and Optic. Technol. Lett.*, Vol. 6, No. 2, 112–115, 1993.

51. Semchenko, I. V., "Gyrotropic properties of superlattices in the long-wave approximation," *Kristallografiya*, Vol. 35, No. 5, 1047–1050, 1990.

52. Gaishun, V. E., I. V. Semchenko, and A. N. Serdyukov, "Gyrotropic properties of superlattices with magnetic structure in the long-wave approximation", *Kristallografiya*, Vol. 38, No. 3, 24–27, 1993.

53. Barkovskii, L. M., G. N. Borzdov, and F. I. Fedorov, "Evolution operators in the electrodynamics of spatially dispersive media," *J. Mod. Opt.*, Vol. 37, No. 2, 85–97, 1990.

54. Barkovskii, L. M., G. N. Borzdov, and A. B. Lavrinenko, "Fresnel reflection and transmission operators for plane layered bianisotropic media," *Izvestiya Akademii Nauk BSSR, Seriya Fiziko-Matematicheskikh Nauk*, No. 2, 79–85, 1986.

55. Barkovskii, L. M., G. N. Borzdov, and A. B. Lavrinenko, "Fresnel's reflection and transmission operators for stratified gyroanisotropic media", *J. of Physics A: Math. and Gen.*, Vol. 20, No. 5, 1095–1106, 1987.

56. Barkovskii, L. M., and Fo Tkhi Nguen Hang, "On liaisons between energy flows and tensor impedances in anisotropic and gyrotropic media", *Kristallografiya*, Vol. 36, No. 6, 1338–1341, 1991.

57. Bokut, B. V., "On the polarization plane rotation in gyrotropic crystals", *Doklady Akademii Nauk BSSR*, Vol. 34, No. 9, 790–793, 1990.

58. Bokut, B. V., "Polarization of electromagnetic waves propagating along optical axes in biaxial gyrotropic crystals", *Doklady Akademii Nauk BSSR*, Vol. 34, No. 11, 986–989, 1990.

59. Hatkevich, A. G., "Propagation of pulses and beams in dispersive gyrotropic crystals", *Zhurnal Prikladnoi Spektroskopii*, Vol. 46, No. 2, 285–291, 1987.

60. Hatkevich, A. G., and S. N. Kurilkina, "Conical refraction in gyrotropic crystals", *Zhurnal Prikladnoi Spektroskopii*, Vol. 51, No. 6, 1005–1009, 1989.

61. Hatkevich, A. G., "Polarization and propagation velocities of waves in absorbing crystals", *Zhurnal Prikladnoi Spektroskopii*, Vol. 43, No. 5, 787–793, 1985.

62. Okorochkov, A. I., A. F. Konstantinova, Z. B. Perekalina, B. N. Grechushnikov, L. V. Soboleva, and L. I. Hapaeva, "Peculiarities in measurements of optical activity in transparent biaxial crystals near an optical axis", *Kristallografiya*, Vol. 28, No. 4, 736–740, 1983.

63. Okorochkov, A. I., A. F. Konstantinova, L. V. Soboleva, and L. I. Hapaeva, "Study of optical activity in biaxial absorbing crystal of $Er(HCOO)_3 \cdot 2H_2O$", *Kristallografiya*, Vol. 29, No. 6, 1102–1108, 1984.

64. Okorochkov, A. I., and A. F. Konstantinova, "Eigenwaves in gyrotropic absorbing rombic crystals", *Kristallografiya*, Vol. 29, No. 5, 841–848, 1984.

65. Okorochkov, A. I., and A. F. Konstantinova, "Influence of non-orthogonality of eigenwaves in a crystal on polarization of transmitted light", *Kristallografiya*, Vol. 30, No. 1, 105–113, 1985.

66. Konstantinova, A. F., A. N. Stepanov, B. N. Grechushnikov, and I. T. Uluchanov, "Study of orientation dependence of transmission through a system "polarizer-crystal-analizer" ", *Kristallografiya*, Vol. 35, No. 2, 429–432, 1990.

67. Konstantinova, A. F., I. T. Uluchanov, and B. N. Grechushnikov, "A method for determination of optical parameters of gyrotropic crystals", *Kristallografiya*, Vol. 36, No. 3, 686–692, 1991.

68. Bokut, B. V., and S. S. Girgel, "Electromagnetic waves in magnetic crystals with optical activity", *Optika i Spektroskopiya*, Vol. 49, No. 4, 738–741, 1980.

69. Girgel, S. S., "Refractive indices and refraction vectors of eigenwaves in optically active magnetic crystals", *Optika i Spektroskopiya*, Vol. 60, No. 4, 777–780, 1986.

70. Bokut, B. V., and S. S. Girgel, "On light propagation along singular elliptic directions in absorbing gyrotropic crystals", *Zhurnal Prikladnoi Spektroskopii*, Vol. 36, No. 4, 666–669, 1982.

71. Gelikonov, G. N., and M. A. Novikov, "On magneto-gyration effect", *Optika i Spektroskopiya*, Vol. 74, No. 6, 1123–1125, 1993.

72. Mityurich, G. S., "Photo-acoustic effects in optically active media", *Doklady Akademii Nauk BSSR*, Vol. 26, No. 5, 414–417, 1982.

73. Mityurich, G. S., and V. B. Zelenyi, "Photo-acoustic transformation in gyrotropic piezoelectric crystals", *Pis'ma v Zhurnal Teknicheskoi Fiziki*, Vol. 14, No. 20, 1879–1883, 1988.

74. Mityurich, G. S., and V. B. Zelenyi, "Termo-optical generation of sound in non-linear piezoelectric crystals", *Kristallografiya*, Vol. 36, No. 5, 1250–1253, 1991.

75. Zelenyi, V. B., and G. S. Mityurich, "Combined effect of photo-acoustic signal registration in isotropic media", *Pis'ma v Zhurnal Teknicheskoi Fiziki*, Vol. 16, No. 9, 44–49, 1990.

76. Mityurich, G.'S., "Photo-acoustic method for measurements of absorption and dichroism in crystals of medium singonies", *Kristallografiya*, Vol. 36, No. 1, 212–213, 1991.

77. Akhramenko, I. N., I. V. Semchenko, and A. N. Serdyukov, "Selective interaction of electromagnetic waves in crystals with spiral-modulated non-stationary dielectric permittivity," *Optika i Spektroskopiya*, Vol. 66, No. 3, 618–622, 1989.

78. Akhramenko, I. N., I. V. Semchenko, and A. N. Serdyukov, "Selective amplification of electromagnetic waves in media with rotating uniaxial structure," *Zhurnal Prikladnoi Spektroskopii*, Vol. 49, No. 2, 304–308, 1988.

79. Akhramenko, I. N., I. V. Semchenko, and A. N. Serdyukov, "Peculiarities of electromagnetic waves propagation in absorbing naturally gyrotropic crystal with induced uniaxial non-stationary anisotropy," *Kristallografiya*, Vol. 34, No. 6, 1349–1353, 1989.

80. Akhramenko, I. N., I. V. Semchenko, and A. N. Serdyukov, "On a possibility of polarization-dependent bleaching at parametric interaction of electromagnetic waves in gyrotropic crystals," *Optika i Spektroskopiya*, Vol. 68, No. 3, 597–601, 1990.

81. Akhramenko, I. N., and I. V. Semchenko, "On a possibility of formation of a non-uniform spiral non-stationary anisotropy of dielectric permittivity in non-linear crystals of different symmetry classes," *Kristallografiya*, Vol. 36, No. 5, 1284–1286, 1991.

82. Golubkov, A. A., and V. A. Makarov, "Polarization spectroscopy of non-linear rotation and deformation of polarization ellips of light transmitted throught non-linear gyrotropic crystals," *Vestnik Moskovskogo Universiteta*, Series 3, Vol. 30, No. 2, 54–60, 1989.

83. Golubkov, A. A., and V. A. Makarov, "Dual-wave spectroscopy of non-linear rotation and polarization ellips of light reflected from non-linear gyrotropic crystals," *Optika i Spektroskopiya*, Vol. 69, No. 3, 622–626, 1990.

84. Gheludyev, N. I., and A. D. Petrenko, "Physical mechanisms of non-linear optical activity in crystals," *Kristallografiya*, Vol. 29, No. 6, 1045–1053, 1984.

85. Gheludyev, N. I., A. D. Petrenko, Yu. P. Svirko, and G. S. Filippova, "Non-linear optical activity in weakly and strongly non-linear media: direct and cascade processes, bistability and chaos," *Izvestiya Akademii Nauk SSSR*, Physical Series, Vol. 48, No. 3, 603–610, 1984.

86. Terlezkii, A. Ya., "Quasi-linear plane waves in optically active non-linear media," *Izvestiya Vys'shikh Uchebnykh Zavedenii, Fizika*, Vol. 29, No. 12, 94–96, 1986.

87. Belyi, V. N., and G. V. Kulak, "Quasi-linear acousto-optical diffraction in gyrotropic absorbing crystals," *Zhurnal Prikladnoi Spektroskopii* , Vol. 40, No. 5, 821–824, 1984.

88. Kulak, G. V., "Diffraction of light by ultrasound in gyrotropic and non-linear media," *Izvestiya Akademii Nauk BSSR, Fiziko-Matematicheskaya Seriya*, No. 1, 63–65, 1985.

89. Kulak, G. V., "Diffraction of light by ultrasound in gyrotropic media," *Zhurnal Prikladnoi Spektroskopii* , Vol. 43, No. 2, 299–302, 1985.

90. Petrov, M. P., S. I. Stepanov, and A. B. Homenko, *Photosensitive Electro- Optical Media in Holography and Optical Information Processing*, Nauka, Leningrad, 287 , 1983.

91. Belyi, V. N., and V. V. Shepelevich, "Diffraction of light by ultrasound in absorbing gyrotropic media," *Optika i Spektroskopiya*, Vol. 52, No. 5, 842–846, 1982.

92. Belyi, V. N., G. V. Kulak, and V. V. Shepelevich, "Raman-Nath diffraction of light by ultrasound in gyrotropic cubic crystals," *Optika i Spektroskopiya*, Vol. 65, No. 3, 636–640, 1988.

93. Kurilkina, S. N., "Characteristic features of electrooptic interaction in uniaxial gyrotropic crystals", *Optika i Spektroskopiya*, Vol. 74, No. 2, 310–314, 1993.

94. Kotov, V. M., "On diffraction of light by ultrasound in uniaxial gyrotropic crystals", *Zhurnal Tekhnicheskoi Fiziki*, Vol. 63, No. 2, 167–170, 1993.

95. Belyi, V. N., P. I. Ropot, and V. V. Shepelevich, "Influence of electro-induced optical anisotropy on acousto-optical Bragg interaction in cubic gyrotropic crystals", *Optika i Spektroskopiya*, Vol. 71, No. 1, 70–77, 1991.

96. Belyi, V. N., and S. N. Kurilkina, "Characteristic features of acoustooptic interaction in uniaxial gyrotropic crystals in external electric field", *Optika i Spektroskopiya*, Vol. 74, No. 4, 7726–731, 1993.

97. Ropot, P. I., "Determination of photo-elastic parameters of optically active cubic crystals by a polarization method", *Optika i Spektroskopiya*, Vol. 70, No. 2, 371–376, 1991.

98. Ropot, P. I., "Elasto-optical parameters of silico-sillenit", *Pis'ma v Zhurnal Technicheskoi Fisiki*, Vol. 17, No. 8, 81–84, 1991.

99. Kulak, G. V., P. I. Ropot, and G. A. Pashkevich, "Polarization-independent light modulation in paratellurit", *Doklady Aka–demii Nauk BSSR*, Vol. 35, No. 5, 429–433, 1991.

100. Kotov, V. M., "Bragg polarization splitting", *Pis'ma v Zhurnal Technicheskoi Fisiki*, Vol. 17, No. 9, 12–15, 1991.

101. Karplyuk, K. S., and E. G. Levchenko, "On the influence of gyrotropy on acousto-optical parameters of paratellurit", *Optika i Spektroskopiya*, Vol. 68, No. 6, 1340–1343, 1990.

102. Zadorin, A. S., and S. N. Sharangovich, "Peculiarities of light modulation by sound in optically active media", *Avtometriya*, No. 5, 10–17, 1989.

103. Zadorin, A. S., and S. N. Sharangovich, "Acousto-optical interaction of wave beams in birefringent crystals", *Izvestiya Vys'shikh Uchebnykh Zavedenii, Radiofizika*, Vol. 29, No. 10, 1228–1237, 1986.

104. Zadorin, A. S., and S. N. Sharangovich, "Transformation of correlation and polarization parameters of light beams at acousto-optical interactions in gyrotropic media", *Optika i Spektroskopiya*, Vol. 70, No. 1, 111–117, 1991.

105. Zilberman, G. E., L. F. Kupchenko, A. A. Bykov, G. F. Goltvyanskaya, and V. Yu. Vdovenkov, "New Bragg angles in optically active crystals", *Radiotekhnika i Elektronika*, Vol. 32, No. 9, 1993–1994, 1987.

106. Zilberman, G. E., L. F. Kupchenko, and G. F. Goltvyanskaya, "Diffraction of light by ultrasound in uniaxial optacally active crystals", *Radiotekhnika i Elektronika*, Vol. 29, No. 12, 2449–2454, 1984.

107. Zilberman, G. E., L. F. Kupchenko, Yu. V. Astashev, A. M. Reznichenko, and I. N. Goltvyanskii, "Theory of the second order Bragg diffraction of light by ultrasound in optically active crystals", *Radiotekhnika i Elektronika*, Vol. 36, No. 8, 1582–1585, 1991.

108. Shepelevich, V. V., "On mutual transformation of electromagnetic waves at recording in volume hologramms with optically active media", *Zhurnal Tekhnicheskoi Fiziki*, Vol. 54, No. 11, 2177–2183, 1984.

109. Shepelevich, V. V., "On the process of holographic grids formation in plane gyrotropic layers", *Optika i Spektroskopiya*, Vol. 54, No. 6, 1064–1071, 1983.

110. Shepelevich, V. V., "Coupled waves equation for optically active media", *Optika i Spektroskopiya*, Vol. 58, No. 6, 1366–1368, 1985.

111. Mandel, A. E., S. M. Shandarov, and V. V. Shepelevich, "Influence of piezoelectric effect on reading from hologramms in photorefractive crystals", *Pis'ma v Zhurnal Tekhnicheskoi Fiziki*, Vol. 14, No. 23, 2147–2151, 1988.

112. Mandel, A. E., S. M. Shandarov, and V. V. Shepelevich, Influence of piezoelectric effect and gyrotropy on light diffraction in cubic photorefractive crystals, *Optika i Spektroskopiya*, Vol. 67, No. 4, 819–822, 1989.

113. Shepelevich, V. V., "Diffraction of light by holographic grids in cubic photorefractive crystals", *Optika i Spektroskopiya*, Vol. 71, No. 6, 1044–1049, 1991.

114. Shepelevich, V. V., and E. M. Hramovich, "Influence of the lattice vector orientation on light diffraction in gyrotropic cubic photorefractive crystals", *Optika i Spektroskopiya*, Vol. 65, No. 2, 403–408, 1988.

115. Shepelevich, V. V., and E. M. Hramovich, "Diffraction of two light waves by holographic lattices in cubic gyrotropic photo-refractive crystals", *Optika i Spektroskopiya*, Vol. 70, No. 5, 1054–1059, 1991.

116. Shepelevich, V. V., and N. N. Egorov, "Diffraction of two light waves by holographic lattices in cubic gyrotropic photo-refractive piezo-crystals", *Pis'ma v Zhurnal Tekhnicheskoi Fiziki*, Vol. 17, No. 5, 24–28, 1991.

117. Baturin, N. A., "A study of electromagnetic wave transmission through a non-uniform isotropic and gyrotropic sample", *Kristallografiya*, Vol. 37, No. 6, 1508–1516, 1992.

118. Viitanen, A. J., "Generalized WKB approximation for stratified bi-isotropic media", *Journ. Electromagn. Waves Applic.*, Vol. 5, No. 4, 71–83, 1992.