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Uncertainty Analysis Method for EMC Simulation Based on the Complex Number Method of Moments

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ABSTRACT: The Method of Moments (MoM) is a non-embedded uncertainty analysis method that has been widely used in Electromagnetic Compatibility (EMC) simulations in recent years due to its two major advantages of high computational efficiency and immunity from dimensional disaster. A random variable sensitivity calculation method based on the Complex Number Method of Moments (CN-MoM) is proposed in this paper to improve the accuracy of the MoM in standard deviation prediction and thereby enhance the credibility of EMC simulation uncertainty analysis results. In the parallel cable crosstalk prediction example in the literature, the result of the Monte Carlo Method (MCM) is used as the standard, and the accuracy of the new method proposed in this paper is quantitatively verified using the Feature Selective Validation (FSV) method. Compared with the MoM, the proposed method can significantly improve the calculation accuracy of the standard deviation results without sacrificing simulation efficiency.

1. INTRODUCTION

In order to effectively deal with random factors in the actual electromagnetic environment, such as manufacturing tolerances and lack of knowledge about physical properties and materials, uncertainty analysis methods are gradually introduced into EMC field, aiming to improve the credibility and practicality of simulation results [1–3].

In practical engineering applications, in order to achieve high reliability EMC simulation, the geometric and material parameters in three-dimensional space need to be finely modeled. At this time, the finite element analysis technology in commercial electromagnetic simulation software usually needs to be utilized, such as COMSOL and CST. At this point, due to the lack of open-source deterministic simulation methods in commercial electromagnetic simulation software, embedded uncertainty analysis methods lose competitiveness, such as Perturbation Method [4] and Stochastic Galerkin Method [5, 6]. At the same time, finite element analysis techniques often cause uncertainty analysis methods with longer single deterministic EMC simulation time, poor convergence, and low computational efficiency to lose competitiveness in practical applications due to time cost issues, such as Monte Carlo Method (MCM) [7–9]. It is worth noting that MCM has almost the best accuracy, so in theoretical research, the uncertainty analysis results of MCM are usually used as standard data to verify the accuracy of other uncertainty analysis methods, which is a recognized standard within the industry.

Dimensional disaster is another common issue encountered in implementing uncertainty analysis in EMC simulations. When the random events used to describe EMC problems are more complex, more random variables need to be used for description, and some uncertainty analysis methods may lose competitiveness due to a significant decrease in computational efficiency, such as Stochastic Collocation Method (SCM) [10, 11]. For SCM, as the number of random variables increases, the number of required collocation points will increase exponentially, and the number of deterministic EMC simulations will also increase, leading to the problem of high time costs that cannot be achieved.

At this stage, there are three types of uncertainty analysis methods that not only meet the requirements of non-embedded computing, but also meet the requirements of high computing efficiency and are not affected by dimensional disasters. They are the Method of Moments (MoM) [12, 13], Stochastic Reduced Order Models (SROM) [14], and Kriging surrogate model method [15]. The main problem with SROM is the inability to accurately determine convergence, which will cause errors and waste of computational resources in the actual simulation process [16]. The Kriging surrogate model method is an uncertainty analysis method based on the continuity assumption. When the EMC simulation uncertainty analysis model is more complex, there will be a problem of low computing efficiency [15].

The first-order Taylor expansion formula is used by the MoM to calculate expected values and variances, which has the advantages of high computational efficiency and easy implementation, and its number of calculations is linearly proportional to the dimensionality of random variables. It has great advantages in dealing with large-scale electromagnetic calculation problems and is currently widely used in the field of EMC uncertainty analysis. The main drawback of the MoM is its poor accuracy in implementing nonlinear EMC simulations. Ref. [17] has solved the accuracy problem of the MoM in predicting mean results through clustering algorithms. Therefore, if the accuracy of MoM in calculating standard deviations can be fur-

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ther improved, the application of the MoM in EMC simulation uncertainty analysis will be highly competitive. In the process of standard deviation calculation, the estimation accuracy of the sensitivity of random variables is crucial. This paper proposes a sensitivity calculation method based on the CN-MoM to improve the standard deviation prediction accuracy of the MoM.

The structure of this paper is arranged as follows. Section 2 provides a brief overview of the application of the MoM in EMC simulation uncertainty analysis. Section 3 provides a detailed introduction to the process of calculating the sensitivity of random variables using the CN-MoM. Section 4 quantitatively verifies the effectiveness of the proposed new method using the parallel cable crosstalk prediction example. Section 5 provides the conclusion of this paper.

2. BRIEF OVERVIEW OF THE MOM

For the random factors in the actual electromagnetic environment, random variable vectors ξ are usually used to model them, as shown below.

$$\xi = \{\xi_1, \xi_2, ..., \xi_n\} \tag{1}$$

Among them, each ξ_i is a random variable and has mutual independence. If the random variables in formula (1) are not mutually independent, the KarhunenLoeve decomposition technique [18, 19] can be used to decouple the vector space of random variables.

The results of uncertainty analysis are mean and variance. For the traditional MoM, the calculation of mean results is relatively straightforward, that is, a single deterministic EMC simulation can be performed at the mean of each random variable, as shown in the following formula.

$$E(y) \approx y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i}, \cdots, \overline{\xi_n}\right) \tag{2}$$

Among them, $\overline{\xi_i}$ is the mean of the random variable ξ_i, and $y_{EM}(\,)$ represents the result of EMC simulation at a certain point. In 2022, the Clustering Method of Moments is proposed to improve the accuracy of the MoM mean prediction, and specific details can be found in [17].

The formula for calculating the standard deviation of the MoM is as follows.

$$\sigma(y) = \sqrt{\left(\frac{dy}{d\xi_1}\right)^2 \sigma_{\xi_1}^2 + \cdots + \left(\frac{dy}{d\xi_i}\right)^2 \sigma_{\xi_i}^2 + \cdots + \left(\frac{dy}{d\xi_n}\right)^2 \sigma_{\xi_n}^2} \tag{3}$$

$$\frac{dy}{d\xi_i} = \frac{y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i} + h, \cdots, \overline{\xi_n}\right) - y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i}, \cdots, \overline{\xi_n}\right)}{h} \tag{4}$$

Among them, $\sigma_{\xi_i}^2$ represents the variance of each random variable ξ_i, and $\sigma(y)$ is the standard deviation result of the overall uncertainty analysis. $\frac{dy}{d\xi_i}$ refers to the sensitivity of random variable ξ_i, which is given by formula (4). h refers to the small perturbation. The formula uses the principle of difference form approximation instead of differential form. It can be seen that the accuracy of sensitivity results will directly affect the accuracy of standard deviation prediction results, which is determined by the accuracy of the approximate difference form in formula (4).

Next, based on the Taylor formula expansion of one-dimensional random variables, the accuracy of formula (4) is analyzed in this section. The Taylor expansion formula is as follows.

$$y(\xi_1) = y(\overline{\xi_1}) + \left.\frac{dy}{d\xi_1}\right|_{\xi_1=\overline{\xi_1}} \times \left(\xi_1 - \overline{\xi_1}\right) + \left.\frac{d^2y}{d\xi_1^2}\right|_{\xi_1=\overline{\xi_1}} \times \frac{\left(\xi_1 - \overline{\xi_1}\right)^2}{2} + o\left[\left(\xi_1 - \overline{\xi_1}\right)^2\right] \tag{5}$$

Among them, the random variable ξ_1 is considered as the independent variable, and y is the dependent variable. If $\xi_1 = \overline{\xi_1} + h$, formula (5) can be transformed into the following form.

$$y\left(\overline{\xi_1} + h\right) = y(\overline{\xi_1}) + \frac{dy}{d\xi_1} \times h + \frac{d^2y}{d\xi_1^2} \times \frac{h^2}{2} + o\left(h^2\right) \tag{6}$$

Among them, $o(\,)$ represents high-order infinitesimal, which can be organized as follows

$$\frac{y\left(\overline{\xi_1} + h\right) - y\left(\overline{\xi_1}\right)}{h} = \frac{dy}{d\xi_1} + \frac{1}{2}\frac{d^2y}{d\xi_1^2} h + o(h) \tag{7}$$

By organizing again, it can be concluded that

$$\frac{dy}{d\xi_1} = \frac{y\left(\overline{\xi_1} + h\right) - y(\overline{\xi_1})}{h} + o(1) \tag{8}$$

Therefore, the sensitivity calculation accuracy based on formula (4) is $o(1)$, which can also characterize the calculation accuracy of MoM standard deviation prediction. Due to 1 EMC simulation conducted in formula (2) and n EMC simulations conducted in formula (4), a total of $n + 1$ EMC simulations are required.

3. RANDOM VARIABLE SENSITIVITY CALCULATION BASED ON THE CN-MOM

The CN-MoM is used in this section to improve formula (4), which can enhance the accuracy of sensitivity calculation results. The input random variable ξ_i is declared in complex form by the CN-MoM, and complex perturbation is applied to sensitivity calculation. For more details on the CN-MoM, please refer to [20].

According to formula (6), if the perturbation h is converted into the complex form hi, the Taylor expansion formula in the following form can be obtained, where i is the imaginary unit.

$$y\left(\overline{\xi_1} + ih\right) = y(\overline{\xi_1}) + \frac{dy}{d\xi_1} \times ih - \frac{d^2y}{d\xi_1^2} \times \frac{h^2}{2} - \frac{d^3y}{d\xi_1^3} \times \frac{ih^3}{6} + o(h^3) \tag{9}$$

If the imaginary parts are taken on both sides of Equation (9), the following formula can be obtained.

$$\text{Im}\left[y\left(\overline{\xi_1} + ih\right)\right] = \frac{dy}{d\xi_1} \times h - \frac{d^3y}{d\xi_1^3} \times \frac{h^3}{6} + o\left(h^4\right) \tag{10}$$

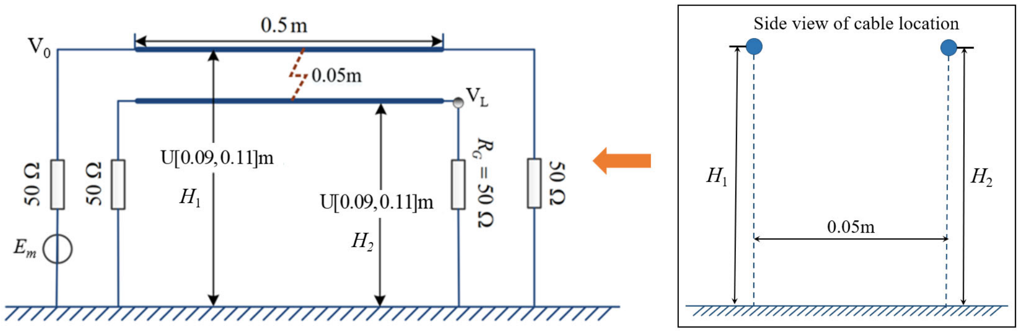

FIGURE 1. Example of parallel cable crosstalk prediction in References [21] and [22].

Divide the two sides of the equation by h and organize

$$\frac{\text{Im}\left[y\left(\overline{\xi_1}+ih\right)\right]}{h}=\frac{dy}{d\xi_1}-\frac{1}{6}\frac{d^3y}{d\xi_1^3}\times h^2+o\left(h^3\right) \tag{11}$$

After reorganization, it can be concluded that

$$\frac{dy}{d\xi_1}=\frac{\text{Im}\left[y\left(\overline{\xi_1}+ih\right)\right]}{h}+o(h) \tag{12}$$

Therefore, if formula (12) is used for sensitivity estimation, the accuracy can be improved from $o(1)$ to $o(h)$. The sensitivity calculation formula in the form of formula (4) is as follows.

$$\frac{dy}{d\xi_i}=\frac{\text{Im}\left[y_{EM}\left(\overline{\xi_1},\cdots,\overline{\xi_i}+hi,\cdots,\overline{\xi_n}\right)\right]}{h} \tag{13}$$

The standard deviation calculation formula remains unchanged, and formula (3) can still be used.

In summary, by comparing formula (8) with formula (12), it can be strictly proven in mathematical mechanics that deriving the CN-MoM can improve the sensitivity calculation accuracy from $o(1)$ to $o(h)$, and the accuracy of sensitivity results directly affects the accuracy of standard deviation prediction, so the CN-MoM can improve the accuracy of MoM standard deviation prediction results. Like MoM, the CNMoM also requires $n+1$ EMC simulations.

4. CALCULATION EXAMPLE

The parallel cable crosstalk prediction example considering geometric randomness shown in Figure 1 is adopted in this section to verify the effectiveness of the CN-MoM. The source voltage in this example is a sinusoidal alternating current with an amplitude of 1 V. This example is a standard example from [21, 22], assuming that the height of parallel cables is an uncertain input parameter, described by the following random variable model:

$$\begin{cases} H_1(\xi_1)=0.1+0.01\times\xi_1\,[\text{m}] \\ H_2(\xi_2)=0.1+0.01\times\xi_2\,[\text{m}] \end{cases} \tag{14}$$

Among them, ξ_1 and ξ_2 are uniformly distributed random variables within the interval $[-1,1]$. The horizontal distance between the two cables is 0.05 m. The frequency range calculated in this example is 1 MHz to 100 MHz, and the output result is the far end crosstalk voltage V_L.

H is the geometric position of the cable, and changes in geometric parameters of H will be converted into changes in electrical parameters such as capacitance and inductance, which in turn affect the results of crosstalk. Taking inductance as an example, the parasitic inductance matrix L per unit length of a cable is:

$$L=\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} \frac{\mu_0}{2\pi}\,\text{In}\left(\frac{2H_1}{r_A}\right) & \frac{\mu_0}{4\pi}\,\text{In}\left(1+\frac{4H_1H_2}{S^2}\right) \\ \frac{\mu_0}{4\pi}\,\text{In}\left(1+\frac{4H_1H_2}{S^2}\right) & \frac{\mu_0}{2\pi}\,\text{In}\left(\frac{2H_2}{r_A}\right) \end{bmatrix} \tag{15}$$

H_1 and H_2 represent the heights of conductor 1 and conductor 2 relative to the grounding plane, respectively. r_A and r_B represent the radius of conductor 1 and conductor 2, respectively. S represents the distance between conductor 1 and conductor 2. μ_0 represents the vacuum magnetic permeability. L_{11} and L_{22} represent the unit length selfinductance of conductor 1 and conductor 2, respectively. L_{12} and L_{21} represent the unit length mutual inductance between conductor 1 and conductor 2, respectively. From the above equation, it can be concluded that the parasitic inductance matrix L per unit length of the cable will change with H. Similarly, the parasitic capacitance matrix C per unit length will also change with H.

This example uses the Finite Difference Time Domain (FDTD) method to solve the ideal multi conductor transmission line equation, which is shown in Equation (16):

$$\begin{cases} \frac{\partial}{\partial z}V(z,t)=-L\frac{\partial}{\partial t}I(z,t) \\ \frac{\partial}{\partial z}I(z,t)=-C\frac{\partial}{\partial t}V(z,t) \end{cases} \tag{16}$$

where L and C are the inductance and capacitance per unit length, and $I(z,t)$ and $V(z,t)$ are the current and voltage at time t and at position z, respectively. From formula (16), it can be seen that when L and C change with the geometric parameter H of the cable, the results of formula (16) also change, indicating that the change in H can affect the results of crosstalk.

This example implements a deterministic simulation solver for the FDTD method in MATLAB environment and replicates

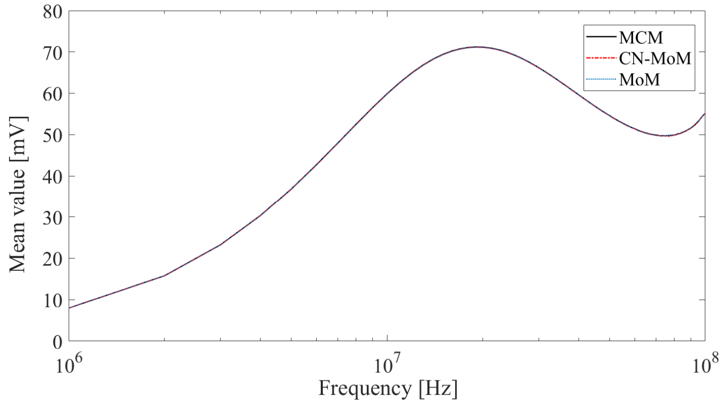

FIGURE 2. Prediction results of V_L mean value from 1 MHz to 100 MHz.

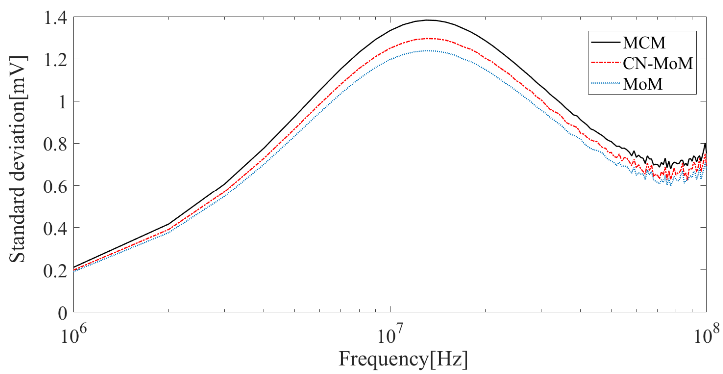

FIGURE 3. Prediction results of V_L standard deviation from 1 MHz to 100 MHz.

the parallel cable crosstalk calculation in mature literature to simulate the capacitance coupling effect and inductance coupling effect between cables, accurately calculating the remote crosstalk voltage. After comparison, the deterministic simulation results are consistent with the results in [21, 22], ensuring the reliability and representativeness of the results.

Figure 2 and Figure 3 show the results obtained by using the MCM, CN-MoM, and MoM to calculate the mean and standard deviation of V_L from 1 MHz to 100 MHz, respectively. The MCM calculation result is used as the standard to evaluate the effectiveness of the mean and standard deviation calculation curves using the Feature Selection Verification (FSV) method. The FSV method is a kind of numerical calculation of the validation rating recommended in IEEE Standard 1597.1, which can give qualitative and quantitative results with regard to the agreement between data sets. It can avoid the subjectivity and non-communicability of human judgment. By using FSV, the total global difference measure (GDM) values between MCM and other methods are calculated. Total-GDM, a value which provides a quantitative description in FSV, indicates the validity of simulation results [23, 24]. There exists a one-to-one correspondence between total-GDM and the qualitative description, as shown in Table 1. The mean and standard deviation FSV results of different methods, as well as the required deterministic EMC simulations, are shown in Table 2. The MCM conducts 1000 deterministic EMC simulations, while the MoM and CN-MoM only require 3 deterministic EMC simulations because the input dimension n of this example is 2, indicating that the MoM and CN-MoM have extremely high computational efficiency compared to the MCM. When calculating the mean, the mean prediction results of the 3 methods almost overlap. When calculating the standard deviation, the FSV value of the CN-MoM is 0.1572, and the FSV value of the MoM is 0.1807, indicating that the calculation accuracy of the CN-MoM is higher than that of the MoM when predicting the standard deviation, achieving a significant improvement on the MoM.

TABLE 1. Relationship between total-GDM and quantitative description.

Total-GDM (quantitative)	FSV interpretation (qualitative)
Less than 0.1	Excellent
Between 0.1 and 0.2	Very Good
Between 0.2 and 0.4	Good
Between 0.4 and 0.8	Fair
Between 0.8 and 1.6	Poor
Greater than 1.6	Very Poor

TABLE 2. The mean and standard deviation FSV results of different methods and their required deterministic EMC simulation times.

Uncertainty analysis method	FSV results of standard deviation	FSV results of mean	Deterministic simulation times
MCM	-	-	1000
CN-MoM	0.1572	0.0115	3
MoM	0.1807	0.0143	3

5. CONCLUSION

Based on the CN-MoM, a novel random variable sensitivity calculation method for traditional nonembedded uncertainty analysis method named MoM is proposed in this paper, which improves the reliability of uncertainty analysis results by improving the accuracy of MoM's standard deviation prediction results. Through the theoretical derivation of Taylor's formula expansion, it is verified that the CN-MoM can improve the sensitivity calculation accuracy from $o(1)$ to $o(h)$. In the parallel cable crosstalk prediction example, the FSV method is used to quantitatively verify the effectiveness of the CN-MoM in improving the accuracy of standard deviation prediction results, proving that the EMC simulation uncertainty analysis method proposed in this paper can significantly improve the accuracy of the MoM in calculating standard deviation results while maintaining simulation efficiency.

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