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Reinforcement Machine Learning for Sparse Array Antenna Optimization with PPO

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ABSTRACT: This paper focuses on optimizing the radiation pattern of sparse array antennas using reinforcement learning, with many algorithms. The paper aims to leverage Proximal Policy Optimization's (PPO's) advantages in optimization and its effectiveness in handling stochastic transitions and rewards to achieve a reduced number of elements while maintaining desired signal performance and minimizing unnecessary side lobe signals. By removing a few of the antennas using reinforcement learning and PPO optimization, the same results as a complete array have been obtained. The anticipated outcomes of this research hold the promise of significantly enhancing the effectiveness and utility of sparse array antennas in communication systems.

1. INTRODUCTION

Antenna array design serves the crucial purpose of achieving the desired radiation pattern with the minimum number of antenna components, a vital consideration in applications where weight and size limitations are critical, such as in phased array radar and satellite communications [1, 2]. Array antennas play a crucial role in processing signals arriving from different paths, and one of the primary objectives in their design is to create a suitable geometric structure to achieve the desired radiation pattern with high gain and compact dimension [3]. Consequently, the design and optimization of array antennas have garnered considerable attention due to their wide-ranging applications and the need to achieve optimal designs.

The aim of optimization in array antenna design is to find the best acceptable solution according to the constraints and needs of the problem. With the introduction of optimization algorithms, the foundation for improving the optimal design of array antennas has been established [4, 5]. Achieving the optimal design of array antennas is a significant challenge due to their wide applications and the need to meet specific design criteria.

Various methods have been proposed to optimize antenna arrays, each with specific goals. For instance, [1, 6, 7] introduce a non-iterative method for linear array synthesis based on the matrix pencil method (MPM), enabling the synthesis of a nonuniform linear array with a reduced number of elements. Similarly, [7] investigates a method to create thinned aperiodic linear phased arrays through the application of genetic algorithms to suppress grating lobes and achieve increased steering angles.

In all these years, the aim has been to improve the performance of the array antenna in different ways. One of the important issues in array antennas is to achieve the desired results despite the reduction of array elements, which is called sparse array antenna. In this method, the number of active elements is reduced, but the same results are obtained when all the elements are active. This method makes the antenna lighter and reduces its costs [8–10].

In recent decades, various approaches have been presented in the literature for sparse array synthesis, initially using basic optimization techniques, then continuing with advanced techniques. Taylor spatial narrowing [11], optimization algorithms such as genetic algorithm [12, 13] are among these methods. Basic optimization techniques may suffer from local minima, and in the presence of multiple minima, finding the optimal solution faces limitations. For this reason, the use of optimization methods such as genetic algorithm is suggested. Ref. [14] used the combination of GA and minimum redundancy method to optimize the random sparse array.

In this paper, a novel reinforcement learning method, specifically Deep Q Learning with PPO algorithm, is introduced to optimize the array factor equation for sparse array antenna. This method is compared with Genetic and PSO and many other algorithms to demonstrate its effectiveness in achieving antenna designs with fewer elements and desired efficiency. By comparing these optimization methods, the goal is to contribute to the existing body of knowledge and provide insight into the potential of reinforcement learning in array antenna design

2. METHODOLOGY

In this section, different optimization methods will be implemented in a practical way by different algorithms in order to clarify the difference between the method used in this article and the other methods. The efficiency of reinforcement learning optimization compared to other methods will be clearly

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seen. The problem to be solved is the design of a dispersive antenna array starting from a uniform linear array antenna. The aim is to remove as many elements as possible from an initial uniform linear array, while keeping the radiation pattern close to the desired pattern, and also optimizing the side lobe level (SLL) of the antennas in addition to achieving the desired radiation pattern. Optimizing the side lobe level is crucial in minimizing unwanted radiation in directions other than the main lobe direction, thereby enhancing the overall performance of the antenna system, using different algorithms optimization. In this problem, the symmetry of the space of antennas has been exploited, and for example, the array of 32 antennas has been optimized by searching in the space of 16.

The array factor for sparse array antennas is obtained from the uniform linear array AF of the antenna by introducing the weighting coefficients [15], X_n with binary values

$$AF_{\text{dBimp}}(\theta) = 20\log\left(\frac{\left|1+\sum_{n=2}^{N} e^{j(n-1)kd\cos\theta}\right|}{|N|}\right) \tag{1}$$

$$AF_{dBsp}(\theta) = 20\log\left(\frac{\left|1+\sum_{n=2}^{N} x_n e^{j(n-1)kd\cos\theta}\right|}{\left|1+\sum_{n=2}^{N} x_n\right|}\right) \tag{2}$$

A weighting factor X_n with a value of zero means that the feeding of the element at position n is cancelled, while a value of 1 means that the element is fed. The error function (ERR) is given by the following expression:

$$\begin{aligned}
&\text{ERR}(x_2,\ldots,x_n,\ldots x_N) \\
&= \sum_{\theta=0}^{\pi}\left(AF_{\text{dB sp}}(\theta) - AF_{\text{dB imp}}(\theta)\right)^2 \\
&= \sum_{\substack{\&\theta=0 \\ \&\theta\neq\frac{\pi}{2}}}^{\pi}\left(10\log\left(\frac{\left|1+\sum_{n=2}^{N} x_n e^{j(n-1)kd\cos\theta}\right|}{\left|1+\sum_{n=2}^{N} x_n\right|}\right)\right. \\
&\left.\quad -10\log\left(\frac{\left|1+\sum_{n=2}^{N} e^{j(n-1)kd\cos\theta}\right|}{|N|}\right)\right)^2
\end{aligned} \tag{3}$$

This error function will be optimized by five different methods: Genetic Algorithm, Particle Swarm Optimization (PSO), Proximal Policy Optimization (PPO), Actor Critic with Kronecker-factored Trust Region (ACKTR), Advantage Actor Critic (A2C) method, in this article to achieve the best results for desired objectives, which are:

•Achieving a suitable beam with optimum side lobe level.

•Reduce the number of array elements as much as possible to achieve better results than other methods.

•The comparison of main method results with other methods.

The choice to utilize reinforcement learning for antenna array optimization stems from its capacity to handle complex, dynamic environments and its ability to learn from interaction with the environment. Reinforcement learning offers the advantage of adaptability and learning from feedback, making it well suited for antenna array optimization tasks that involve intricate trade-offs and dynamic constraints. By employing reinforcement learning, its potential in achieving efficient and adaptive solutions for antenna array design is aimed to be exploited, particularly in scenarios where traditional optimization methods may face challenges in handling complex and evolving design requirements [16–18].

In summary, the methodology used in this paper involves implementing various optimization methods with reinforcement learning, to address the design of a sparse array antenna, with a focus on optimizing the side lobe level and exploiting the symmetry of the antenna space.

3. OPTIMIZATION PROBLEM SOLVING METHOD WITH REINFORCEMENT LEARNING

3.1. Reinforcement Learning for Hybrid Optimization

Solving a combinatorial problem with the RL approach requires an Markov Decision Process (MDP) formulation. The environment is defined by a specific instance of the optimization problem. The states are encoded by a neural network model (e.g., each node has a vector representation encoded by a graph neural network) [19]. The agent is guided by an RL algorithm (e.g., Monte Carlo tree search) and makes decisions that move the environment to the next state (e.g., remove a vertex from a solution set).

3.2. Introducing the Synthesis of Phased Array Antenna as a Combinational Optimization

In Figure 1, the arrangement of antenna array elements for synthesis is shown which must have one of two states on or off. As it is clear in the figure, in the optimization of the array antenna, some elements are turned off, and in other words, they try to get the desired answer with the maximum number of turned off elements.

3.3. Introducing the Reinforcement Learning Optimization

In [20, 21], optimization is introduced, and it includes various optimization problems from hybrid, continuous, mixed discrete/continuous to high dimensional problems, heavy computations, and constrained engineering optimizations. This gives a hint to solve the antenna optimization problems using such methods in this paper, because due to the possibility of using the feature of antennas being parallel in the first to fourth lines of the Python code, first the input vector x is parallelized, and then by adding the parallel vector to x, a new vector with twice the length is created. Therefore, the search space for the optimization algorithm has dimensions $(2:1)$ equal to the dimensions of the array end space. For example, by searching in 32-dimensional space, it is possible to synthesize the vector

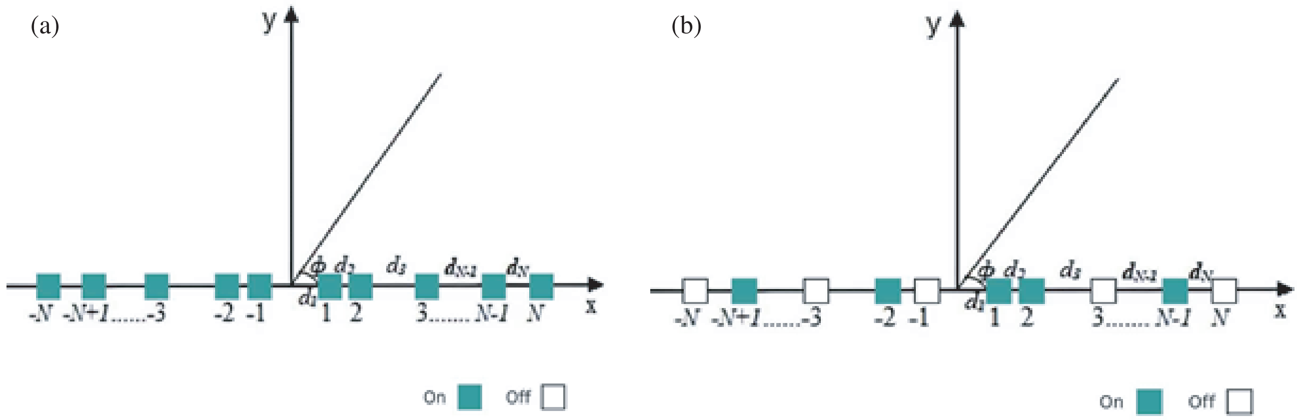

FIGURE 1. Arrangement of antenna array elements, (a) turning on entire antenna elements, (b) turning off some concentrated array elements.

of 64-dimensional antennas and give distinctive and successful results for problems similar to the one that this article discusses.

This paper uses reinforcement learning methods because they have been shown to provide distinctive and successful results for similar problems.

4. RESULTS AND DISCUSSIONS

Different methods were used for the proper design of a sparse array antenna, and some of the results are considered in the following sections. The results of all the used methods will also be presented so that the results can be compared.

4.1. Implementation of Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is one of the bio-inspired algorithms and is a simple algorithm for finding an optimal solution in the solution space. It is particularly useful for antenna array optimization [17, 18]. It differs from other optimization algorithms in that it only needs the objective function and does not depend on the gradient or any differential form of the objective. It also has very few hyperparameters. In [22], Particle Swarm Optimization demonstrates its effectiveness in addressing the challenges of antenna synthesis, showcasing advancements in optimizing antenna performance.

In this section, the givens used for this algorithm are: Time Step $= 16000$ and Episode $= 1000$. The results of running the PSO algorithm in the search space of dimension 32, 64, and 96 elements are shown in Table 1.

Table 1 shows that PSO performed best with 64 antennas, 20 of which were turned off (31.25% of the total). As the number of antennas increased to 96, the percentage decreased to 16.7%. This contradicts the previous method's result.

Figure 2 shows that the first three side lobes increased slightly (first SLL show increase to -14 dB), which is considered a undesirable result in this paper.

4.2. Implementation of Reinforcement Learning Method

To solve discrete optimization problems, the Q reinforcement algorithm method can be used instead of the random search

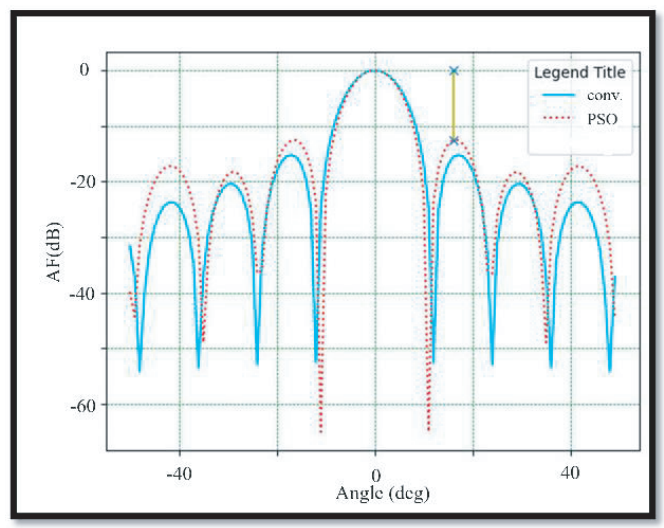

FIGURE 2. AF optimized antenna by PSO (red) non-optimized antenna (blue).

algorithm. To solve the optimization problem, as mentioned in [21, 23], first the environment, agent, actions, and rewards are defined, and then the Q-table and optimal policy should be obtained using the toolbox, then in the order of these implementation steps should be made.

1- To solve the discrete optimization problem after training and obtaining the optimal policy, the learned model is fed in the first step of the current or initial state.

2- From the beginning of the policy, obtain the largest action to carry out the action.

3- Apply the chosen action to the environment.

Repeat this process until reaching the solved or desired state. This article implements several reinforcement learning methods to optimize array antennas and compares their results, as mentioned below.

TABLE 1. The results of running the PSO algorithm.

Number of antennas	Best individual (x) and (y) found	antennas turned off
32	$Y = 131728.12005149468$ $X = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0]$	2
64	$Y = 22819.18791633075$ $X = [1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0]$	20
96	$Y = 280881.98186617147$ $X = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1]$	16

4.2.1. Implementation of Proximal Policy Optimization (PPO) Method

The employment of the PPO algorithm in the optimization of array antennas has yielded remarkable results, particularly in the search space of dimension 32, 64, and 96 antennas with added correlation. Notably, the PPO algorithm has exhibited its superiority over all other methods, especially in antennas of larger dimensions. The forthcoming presentation of graphs and results serves to underscore the unparalleled performance of the PPO algorithm in enhancing array antenna functionality, signifying its potential as a leading approach in this domain.

Table 2 shows that PPO performed very good in two cases with 64 and 96 antennas, with 24 and 40 turned off antennas respectively (37,5% and 41.7% of the total).

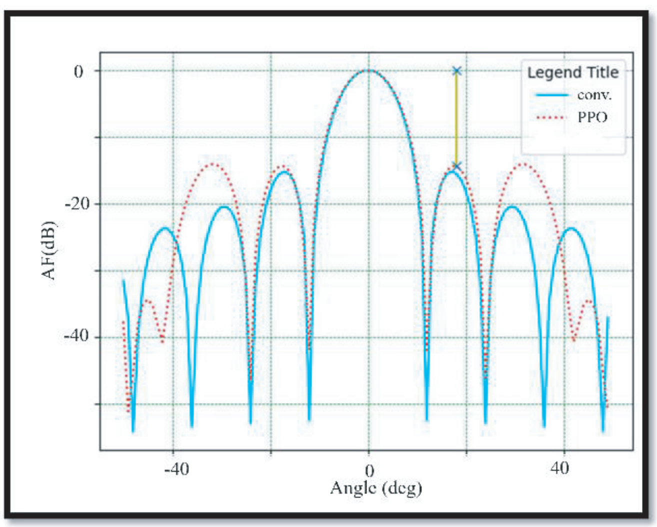

FIGURE 3. Antenna AF optimized by PPO (red) non-optimized and conventional graph (blue).

Figure 3 illustrates that with a slight increase in all side lobe levels, a small increase occurs in the first side lobe level (-15 dB) while the third side lobe level is reduced to -35 dB.

4.2.2. Implementation of Advantage Actor Critic (A2C) Method

Also the results obtained from employing the A2C method in this context are significant, particularly in the search space of dimensions 32, 64, and 96, with the subsequent doubling of correlation. The forthcoming presentation of graphs and results serves to underscore the favorable outcomes attained in both targeted objectives, underscoring the effectiveness of the A2C method in advancing array antenna functionality, but the SLL has become a little less than the usual state.

Table 3 indicates that the A2C algorithm successfully switches off unnecessary antennas when being applied to 96 antennas, achieving a success rate of 41.7%.

Figure 4 shows that all side lobes show an unwanted increase, with the first side lobe show increasing to -11 dB.

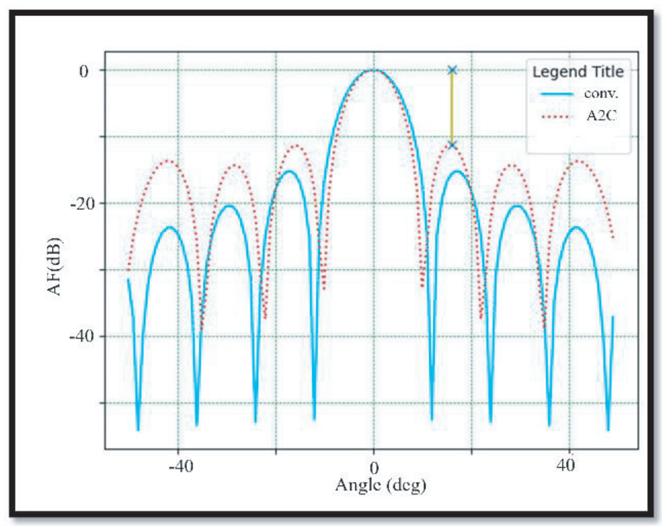

FIGURE 4. Antenna AF optimized by A2C (red), non-optimized graph (blue).

5. COMPARISON AND ANALYSIS OF THE RESULTS

Table 4 shows inclusive results of all methods used in this article to help in comparing between them and find which one had better optimization and achieved this article goals.

In the implementation of the algorithms in case of having a high number of antennas (96 antenna), it is clear that reinforcement learning is superior. especial PPO and A2C which achieved excellent results by switching off 40 antennas of 96 antennas and this make it 41.7% switching off percentage. In the case of 64 antennas PPO has absolute superiority by 37.5% switching off percentage, highlighting the promising potential of reinforcement learning in antenna design. In the other case like 32 antennas reinforcement learning also shows superiority over classical methods like genetic and PSO algorithms. What is remarkable is that ACER and ACKTR make only 25%

TABLE 2. The results of running the PPO algorithm.

Number of antennas	Best individual (x) and (y) found	antennas turned off
32	$Y = 167107.47$ $X = [1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$	4
64	$Y = 122483.82$ $X = [1\ 1\ 1\ 0\ 0\ 1\ 1\ 11\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 1\ 0]$	24
96	$Y = 364100.12$ $X = [0\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 1]$	40

TABLE 3. A2C method output for different number of antennas.

Number of antennas	Best individual (x) and (y) found	antennas turned off
32	$Y = 167107.47$ $X = [1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$	4
64	$Y = 300846.06$ $X = [1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1]$	12
96	$Y = 350162.38$ $X = [1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0]$	40

TABLE 4. Comparison of different methods: Genetic Algorithm, PSO and Reinforcement Learning methods (before adding the term of side lobes in the cost function).

Method	optimization Algorithm	Number of ON antennas (ones) of 32 antennas	Number of ON antennas (ones) of 64 antennas	Number of ON antennas (ones) of 96 antennas	The average ability of this method to turn off antennas (%)	Stability of the answer
1	GA	28	48	64	27.08	Medium
2	PSO	30	44	80	19.79	Good
3	PPO	28	40	56	35.42	Excellent
4	A2C	28	52	56	29.17	v.good
5	ACKTR	24	44	68	29.17	v.good

switching off percentage. The overall result of PPO makes 35.24% as the average ability of this method to switch off antennas with more than 5% superiority from the nearest method which is ACER by (30.20%).

Therefore, the results of the method used in this article, i.e., reinforcement learning with the PPO algorithm, are better than the traditional PSO method ($\sim$ 20% more successful in 64 antennas and 15.63% more successful in overall results) and better than the genetic algorithm too.

6. RESULTS BY ADDING A SIDE-LOBE OPTIMIZATION TERM TO THE COST FUNCTION

In this section, by incorporating the size of the side lobes which represents the value of the side lobe levels (SLLs) — into the cost function, the problem was transformed into a multi-objective one and subsequently solved. Furthermore, the difference between the method employed in this article and other methods becomes clear: the efficiency of the reinforcement learning approach in optimization surpasses that of other methods.

The new cost function = Original AF cost function + SLL inverse value, which is with the following formula:

$$Z = ERR + c * \left(\frac{1}{\text{side\_lobe\_level}}\right)$$

$$\text{ERR}\left(x_2, \ldots, x_n, \ldots x_N\right) = \sum_{\theta=0}^{\pi}\left(AF_{\text{dB sp}}(\theta) - AF_{\text{dB imp}}(\theta)\right)^2 + c*\left(\frac{1}{\text{SLL}}\right) \quad (4)$$

where: Z = new cost function,

TABLE 5. The final comparison table of 3 methods after adding the term of side lobes in the cost function.

Method	optimization Algorithm	Antennas 16		Antennas 32		Antennas 64		Antennas 12	
		SLL	On	SLL	On	SLL	On	SLL	On
1	**PSO**	24	12	24	24	23.7	42	33.3	88
2	**ACKTR**	36	10	29	14	40	32	32	76
3	**PPO**	32	10	31	18	42	36	39	67

ERR $=$ original AF cost function (3),

$c =$ importance coefficient of SLL.

It is clear that the first main criterion is the number of antennas off, and then the amount of the AF error function, the lower the better, which is now the best results for reinforcement learning and specifically PPO method, and this can be clearly seen in Table 4.

The analysis of the graph considers side lobes as the primary criterion. To reduce side lobes, they must be included in the cost function. However, reinforcement learning was able to achieve better side lobe levels than Genetics and PSO and other methods in the first side lobe and by turning off the antenna.

In Table 5, it can be observed that the addition of a term to the cost function has led to the generation of new results for each employed method. It is evident that the PPO method outperforms other approaches, particularly in scenarios involving many antennas (such as high-dimensional designs). It should be emphasized that the focus in this article is on radars equipped with a substantial number of antennas, aiming to achieve goals like price reduction and size optimization. Consequently, if the chosen method yields optimal outcomes even in the presence of a significant number of variables, the desired result will be attained. Additionally, alternative methods can be combined when working with a specific quantity of antennas.

7. CONCLUSION

Finally, it can be concluded that in the problem of optimizing the number of ON antennas and the side lobe level (SLL) for the design of the phased array antenna, the reinforcement learning method (PPO algorithm) gives the best results in all kinds of genetic, PSO, and other reinforcement algorithm implementations, and also in many problems with high dimensions, the reinforcement learning method has achieved much better results than the genetic algorithm, and the PSO method. Reinforcement learning method in problems where there is no accurate model of the system and environment and uncertainties in the model or due to the structure and dynamics with many random parameters, the use of complex stochastic processes to simulate the environment is needed. It can solve discrete optimization problems and can give better results than genetic algorithm and other evolutionary algorithms in problems with high dimensions. In this paper, different methods such as Genetic Algorithm, PSO Algorithm, and Reinforcement Learning of Q-Learning type have been investigated, and finally the Q-Learning algorithm gives better results.

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