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Key Practical Issues of the MoM Using in EMC Uncertainty Simulation

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ABSTRACT: The Method of Moments (MoM) is widely used in Electromagnetic Compatibility (EMC) uncertain simulation due to its advantages such as non-embedded simulation, high computational efficiency, and immunity from dimensional disasters. The theoretical research of the MoM has been relatively complete, but many of its key practical issues have not been fully discussed, which will result in the calculation accuracy in practical engineering applications falling short of theoretical expectations. With the help of the Feature Selective Validation (FSV) method, this paper analyzes and discusses two aspects. One is how to reasonably select the perturbation, and the other is the relationship between the uncertainty input size and the accuracy. By solving key practical issues of the MoM, the aim is to further promote it in the EMC field.

1. INTRODUCTION

In the actual electromagnetic environment, there are many uncertain factors. If random variables can be used instead of deterministic constants for modeling, and quantitative prediction of uncertainty transfer response can be achieved, the credibility of the EMC simulation results can be greatly improved [1, 2].

MoM is an uncertainty analysis method based on the Taylor's formula expansion principle. It is important to note that the MoM referenced here is not the same as the one mentioned in [3] and [4] for converting systems of integral equations into linear systems. The two MoMs share the same name by coincidence. The first advantage of the MoM proposed in this paper is its non-embedded simulation approach. Because of the complexity of practical situations, existing EMC simulations often require the use of commercial EMC simulation software (e.g., FEKO, CST). However, the algorithmic programs of these software packages are not open source, making the non-embedded simulation method more practical and competitive in the EMC field. This property of MoM makes its applicability better than the Stochastic Galerkin Method [5]. The second advantage of the MoM is that it is highly computationally efficient, for both traditional methods and improved algorithms. This feature makes the MoM more applicable than the Monte Carlo Method (MCM) [6]. The third advantage of the MoM is that it is not subject to the curse of dimensionality. The MoM and its improved algorithms ensure that the number of deterministic EMC simulations is linearly related to the number of random variables required for uncertainty analysis, or more precisely, double or triple that number. Therefore, the increase in random variables does not substantially affect the computational efficiency of the MoM, a property that makes it more widely used than the Stochastic Collocation Method [7].

The main feature of traditional MoM is its high computational efficiency, but its accuracy diminishes when the nonlinearity between the simulation input and output is large [8–10]. In 2016, an improved MoM (IMoM) was proposed [11]. The Richardson extrapolation-based method enhances the accuracy of calculating the sensitivity of random variables when the nonlinearity is large, thus improving the accuracy of the standard deviation of simulation results. However, the limitation of this improvement is that the accuracy of the simulation results depends on the exact perturbation amplitude. Determining the appropriate perturbation amplitude requires further study. In 2022, the clustered MoM was proposed [12], which improved the average computational accuracy of MoM when nonlinearity is large. However, the problem remains that determining the applicability of this method has not been confirmed.

In this paper, a quantitative analysis method based on FSV [13–15] is applied to solve two practical problems to prevent the accuracy of the measurement model from falling below the theoretical accuracy due to improper use in practice. The first major practical issue is determining the applicability of MoM. The uncertainty range of EMC analog inputs should not be too large because MoM inherently has a limited ability to handle nonlinearities. Determining the applicability range based on the magnitude of uncertainty is the first issue to tackle. The second major practical issue is determining the magnitude of perturbation to enhance the simulation accuracy of the standard deviation results simulation model.

The structure of this article is as follows. Section 2 gives a brief overview of the MoM. Section 3 provides an introduction to the one-dimensional wave propagation calculation example required for uncertainty analysis. Section 4 discusses in detail the impact of the selection of perturbation and uncertainty range on the accuracy of the MoM uncertainty analysis results. Section 5 summarizes the paper.

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2. INTRODUCTION OF MOM

In the MoM, the random variable space $\xi = \{\xi_1, \xi_2, \ldots, \xi_n\}$ is used to describe the randomness factors in EMC simulation input. Uncertainty analysis refers to how to quantify the impact of this uncertainty on EMC simulation output.

The quantitative calculation formula for the mean result of the traditional MoM is as follows.

$$E\left[y_{\text{out}}\right] = \text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots, \overline{\xi_i}, \ldots, \overline{\xi_n}) \quad (1)$$

Among them, y_{out} represents the EMC simulation output, and $E[y_{\text{out}}]$ is its mean calculation result. $\overline{\xi_i}$ is the mean of the random variable ξ_i, and $\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots, \overline{\xi_i}, \ldots, \overline{\xi_n})$ represents the deterministic EMC simulation results at the mean point. At this point, EMC simulation can be treated entirely as a black box model to achieve non-embedded uncertainty analysis. The formula for calculating the mean result of the Clustering MoM is as follows.

$$E\left[y_{\text{out}}\right] = \sum_{i=1}^{M} p_i \times \text{EMC}_{\text{sim}}[N_i(\xi)] \quad (2)$$

Among them, $N_i(\xi)$ is the representative sampling point selected based on the random variable space ξ, and the total number of sampling points is M. p_i is the percentage of each representative sampling point in the overall sampling point after clustering calculation. The specific details of the Clustering MoM can be found in [12].

The standard deviation calculation formula for the traditional MoM is as follows.

$$\sigma\left[y_{\text{out}}\right] = \sqrt{(A_1)^2\sigma_{\xi_1}^2 + \ldots + (A_i)^2\sigma_{\xi_i}^2 + \ldots (A_n)^2\sigma_{\xi_n}^2} \quad (3)$$

$$A_i \approx \frac{dy}{d\xi_i} \quad (4)$$

$$\frac{dy}{d\xi_i} = \lim_{\delta_i \to 0} \frac{\begin{array}{c}\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots \overline{\xi_i} + \delta_i \ldots, \overline{\xi_n}) \\ -\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots \overline{\xi_i} \ldots, \overline{\xi_n})\end{array}}{\delta_i} \quad (5)$$

$$A_i = \frac{\begin{array}{c}\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots \overline{\xi_i} + \delta_i \ldots, \overline{\xi_n}) \\ -\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots \overline{\xi_i} \ldots, \overline{\xi_n})\end{array}}{\delta_i} \quad (6)$$

Among them, σ_{ξ_i} represents the standard deviation of each random variable ξ_i, and $\sigma[y_{\text{out}}]$ is the standard deviation calculation result of the EMC simulation output parameters. A_i represents the sensitivity of the random variable ξ_i, which is estimated using the formula (4)–(6). δ_i is a small perturbation, and its value can directly determine the calculation accuracy of formula (6). In existing reference, there are cases where the perturbation value is assigned as σ_{ξ_i}, as well as cases where it is assigned as the ratio between the uncertainty range and the mean of the simulation input parameters [10–12]. Formula (6) has been improved through Richardson Extrapolation Method [11], and its calculation principle is as follows.

$$A_i = 2 \times \frac{\begin{array}{c}\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots \overline{\xi_i} + \frac{\delta_i}{2} \ldots, \overline{\xi_n}) \\ -\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots \overline{\xi_i} \ldots, \overline{\xi_n})\end{array}}{\frac{\delta_i}{2}} - \frac{\begin{array}{c}\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots \overline{\xi_i} + \delta_i \ldots, \overline{\xi_n}) \\ -\text{EMC}_{\text{sim}}(\overline{\xi_1}, \ldots \overline{\xi_i} \ldots, \overline{\xi_n})\end{array}}{\delta_i} \quad (7)$$

At this point, the overall standard deviation calculation formula remains unchanged as formula (3). The principles and formulas of three types of the MoM are compared in Table 1.

TABLE 1. Comparison of the principles and formulas of three types of the MoM.

	Mean formula	Standard deviation formula
MoM	Formula (1)	Formula (3) and (6)
IMoM	Formula (1)	Formula (3) and (7)
Clustering MoM	Formula (2)	Formula (3) and (7)

The traditional MoM requires a total of $n + 1$ deterministic EMC simulations, while the IMoM requires $2n + 1$. The Clustering MoM is the least computationally efficient and requires $2n + M + 1$ simulations. From this, it can be seen that the MoM and its improved algorithms have the advantage of high computational efficiency and are not affected by dimensional disasters.

In the view of the MoM, the mean and standard deviation are important uncertainty analysis results provided by it.

3. ONE-DIMENSION WAVE PROPAGATION CALCULATION EXAMPLE

An example of one-dimensional wave propagation calculation is presented in this section, and it is published in both [11] and [16]. Since [11] is an important theoretical improvement of the MoM, that is, the content in formula (7), this article chooses the same calculation example. The schematic diagram of the calculation example is shown in Figure 1, which describes the wave propagation using a one-dimensional Maxwell's equation system. The Finite Difference Time Domain Method (FDTD) needs to be used to solve Maxwell's equations. The overall length of the space is 2.25 meters, equidistant into 150 discrete points. At the first discrete point, there is a sine excitation source for electric field intensity, with amplitude $E_M = 2.7 \times 10^{-3}$ [V/m] and frequency $\omega_0 = 1.0 \times 10^9$ [Hz].

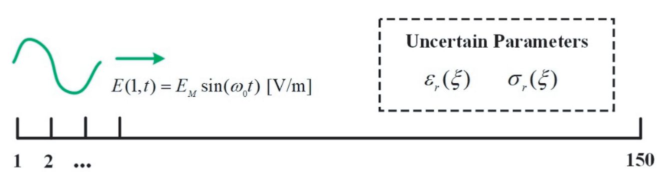

FIGURE 1. Schematic diagram of calculation example.

The calculation example contains two uncertain material parameters, the dielectric coefficient $\varepsilon_r(\xi)$ and the conductivity $\sigma_r(\xi)$, which are modeled using the following formula.

$$\varepsilon_r(\xi) = \varepsilon_r^* \times (1 + h \times \xi_1) \text{ [F/m]} \quad (8)$$

$$\sigma_r(\xi) = \sigma_r^* \times (1 + h \times \xi_2) \text{ [S/m]} \quad (9)$$

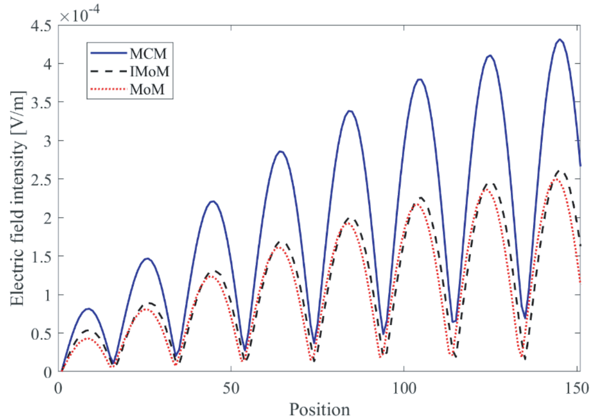

FIGURE 2. Standard deviation prediction results when $h = 0.05$ and $\delta_i = \sigma_{\xi_i}$.

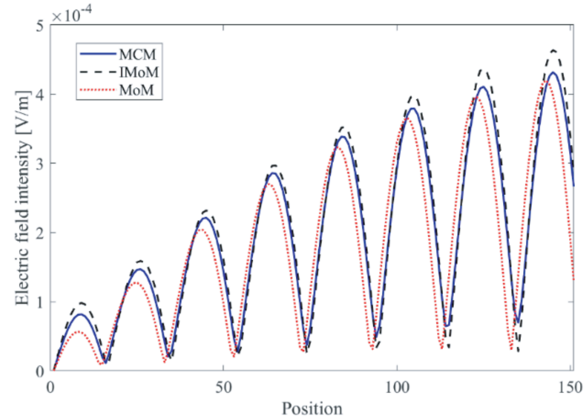

FIGURE 3. Standard deviation prediction results when $h = 0.05$ and $\delta_i = h$.

Among them, ξ_1 and ξ_2 are uniformly distributed random variables with a value range of $[-1,1]$, and $\varepsilon_r^* = 1.5\,[\text{F/m}]$, $\sigma_r^* = 5 \times 10^{-3}\,[\text{S/m}]$. The magnitude of the value h directly determines the uncertainty range of the simulation input parameters.

The absorbing boundary condition throughout the simulation is a perfectly matched layer. The time discrete step length of FDTD is 2×10^{-11} s, and the termination condition is 600 time steps. So, the simulation output result is the electric field intensity value of the entire space after 12×10^{-9} s of electromagnetic wave propagation. For more details on this calculation example, please refer to [11].

4. ANALYSIS OF KEY PRACTICAL ISSUES OF THE MOM IN UNCERTAINTY ANALYSIS

4.1. Issue 1: How to Select Perturbation?

For the calculation example in Figure 1, this section selects the following three types of perturbations for uncertainty simulation, and then compares and analyzes their results. For case 1, the perturbation is assigned as the standard deviation of the random variable, i.e., $\delta_i = \sigma_{\xi_i}$. The selection method of this perturbation quantity is derived from [9]. All current research on the MoM in the field of computational electromagnetics adopts the same approach. The selection strategies of the latter two perturbations are proposed in this article. For case 2, the perturbation assignment is the ratio of the uncertainty range of the EMC simulation input parameters to the mean, i.e., $\delta_i = \frac{h}{1} = h$. Since the principle of the MoM is the Taylor's expansion formula, the more accurately the truncated Taylor's formula describes the nonlinear situation of the sensitivity of random variables, the more accurate the results of the MoM will be. One of the criteria that determines the quality of nonlinearity description is whether the perturbation range $[-\delta_i, \delta_i]$ can completely cover the value range of the random variable. Therefore, h is more appropriate than σ_{ξ_i} for the perturbation amount. For case 3, the perturbation is corrected by the ratio in case 2, i.e., $\delta_i = \frac{h}{1+h}$. When the value of h is small, $\frac{h}{1+h}$ is approximately equal to h, but slightly smaller than h. Since h is the boundary of the uncertainty range, case 2 wastes the coverage outside the boundary, so the perturbation result of case 3 is theoretically better.

Taking $h = 0.05$, Figure 2 shows the predicted electric field intensity standard deviation of the MoM and the IMoM with perturbation of case 1, and the MCM results are also provided as standard data. Similarly, Figure 3 and Figure 4 provide the standard deviation prediction results for case 2 and case 3, respectively.

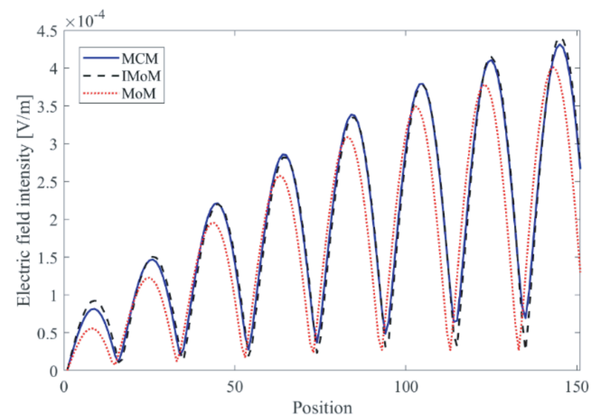

FIGURE 4. Standard deviation prediction results when $h = 0.05$ and $\delta_i = \frac{h}{1+h}$.

Using the results of the MCM as standard data, the FSV is introduced to quantitatively calculate the similarity between simulation results and the standard data to determine the accuracy of the algorithm to be evaluated. For more details on the FSV method, please refer to [13–15]. In the accuracy determination process of the remaining results in this article, the MCM method is used as the standard data.

The accuracy evaluation results using the FSV method are shown in Table 2. From the results in Table 2, it can be seen that the FSV value of the IMoM is significantly lower than that of the MoM, proving that the standard deviation calculation accuracy of the IMoM is indeed higher than that of the traditional MoM. Meanwhile, the standard deviation calculation results in case 3 (corresponding to Figure 4) are more accurate, and es-

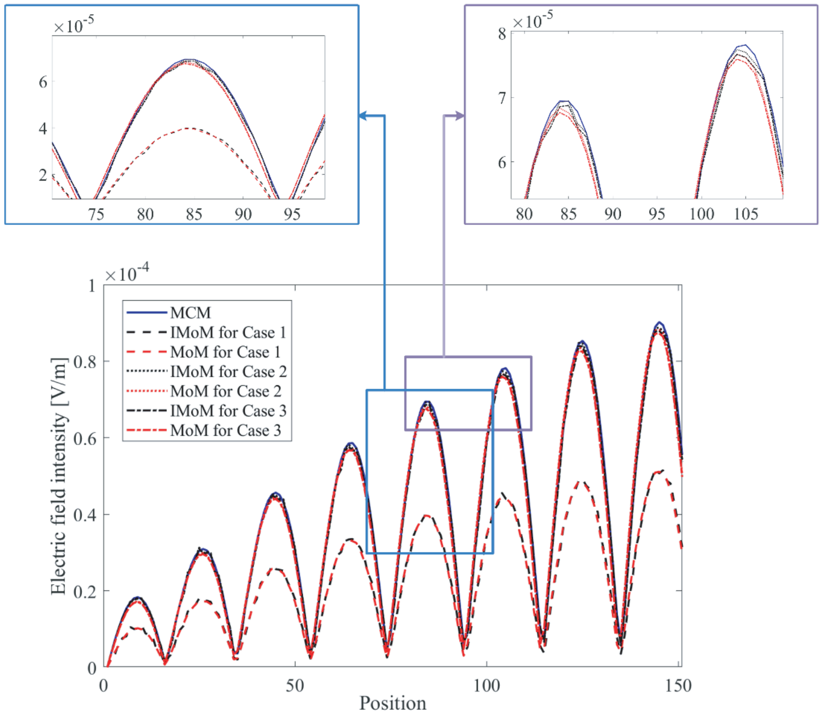

FIGURE 5. Standard deviation prediction results when $h = 0.01$.

TABLE 2. Accuracy evaluation result of different perturbations when $h = 0.05$.

	Case 1	Case 2	Case 3
IMoM	0.7060	0.1507	0.1061
MoM	0.8153	0.3795	0.3728

pecially the FSV value provided by the IMoM is 0.1061, which is very close to the "Excellent" rating. On the contrary, case 1 only showed accuracy at the "Fair" level or even worse, indicating that the perturbation in this case is not suitable for the MoM. Qualitative evaluation methods such as "Excellent" and "Fair" originate from [13–15].

It is worth noting that the perturbation only affects the standard deviation calculation results, and the standard deviation calculation results of the Clustering MoM and IMoM are completely consistent, so it is not necessary to provide them.

When $h = 0.01$, the uncertainty analysis is performed respectively, and the standard deviation of the obtained electric field strength is shown in Figure 5. The quantitative analysis based on the FSV method yields the results in Table 3. It can be seen that the obtained results are in complete agreement with the conclusions of the previous analysis. The only difference is that MoM and IMoM show excellent accuracy in both case 2 and case 3, which indicates that the uncertainty analysis is simpler when $h = 0.01$. Therefore, good accuracy can be obtained with a reasonable choice of perturbations.

Figure 6 shows the mean simulation results for $h = 0.05$ and $h = 0.01$, with the FSV method analysis results of 0.0380 and 0.0021, both at the "Excellent" level. When $h = 0.05$, a higher FSV value indicates that the uncertainty analysis problem is more difficult at this time. According to the sensitivity calculation process in formula (4), the standard deviation calculation process needs to use the mean prediction result, so the accuracy of the mean result will directly affect the accuracy of the standard deviation result.

TABLE 3. Accuracy evaluation results of different perturbations when $h = 0.01$.

	Case 1	Case 2	Case 3
IMoM	0.8020	0.0371	0.0347
MoM	0.8150	0.0900	0.0858

In summary, when the perturbation is selected as $\delta_i = \frac{h}{1+h}$, the standard deviation calculation accuracy of the MoM is the best, especially when using the IMoM.

4.2. Issue 2: The Relationship between the Uncertainty Range of EMC Simulation Input and the Accuracy of the MoM

Continuing to increase the uncertainty range of EMC simulation, Figure 7 and Figure 8 provide the standard deviation calculation results of the electric field intensity at $h = 0.1$ and $h = 0.2$, respectively. It is worth noting that 0.1 and 0.2 are already large uncertainty input ranges in practical engineering. Due to the poor simulation results of case 1, only the results of case 2 and case 3 are presented here. The quantitative evaluation results of the FSV method are shown in Tables 4 and 5. From Table 4, it can be seen that when $h = 0.1$, the IMoM can still maintain accuracy at the "Good" level, but the MoM is only at the "Fair" level. In Table 5, even the IMoM is at the "Poor" level. This indicates that the MoM, due to its inherent

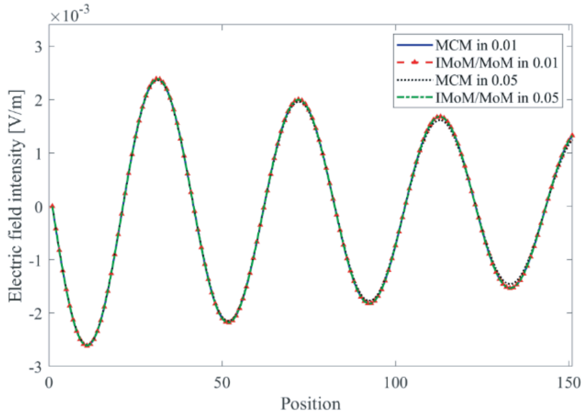

FIGURE 6. Mean prediction results of the MoM ($h = 0.05$ and $h = 0.01$).

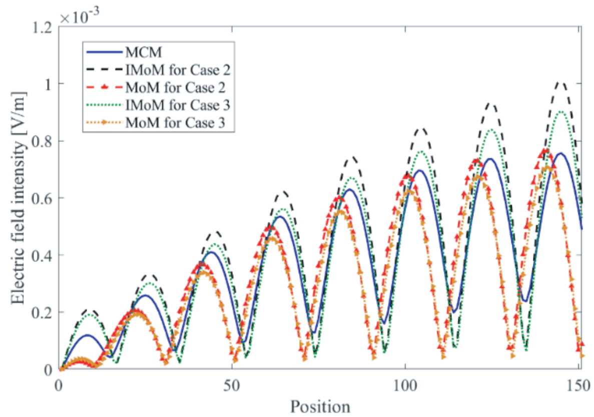

FIGURE 7. Standard deviation prediction results when $h = 0.1$.

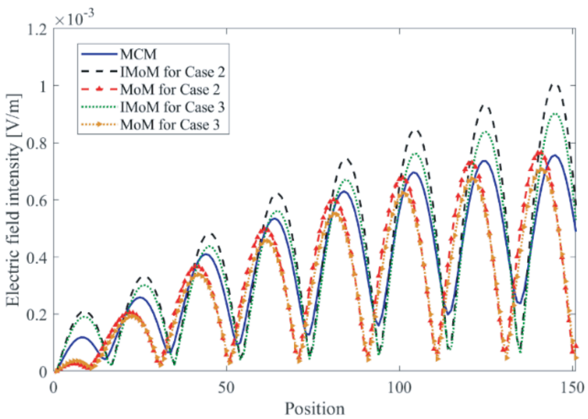

FIGURE 8. Standard deviation prediction results when $h = 0.2$.

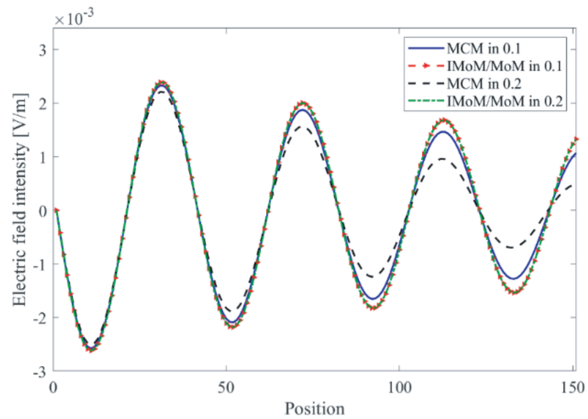

FIGURE 9. Mean prediction results of the MoM ($h = 0.1$ and $h = 0.2$).

weakness in handling nonlinearity, cannot maintain high computational accuracy in large uncertainty ranges. Therefore, to maintain good calculation accuracy of the MoM, the ratio between the uncertainty range of input parameters and the mean should not exceed 0.1.

TABLE 4. Standard deviation accuracy evaluation result when $h = 0.1$.

	Case 2	Case 3
IMoM	0.4828	0.3615
MoM	0.6159	0.6085

TABLE 5. Standard deviation accuracy evaluation result when $h = 0.2$.

	Case 2	Case 3
IMoM	0.9642	0.8235
MoM	0.8710	0.8712

Figure 9 shows the mean simulation results for $h = 0.1$ and $h = 0.2$, with the FSV method analysis results of 0.1342 and 0.4597. It is also proven that the larger the uncertainty input range is, the more complex the uncertainty analysis problem is solved, and the worse the mean prediction result of the MoM is, resulting in poor accuracy of the final standard deviation prediction.

In summary, when the ratio between the uncertainty range of input parameters and the mean does not exceed 0.1, the MoM can ensure high calculation accuracy, and the smaller the ratio, the better the accuracy of the MoM.

5. CONCLUSION

This article uses the FSV method and its quantitative evaluation method to solve the key practical issues of the MoM in EMC simulation uncertainty analysis. The following conclusions are obtained based on the example of one-dimensional wave propagation calculation. Firstly, when the perturbation is selected as $\delta_i = \frac{h}{1+h}$, the accuracy of the MoM is more ideal. If the IMoM is used, the calculation accuracy of the standard deviation will be better. It is recommended that other designers prioritize this specific perturbation $\frac{h}{1+h}$ when selecting the perturbation value of the MoM. Secondly, the smaller the h is, the higher the calculation accuracy is. It is recommended that other designers decide whether to choose the MoM for uncertainty analysis based on the actual size of h to avoid causing significant errors. Through the research in this article, further promotion and application of the MoM in EMC simulation can be achieved, fully leveraging its advantages such as non-embedded simulation, high computational efficiency, and immunity from dimensional disasters.

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