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Broadside-Incidence Extinction Measurements of Thin Copper Circular Discs and the Extinction Paradox at 35 GHz

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ABSTRACT: This article presents millimeter wavelength measurements of the mass normalized extinction cross section (extinction efficiency) of thin copper circular discs at broadside incidence. The extinction efficiencies of the discs were measured as a function of diameter and thickness at a fixed frequency of 35 GHz. The measurements cover a wide range of diameters and thicknesses and were compared with the approximate numerical solution of the problem provided by the CWW code. A good agreement between the measurements and CWW code was achieved after applying the extinction paradox for small particles with high index of refraction to the CWW code calculations.

1. INTRODUCTION

Numerical and experimental investigation of the problem of electromagnetic scattering and absorption by two dimensional particles (disc shape particles) has been established for decades [1–10]. For circular thin discs, Willis and Weil developed a moment method code (CWW code) that calculates the scattering, absorption, and extinction cross sections [1]. Although CWW code has restrictions of use to be accurate, the parameters of measurements in this article are within the applicability of the code. We have theoretically investigated disc particles in the visible spectral region, where we showed a detailed picture of their response to visible wavelength [10].

The extinction paradox for more than 60 years has been connected to the asymptotic limit of the extinction cross section of large particles compared with the wavelength [11, 12]. The geometrical optics limit for a large particle compared with the wavelength predicts an extinction cross section that is equal to its geometrical cross section, but a proper measurement or an exact theory calculation of the extinction cross section yields twice the geometrical cross section. A canonical explanation of the paradox has been adopted for many years which was based on the idea of a combination of diffraction and geometrical optics [13]. This explanation does have problems as discussed by Berg et al. [14], and a new explanation of the paradox that is based on connecting the cross-section calculation to the Ewald-Oseen theorem was introduced. The new explanation shows that the paradox happens even for small particles given that the phase shift parameter $\rho = \mathrm{Re}\,(m-1)\,.2x \gg 1$, where m is the complex index of refraction of the particle, and x is the size parameter.

In this study, the extinction cross section for circular copper thin discs was measured as a function of disc diameter and thickness at millimeter wavelength, and the results are compared with the numerical solution provided by the CWW code. The measured diameters span a wide range that covers well the prime resonance of the discs and for all of them $\rho \gg 1$. The extinction paradox, which is based on the new explanation presented by Berg et al. [14], is used to achieve a good agreement between theory and measurements. As predicted, a factor of two differences between measurements and calculations is applied to the CWW code calculations in order to achieve a good agreement.

2. MEASUREMENTS

2.1. Discs Fabrication

The copper discs were plated in a high vacuum plating system (Cooke Vacuum Products, Inc.) with an Edwards FTM-5 Coating Thickness Monitor on 10 mil mylar diaphragms using a set of masks. Figure 1 shows sample pictures of the plated copper discs on mylar diaphragms.

2.2. Measurement System

The extinction measurements were made at 35 GHz using a rotating platform system, which was developed by the authors for the purpose of measuring single fibrous particles. In that case, the fiber is rotated in a plane perpendicular to the direction of propagation of the electromagnetic wave. As the fiber rotates, a high attenuation signal is produced when the fiber is aligned to the electric field vector and zero signal when being perpendicular to the electric field vector of the electromagnetic wave producing a high sensitivity for the tenuous fibers. For much larger circular discs, the attenuation signal produces a substantial fixed, broadside signal while the disc is situated (centered on the platform) in the perpendicular plane because of symmetry. In both cases, measurements were calibrated using a set

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FIGURE 1. Sample pictures of the plated copper discs on mylar diaphragms.

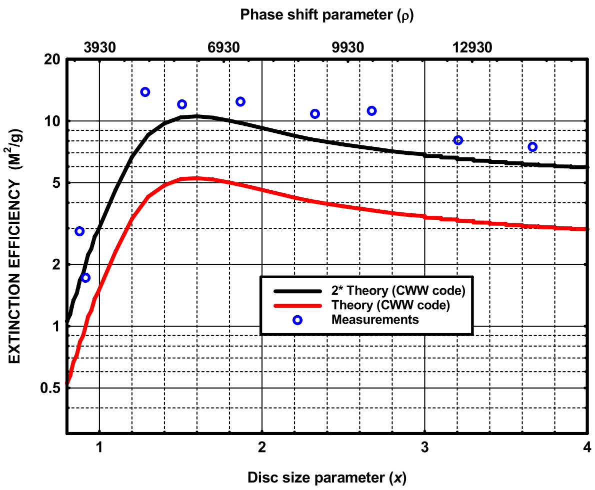

FIGURE 2. Extinction efficiency as a function of size parameter and phase shift parameter for copper discs with thickness of 72 nm for broadside incidence at 35 GHz.

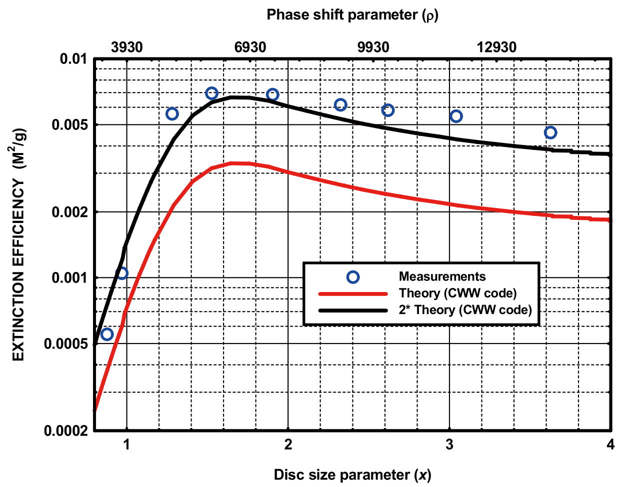

FIGURE 3. Extinction efficiency as a function of size parameter and phase shift parameter for copper discs with thickness of 127 nm for broadside incidence at 35 GHz.

of stainless-steel fibers with 4 µm diameter. See [15] for details and pictures of the measurement system. The reason for using the millimeter wavelengths for the measurements is that: discs, scaled to such a size range can easily be fabricated and measured using available equipment.

3. RESULTS AND DISCUSSION

Figures 2 and 3 show the measured extinction cross section per unit mass (extinction efficiency) as a function of disc size parameter $x = ka$, where a is the radius of the disc, and k is the wave number, for copper discs of thickness $t = 72$ nm and 127 nm, respectively. The measurements were made for broadside incidence at 35 GHz. Using the Drude model and a copper conductivity of $\sigma = 5.8 \times 10^7$ mho/m, the copper complex index of refraction at 35 GHz value is $m = 3862 + 3862i$. All the measured discs are not large compared with the wavelength but satisfy the criteria that $\rho \gg 1$. The corresponding ρ values of the discs are also shown in Figures 2 and 3. In the figures, the calculated extinction efficiency using the CWW code is shown. As we can see in Figures 2 and 3, there is a factor of two difference between measurement and calculations which support the extinction paradox explanation presented in [14]. If this factor of two is applied to the calculations provided by the CWW code, a good agreement is achieved as shown in the figures.

Figure 4 shows the extinction efficiency as a function disc thickness for a fixed size parameter $x = 1.83$, and again a good agreement is achieved after applying a factor of two to the numerical calculations. Although the discs were measured while being plated on mylar diaphragms, an effective surrounding medium correction should be applied to the measurements, and this correction was found to be negligible using the CWW code calculations.

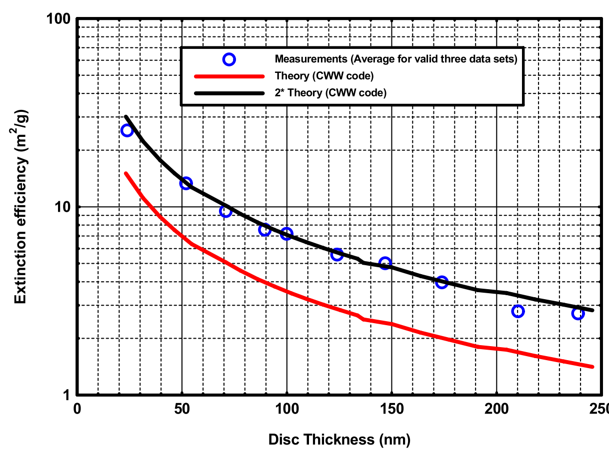

FIGURE 4. Extinction efficiency as a function of thickness for copper discs of size parameter $x = 1.83$, for broadside incidence at 35 GHz.

4. CONCLUSION

The extinction paradox was applied to achieve a good agreement between the numerical calculation of the extinction efficiency of copper thin discs and our measurements for broadside incidence at 35 GHz. All the discs measured in this article satisfy the condition of the paradox. The results in this article show that the CWW code underestimates the extinction efficiency by a factor of two for small particles with high index of refraction in agreement with the new explanation mentioned in [14], and for such cases a multiplication by a factor of two should be applied to the CWW code calculations to achieve true results comparable with measurements.

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