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An Optimized Phase-Only Trapezoid Taper Window for Array Pattern Shaping

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ABSTRACT: Generally, the array pattern synthesizing can be shaped by controlling the excitation amplitude and phase of individual elements of the antenna array which they could be controlled either separately or jointly to provide most flexible solutions for the desired pattern shaping. In this paper, a new controllable trapezoid phase-only taper is proposed. In practical applications, phase-only tapers are more preferable than amplitude-only tapers due to their desirable advantages. The required pattern shaping with fulfilled user-defined constraints on the sidelobe peaks, beam widths, and steered nulls can be achieved by optimizing only the excitation phases of the trapezoidal taper. More importantly, the proposed trapezoidal taper offers the best tradeoff between the array directivity and undesirable sidelobe pattern. In addition, the element excitation amplitudes of the proposed phase-only trapezoid taper are made constant and equal to that of the original trapezoid taper function. Thus, it enjoys low array complexity. Moreover, the manipulated phases are assumed to be symmetric to further simplify the array feeding network. The genetic algorithm was used to optimize only the half number of the elements' phases. The results show that the phase-only trapezoid taper yields identical main beam shape to that of the amplitude-only trapezoid taper and much better than the other conventional tapers. Furthermore, it is found that the trapezoid phase-only method needs more variable elements than the trapezoid amplitude-only method to achieve almost the same performance. Thus, the complexity reduction percentage of the phase-only method is lower than that of the amplitude-only method.

1. INTRODUCTION

Many applications such as wireless communications, radar, and remote sensing use phased arrays that have capability to generate a concentrated beam of radio waves and electronically scan it to a point in any direction by means of phase shifters. However, such beam patterns are usually associated with undesirable sidelobes that cause leakage of the transmitted/received radio waves power to directions other than the intended ones [1].

In general, far-field antenna patterns and their aperture illumination functions are related to each other by direct and inverse Fourier Transform equations [2]. Tapering the amplitudes and/or the phases of the array's elements can reduce the undesirable sidelobe levels and shape the arrays patterns. In the literature, for example see [3, 4], there are various amplitude tapering windows that have been proposed, and they can be used to design the antenna arrays and also in the spectral analysis of the signals.

Alternatively, phase-only tapering control can be exploited to shape the array's far-field pattern and also to reduce the undesirable sidelobe peaks. These phase-only control methods have many advantages compared to that of amplitude-only tapering control, and thus, they become more preferable and widely reported in the literature [5–8]. In some applications of active electronically steered antenna arrays, implementing an amplitude-only tapering control may be difficult where the signals amplitudes cannot be varied due to the used power amplifiers in which their efficiencies were maximized only when they operate in compression mode. In this case, the signal amplification becomes nonlinear, and thus, signal amplitude tapering can be problematic. Therefore, manipulating phases instead of amplitudes can be a better choice for shaping array patterns.

Another important issue with parametric amplitude-only tapers is a trade-off between sidelobe peaks and main beamwidths. The beamwidth can be significantly widened, thus causing array's directivity reduction when the sidelobe peak is reduced. Therefore, optimization algorithms such as genetic algorithm [9], particle swarm optimization [10], convex optimization [11], and many other algorithms have been used to optimize the amplitude and/or phase of tapered elements to get the optimum performance and minimized sidelobes.

Amplitude and/or phase tapering controls have also been implemented at subarray output [12–14]. Controlling the signal's amplitude and phase at the subarray output reduces the number of needed hardware components, but it may cause distortion in the subarray's far-field pattern [15, 21].

Partially elemental amplitude and phase tapering adjustments by either controlling a few existing side elements [16, 17] or adding extra edge elements [18, 19] have also been suggested for sidelobe depression. It is proved that partially elemental tapers can maintain beamwidths of the resultant patterns undistorted and their directivities unchanged. Thus, they are found to be more preferable than those conventional fully-elemental tapers especially in large arrays.

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In this paper, the amplitude-only trapezoid tapering method presented in [20] is further developed and extended to include phase-only adjustments. Here, we use the original trapezoid taper window as a fixed and preexisting amplitude-taper for element excitation amplitudes. Thus, only element excitation phases are the optimization variables that need to be optimized to get the shaped patterns under some prespecified user-defined constraints on sidelobe peaks, beamwidths, and steered nulls. For easy implementation and also to maintain the corresponding magnitude pattern an even symmetry, the element excitation phases are assumed to be symmetric about the array center. The proposed symmetrical taper has been applied to one-dimensional linear and two-dimensional planar arrays to assess its validation and superiority compared to all other existing phase-only taper windows.

2. THE AMPLITUDE-ONLY TRAPEZOID TAPER WITH ZERO PHASES

Consider a linear antenna array composed of N total number of elements that are symmetrically distributed about the array center in the x axis with uniform inter-element spacing $d = \frac{\lambda}{2}$. The element excitation amplitudes and phases are assumed to be symmetric around the array center. Consequently, the resulting normalized far-field Fourier transform pattern is symmetric, and it can be given by

$$FF_o(\theta) = \sum_{n=1}^{N/2} w_n \cos\left[\frac{(2n-1)}{2} kd \sin\theta\right] \tag{1}$$

where $k = \frac{2\pi}{\lambda}\lambda$ is the wave length, θ the direction of arrival angle from the broadside, w_n the complex element excitation of the nth element, and a_n and p_n represent the amplitude and phase of element excitation w_n. In the work presented in [20], a_n was first chosen as a trapezoid taper with partially controlled elements. Then, the controllable elements were optimized using an efficient constrained-optimization algorithm, while all the elements' phases p_n were set to zero. In this work, the values of a_n are assumed to be fixed according to the trapezoid taper function, and the phases are partially optimized (i.e., the phases of sided elements are only optimized while those at the central region of the taper are made zero). The trapezoid taper used for calculating the values of a_n for the amplitude-only taper is something between standard rectangular and triangular taper windows. It has a certain number of unit-amplitude elements at the central region of the trapezoid window, and the remaining number of elements has a tapered-amplitude at the two edges of the trapezoid window. Like all other common tapers, this taper function is parametric, and its amplitudes can be determined in Matlab by

$$a_n = trapmf\left(n, \left[1 \ \frac{N}{2} - M \ \frac{N}{2} + M \ N\right]\right) \tag{2}$$

where $n = 1, 2, \ldots, N$ is the element index, and M is the number of the unit-amplitude elements at the central region of the trapezoid window. There is another method to generate the trapezoid function by simply taking the difference between two triangle windows. In Matlab, the far-field array pattern of the trapezoid taper can be obtained by taking the Fourier transform (FT) of (2) as follows

$$FF_o(\theta) = fftshift(fft(a_n, S)) \times EP \tag{3}$$

where S is a total number of the Fourier transform points, and EP is the element pattern of the individual array element. Fig. 1 shows the trapezoid taper for $N = 40$ and $M = 18$ that has been generated by using (2). For comparison, the standard rectangular and triangular taper windows have also been plotted. As mentioned, this trapezoid amplitude taper will be used with the proposed phase-only taper where the phases of sided elements only need to be optimized to get the required shape of the array pattern.

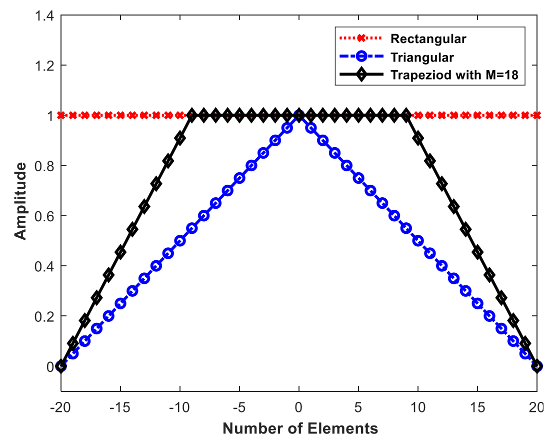

FIGURE 1. Typical rectangular, triangular, and trapezoid tapers for $N = 40$ and $M = 18$.

3. THE PHASE-ONLY TRAPEZOID TAPER WITH FIXED AMPLITUDES

The given trapezoid window (see Fig. 1) has two different regions. The first region is located at the top-center of the taper, and it has a number of elements with unit-amplitude equal to M elements, while the second region is the two linear tapers at the array sides which contains a number of tapered elements equal to $N - M$. The element excitation amplitudes across the whole trapezoid window, a_n, can be given by

$$a_n = \begin{cases} \frac{n+\frac{N}{2}}{-\frac{M}{2}+\frac{N}{2}} & -\frac{N}{2} \le n \le -\frac{M}{2} \\ 1 & -\frac{M}{2} \le n \le \frac{M}{2} \\ \frac{\frac{N}{2}-n}{\frac{N}{2}-\frac{M}{2}} & \frac{M}{2} \le n \le \frac{N}{2} \end{cases} \tag{4}$$

We can now define the vector, $w_n = a_n e^{jp_n}$ of prefixed trapezoid amplitudes, a_n for $1 \le n \le N$ and varying phases, p_n for $1 \le i \le (N - M)$ where $N - M$ elements are assumed to be adjusted in their phases. Therefore, the updated phases can be written as

$$\emptyset_i = p_i + \Delta_i \quad i = 1, 2, \ldots, (N - M) \tag{5}$$

where Δ_i are the phase adjustments of $N - M$ elements. Here, we assume that the phase adjustments are symmetrically even around the array center. Then, the resulting array pattern can

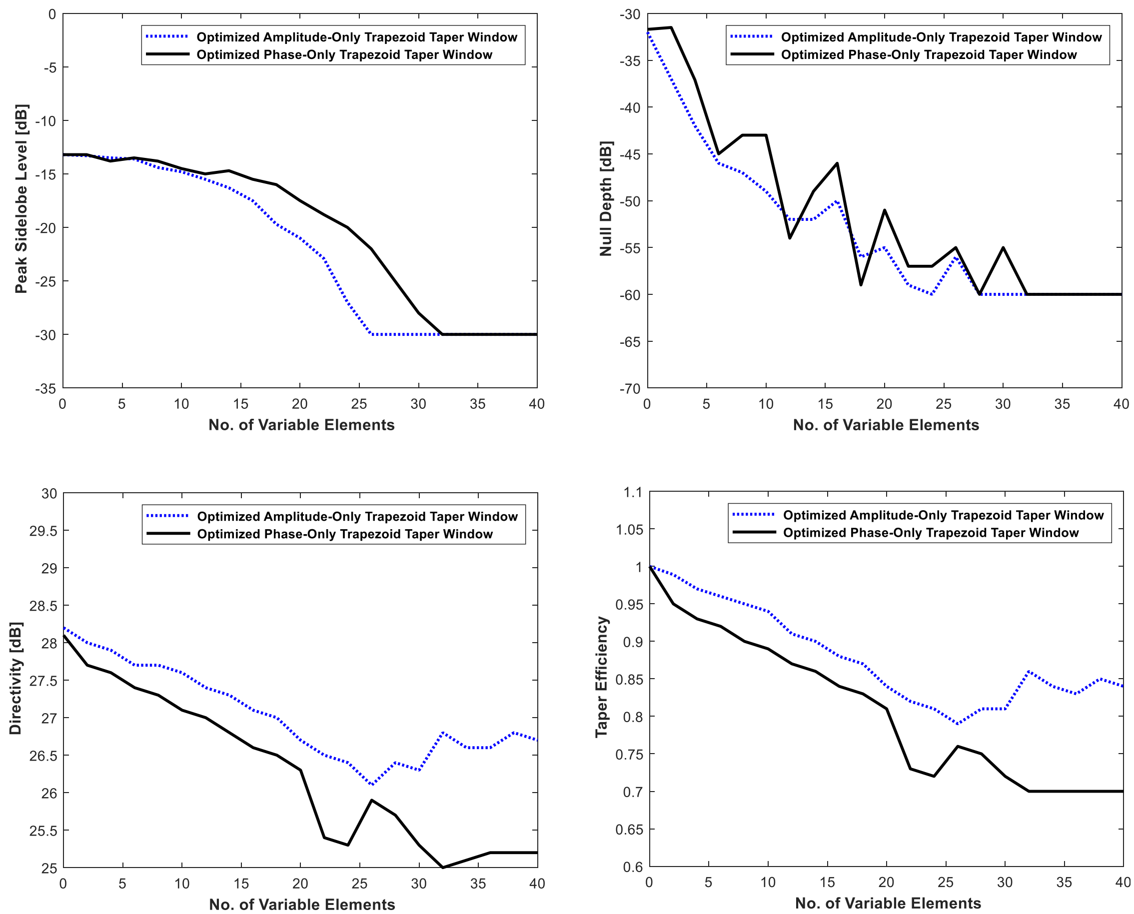

FIGURE 2. Performance comparison of the amplitude-only and phase-only trapezoid tapers.

be written as

$$FF(\theta) = \underbrace{\sum\nolimits_{n=1}^{M/2} a_n \cos\left[\frac{(2n-1)}{2}kd\sin\theta\right]}_{\textit{Central Region of Trapezoid Taper}} + \underbrace{\sum\nolimits_{i=M/2+1}^{N/2} a_i e^{j\emptyset_i} \cos\left[\frac{(2i-1)}{2}kd\sin\theta\right]}_{\textit{Side Regions of Trapezoid Taper}} \quad (6)$$

As mentioned, the element amplitudes were chosen according to the trapezoid taper as given in (4) for $1 \leq n \leq N$. Rearranging (6), it becomes

$$FF(\theta) = \underbrace{FF_o(\theta)}_{\textit{Original Trapezoid Pattern}} + \underbrace{\sum\nolimits_{i=M/2+1}^{N/2} \Delta_i \cos\left[\frac{(2i-1)}{2}kd\sin\theta\right]}_{\textit{Phase-Only Adjustment Pattern}} \quad (7)$$

From (7) it is clear that there are only $N-M$ variable phases to be optimized instead of N total array variables for pre-specified sidelobe peaks, beamwidths, and steered nulls. These pre-specified user-defined constraints (UDCs) can be given by

$$UDCs(\theta) = \begin{cases} 20\log_{10}(SLL_{limit}), & FNBW \leq |\theta| \leq 90^\circ \\ 20\log_{10}\frac{\mathrm{FF}(\theta)}{\max(\mathrm{FF}(\theta))}, & -FNBW \leq \theta \leq FNBW \end{cases} \quad (8)$$

where SLL_{limit} is the desired limit on the sidelobe level, and $FNBW$ is the first null-to-null beamwidth of the main beam.

Now, the $N-M$ variable phases need to be optimized such that the phase-only adjusted pattern best approximates the desired user-defined constraints pattern such that the difference between them is minimized in the least mean squared sense as follows

$$E(\theta_s) = \sum\nolimits_{s=1}^{S} |FF(\theta_s) - UDC(\theta_s)|^2 \quad (9)$$

where $s = 1, 2, \ldots, S$ are the sample points between these two patterns. The phase-only adjustment method can now be stated as follows: find the minimum number of variable element phases, $N-M$ (i.e., the maximum number of zero-phases at the central region of the trapezoid taper, M) and their phase adjustments, Δ_i, for a number of array elements equal to $i = 1, 2, \ldots, (N-M)$, which minimizes the difference, $E(\theta_s)$,

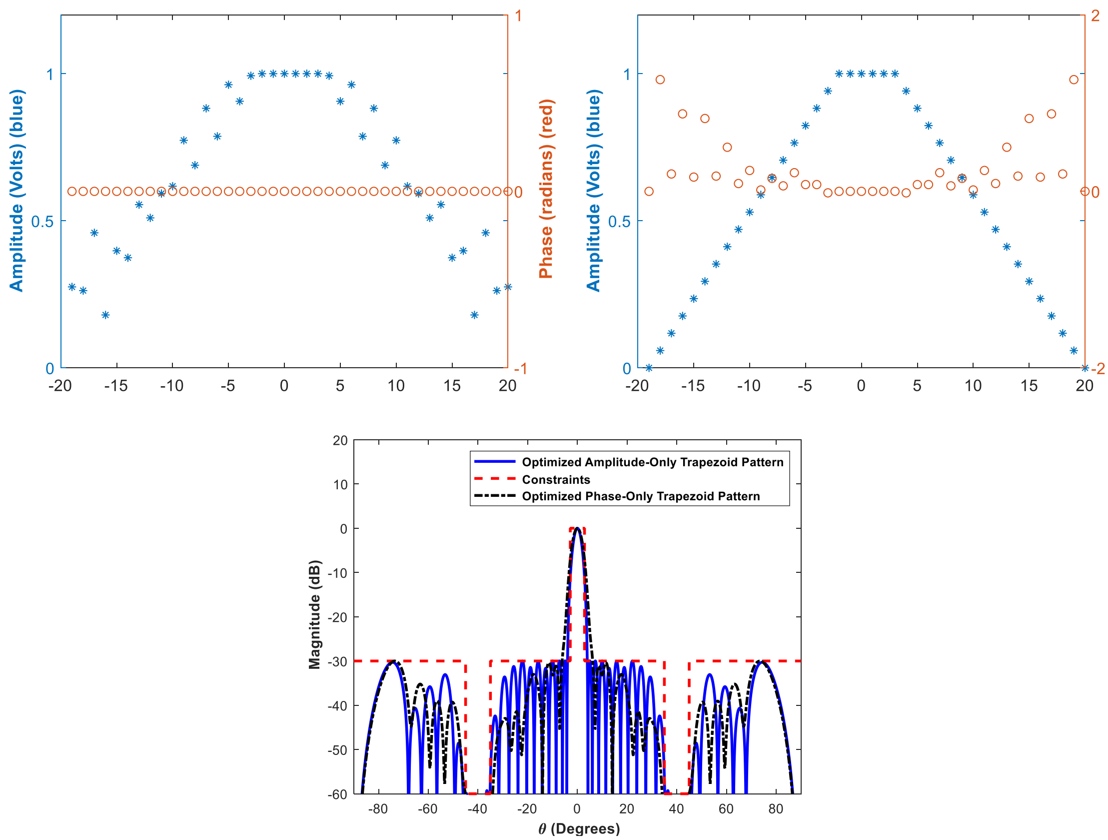

FIGURE 3. Element excitation amplitudes and phases and their corresponding patterns for a number of variable elements, $N-M=34$ and $M=6$.

over the angular range $0 \leq |\theta| \leq 90°$. This problem may be formulated mathematically as

$$\|E(\theta_s)\|_\infty = \min_{\Delta_i}\left[\max_{\theta\in(0,90)} |E(\theta_s)|\right] \tag{10}$$

This is the cost function that has been used with the genetic optimization algorithm. The specification parameters of the used genetic algorithm were: the number of iterations was 1000; the number of population was set to 50; the number of marriages was 25; the number of crossovers was 2; a uniform crossover was chosen; and the selection was tournament. The mutation probability was set to 0.04 and the tournament eligible set to 10.

4. SIMULATION RESULTS

This section carries out various simulations to demonstrate the validity of the proposed phase-only trapezoid taper in shaping the array pattern according to the pre-specified user-defined constraints such as limited sidelobe peak and steered multiple wide and deep nulls using a partial control of the element excitation phases.

In all examples, a symmetrical uniform linear array with $N=40$ elements and equal inter-element spacing $d=\frac{\lambda}{2}$ was examined.

In the first example, the performance metrics in terms of peak sidelobe level (PSLL), null depth (ND), directivity (D), and taper efficiency (TE) of the optimized phase-only trapezoid window under different numbers of variable elements phases, $N-M$ (or a certain number of unit-amplitude elements, M, in the array center) have been numerically studied. Fig. 2 illustrates such results. For comparison purpose, the results of the method of amplitude-only trapezoid taper that was presented in [20] are also included. In addition, the method of the fully phase-only control presented in [6] is also studied. These three methods were compared under the same conditions of user-defined constraints where the required PSLL was set to -30 dB; half power beamwidth was set to $0.05°$; and two symmetrical wide nulls with width equal to $10°$ (i.e., $35° \leq |\theta| \leq 45°$) and centered at $\theta = \pm 40°$ were considered. The fully phase-only control method with prefixed unit-amplitudes fails to provide a satisfactory performance and meet such constraints. Thus, the results of optimized amplitude-only and phase-only trapezoid windows were shown and discussed. It can be seen from these results that the PSLL and ND are both improveD with increased number of variable elements, $N-M$. In other words, they are both getting worse with increased number of unit-amplitude elements, M, until they reach the typical rectangular window values at $M=N=40$. Whereas, the D and TE values are both degrading when increasing the number of variable elements, $N-M$. They are both at their highest values only when $M=N=40$ which corresponds to the typical rectangular taper window as mentioned before.

In the second example, the array patterns of the amplitude-only trapezoid taper and the proposed phase-only trapezoid ta-

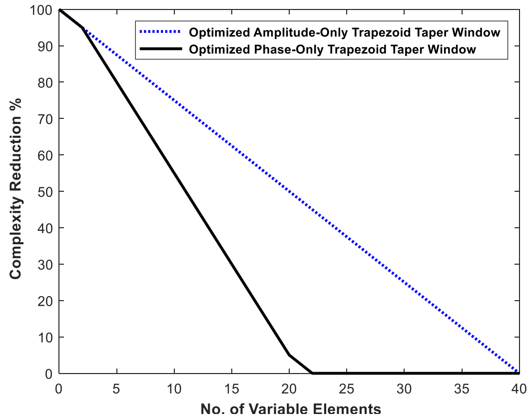

FIGURE 4. Complexity reduction percentages of the amplitude-only and phase-only trapezoid tapers.

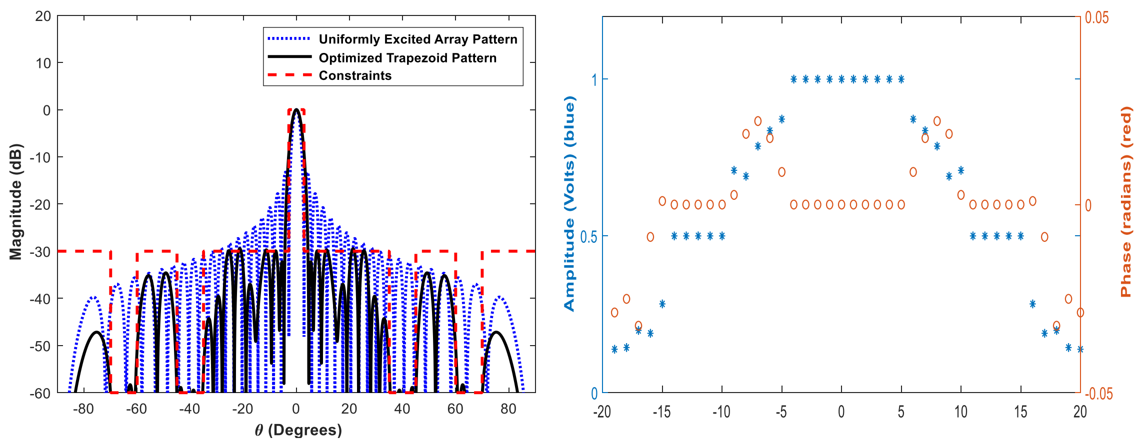

FIGURE 5. Complex amplitude and phase Trapezoid taper and its corresponding array pattern for a number of variable amplitude-phase elements, $N-M=20$ and $M=20$.

per with fixed amplitudes for $N-M=34$ and $M=6$ elements on both sides of the array center are illustrated. Fig. 3 illustrates the optimized phase-only trapezoid pattern and compares it to the optimized amplitude-only trapezoid taper with zero phases that was presented in [20]. The user-defined constraints are highlighted in this figure by the dashed red-line where the limited sidelobe level is assumed to not exceed $SLL_{limit}=-30$ dB, and two wide and deep nulls centered at $\theta=\pm 40^\circ$ are considered. This figure also shows their corresponding taper functions. The PSLL, wide and deep nulls, and main beamwidths were successfully kept within the user-defined constraints in both methods.

In the third example, the array feeding complexity in terms of number of nonunit-amplitude and nonzero-phase, $N-M$, in the taper region of the trapezoid window is studied. The complexity reduction (CR) is calculated as

$$CR=\left[\frac{\begin{array}{c}Total\ number\ of\ array\ elements-\\ Total\ number\ of\ variable\ elements\end{array}}{Total\ number\ of\ array\ elements}\right]\times 100\% \tag{11}$$

This equation is applied to the optimized amplitude-only trapezoid taper and optimized phase-only trapezoid taper under various numbers of $N-M$ as shown in Fig. 4. It can be seen that the CR% of the phase-only trapezoid taper is much lower than that of the amplitude-only trapezoid taper. This means that the phase-only method needs more variable elements than the amplitude-only method to achieve the same performance. For $N-M=22$, the CR of the phase-only method becomes zero, while that of the amplitude-only method is 45%. This verifies the effectiveness of the amplitude-only method compared to the phase-only method.

In the next example, the array pattern and its corresponding optimized amplitude and phase of the trapezoid window (i.e., complex element excitation) for $N-M=20$ and $M=20$ are illustrated in Fig. 5 where the number of variable amplitude and phase elements was 20 while the number of unit-amplitude and zero-phase elements was also 20. In this case, there are 20 non-unit amplitudes and 20 non-zero phases, and thus, the total number of variable elements is 40. By applying (8) and calculating the CR%, CR = %. This means that there is no any reduction in the array complexity of the feeding network. Nevertheless, the PSLL was maintained below -30 dB, and four

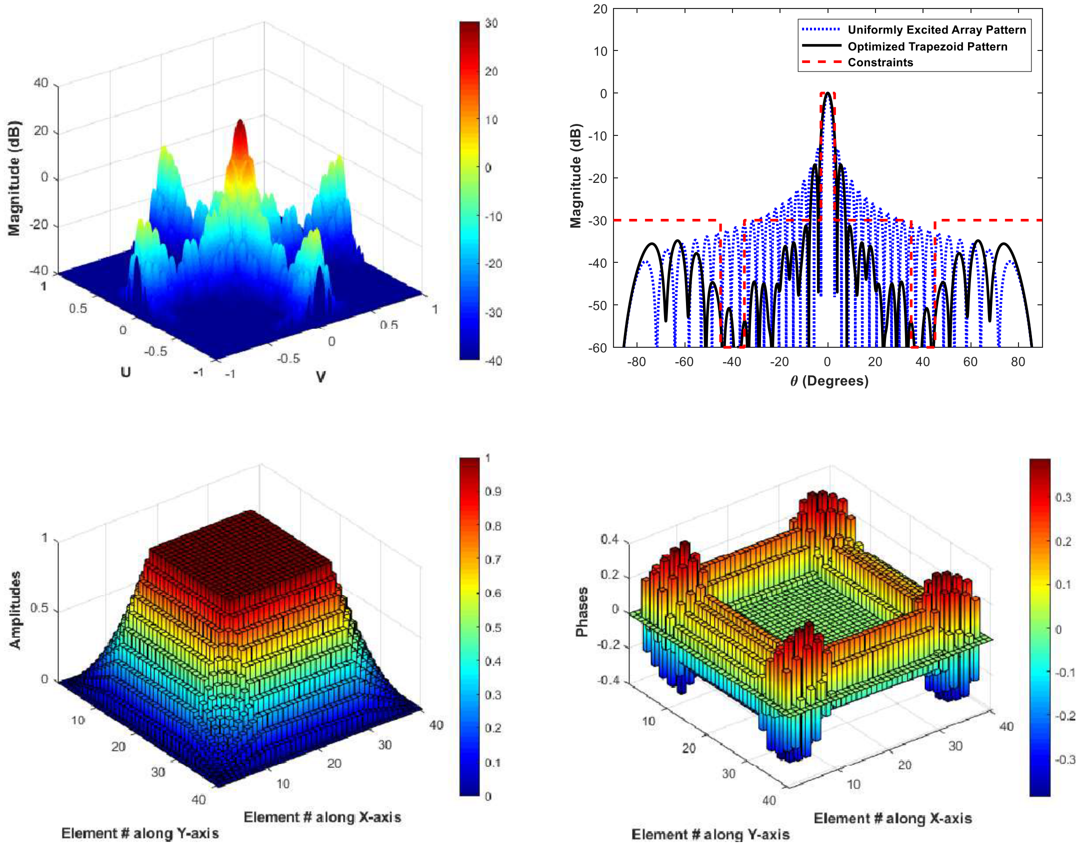

FIGURE 6. Results of two-dimensional phase-only taper with prefixed trapezoid amplitudes.

wide-deep nulls around the centers at $\theta = \pm 40°$ and $\theta = \pm 65°$ with depths more than -60 dB were successfully validated.

Finally, the proposed phase-only trapezoid taper that was previously applied to the one-dimensional linear array is extended to the two-dimensional planar array with a total number of uniformly inter-element spacing elements equal to 40×40. The number of elements of the optimized phase-only trapezoid taper is $(N-M)\times(N-M) = 20\times 20$, and the number of unit-amplitudes and zero-phases is equal to $M \times M = 20 \times 20$. The user-defined constraints were two wide-deep nulls around the centers $\theta = \pm 40°$ and limited $SLL = -30$ dB. The results of this case are illustrated in Fig. 6. Although the pattern was well shaped, the peak of the first SLL located just beside the main beam was at -18 dB. This is mainly due to the availability of a small number of the variable phases or degrees of freedom.

5. CONCLUSIONS

The performances of the amplitude-only trapezoid taper with zero phases and the phase-only taper with pre-fixed trapezoid amplitudes have been assessed and compared. In some applications, there is a difficulty to apply amplitude-only methods, thus, the phase-only methods can be an alternative way to accomplish the required pattern shaping. From the results, it has been shown that the amplitude-only method with sufficient number of sided variableamplitude elements could perform better than the phase-only method even with relatively large number of sided variablephases. This is mainly because the sided element excitation amplitudes have greater impact on the array pattern formulation than the phases. The peak SLL, null depths, directivity, taper efficiency, and the complexity reduction of the amplitude-only method were slightly better than that of the phase-only method. It is worth to mention that the fully phase-only method with prefixed rectangular taper failed to meet the required user-defined constraints. Thus, the proposed phase-only method with prefixed trapezoid amplitudes outperforms many other existing phase-only methods. Moreover, the proposed phase-only taper has been applied to linear and planar antenna arrays to confirm its effectiveness and generality.

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