

DOA Estimation Based on Extended Array Using Cyclic Spectral Components

Zhangsheng Wang*, Shuiwei Liu, and Lei Tang

Gannan Normal University, Ganzhou 340000, China

ABSTRACT: The paper addresses how to improve the degree of freedom of array for DOA (direction of arrival) estimation. According to the DOA estimation model for cyclostationary signal, a method of constructing virtual extended array based on two cyclic spectral components using a single uniform linear array and a method of estimating DOA based on the virtual array are proposed. Firstly, two array receiving data matrices of uniform linear arrays are constructed by using cyclic autocorrelation function of two different cyclic frequencies. Then, the array receiving data matrix of the virtual nested array is constructed by the Kronecker product of the two linear array receiving data matrices. Through virtual expansion, an M^2 -dimensional array receiving data matrix is obtained based on a uniform linear array of M -array elements, so that the DOAs of $M^2 - 1$ sources can be estimated. It breaks the limitation of array degrees of freedom. Finally, the direction finding model for the virtual nested array is formulated, and the compressed sensing algorithm is used to estimate the DOAs of sources. Through computational simulation experiments, the performance of the algorithm is verified.

1. INTRODUCTION

With the development of direction finding technology, array antennas are widely used in wireless positioning, satellite navigation, radar, and many other fields, but the performance of conventional direction finding algorithms is limited by the number of array elements of antenna arrays, that is, the degree of freedom of antenna arrays [1]. For the antenna array of an M -matrix, the degree of freedom of the antenna array is $M - 1$. When there are few antenna array elements, direction finding technology cannot be applied to the complex electromagnetic environment with a larger number of signals. In recent years, virtual array technology has attracted widespread attention from scholars because it has expanded against arrays by constructing virtual array elements. While increasing the degree of freedom of the array and improving the anti-interference performance of the array, it can also expand the aperture of the array and improve the resolution of angle. It has been widely concerned by scholars and becomes a hot research topic in array signal processing [2, 3].

Virtual array technology originated in the mid-1990s. Its purpose is to improve the degree of freedom of array by using virtual array elements under the condition of specific array size, so as to use fewer array elements to process more signals and reduce the complexity of equipment. The commonly used array expansion algorithms include virtual array expansion algorithm based on interpolation transform [4–6], virtual array expansion algorithm based on conjugate virtual array [7], virtual array expansion algorithm based on high-order cumulant [8], etc. The algorithm based on virtual interpolation has the problem of angle sensitivity. When the direction of the incoming wave of interference is not within the range of interpolated variation,

the performance of the algorithm will decrease. Ni et al. proposed a beamforming method based on a conjugate virtual array [9]. The algorithm used the received data of the antenna array to construct the received data matrix by conjugation and realized the virtual extension of the array, but the algorithm was only applied to non-circular signals with real-valued characteristics. The virtual array expansion algorithm based on high-order cumulants can estimate more signal sources than the actual number of array elements, which improves the spatial resolution, but the complexity is too high to be applied in practical engineering. In order to avoid calculating high-order cumulants, scholars have proposed a virtual array construction method based on the properties of Khatri-Rao (KR) product. Hiroyoshi et al. proposed a virtual array technology based on Khatri-Rao product and applied it to ocean surface current radar to minimize the array aperture without reducing the angular resolution [10]. Zhu et al. proposed a single snapshot direction of arrival (DOA) estimation method based on Khatri-Rao product. Using the joint sparse representation of signal subspace, the estimation accuracy is higher than that of traditional Multiple Signal Classification (MUSIC) and Orthogonal Matching Pursuit (OMP) algorithms at low signal-to-noise ratios (SNRs) [11]. Wang proposed a virtual array construction and DOA estimation method based on Khatri-Rao subspace of signal cyclostationarity. Compared with the method based on high-order cumulants, the algorithm has less running time [12].

In addition, in order to further improve the utilization of virtual array elements, scholars have proposed the method of constructing virtual arrays by nonuniform arrays such as coprime arrays and nested arrays [13, 14]. Aimin proposed a DOA estimation method based on a complementary coprime array and verified the improvement of degree of freedom, angle resolu-

* Corresponding author: Zhangsheng Wang (wzs7978@aliyun.com).

tion, and accuracy of complementary coprime array through simulation experiments [15]. Aiming at the problem of coherent source direction finding, Song et al. combined compressed sensing technology with virtual array aperture theory and used convex optimization method to solve the target azimuth. This method has higher spatial resolution than traditional methods under the conditions of low SNR, coherent source, and small snapshot number [16].

In this paper, based on uniform linear array, two different virtual uniform linear arrays are constructed by using two cyclostationary components for the cyclostationary signal. According to the nested virtual array theory, the Kronecker product of the received data matrices of the two virtual uniform linear arrays is proposed as the virtual extended received data matrix, and then the compressed sensing algorithm is used to estimate the DOA of the signal. The algorithm extends the degree of freedom of the array and can estimate the DOAs of $M^2 - 1$ signals with an array composed of M -array elements.

2. CYCLE CYCLOSTATIONARY SIGNAL DIRECTION FINDING MODEL

Assuming an array composed of M -array elements, the position and characteristics of each array element are determined. Considering K far-field signals $s_1(t), s_2(t), \dots, s_K(t)$ incident to the M -array elements of the antenna array from different directions $\theta_1, \theta_2, \dots, \theta_k$, the received signal of the m th array element can be expressed as

$$x_m(t) = \sum_{k=1}^K s_k(t + \tau_m(\theta_k)) + e_m(t). \quad (1)$$

where $s_k(t)$ is signals incident from the k th direction θ_k ; $e_m(t)$ is the random noise on the m th array element; $\tau_m(\theta_k)$ is the time delay of $s_k(t)$ reaching the m th array element.

For uniform linear arrays in which the spacing is d , $\tau_m(\theta_k) = (m - 1)d \sin \theta_k / c$, and c is the speed of light.

$$x_m(t) = \sum_{k=1}^K s_k(t + (m - 1)d \sin \theta_k / c) + e_m(t). \quad (2)$$

Assuming that K far-field signals are cyclostationary signals with cyclic frequency α , noise signal is white Gaussian noise, and there are no correlations among K far-field signals and between K far-field signals and noise at cyclic frequency α . By cyclostationarity theory it is known that

$$\mathbf{R}_{s_k s_i}^\alpha(\tau) = \mathbf{R}_{s_k e_i}^\alpha(\tau) = \mathbf{R}_{e_k e_i}^\alpha(\tau) = \mathbf{R}_{e_k}^\alpha(\tau) = 0, \quad k \neq i. \quad (3)$$

The cyclic autocorrelation function of the received signal $x_m(t)$ of the m th array element is:

$$\begin{aligned} R_{x_m}^\alpha(\tau) = & \left\langle \sum_{k=1}^K s_k(t + \tau/2 + \tau_m(\theta_k)) s_k(t - \tau/2 \right. \\ & \left. + \tau_m(\theta_k)) e^{-j2\pi\alpha t} \right\rangle = \sum_{k=1}^K R_{s_k}^\alpha e^{-j2\pi\alpha\tau_m(\theta_k)} \quad (4) \end{aligned}$$

Set:

$$\mathbf{R}_X^\alpha(\tau) = [R_{x_1}^\alpha(\tau), \dots, R_{x_m}^\alpha(\tau), \dots, R_{x_M}^\alpha(\tau)]^T \quad (5)$$

Then:

$$\mathbf{R}_X^\alpha(\tau) = \mathbf{A}(\alpha, \theta) \mathbf{R}_S^\alpha(\tau) \quad (6)$$

in which, $\mathbf{R}_S^\alpha(\tau) = [\mathbf{R}_{s_1}^\alpha(\tau), \dots, \mathbf{R}_{s_m}^\alpha(\tau), \dots, \mathbf{R}_{s_M}^\alpha(\tau)]^T$, $A(\alpha, \theta) = [a_1(\alpha, \theta_1), \dots, a_k(\alpha, \theta_k), \dots, a_K(\alpha, \theta_K)]$, $a_k(\alpha, \theta_k) = [e^{j2\pi\alpha\tau_1(\theta_k)}, e^{j2\pi\alpha\tau_m(\theta_k)}, \dots, e^{j2\pi\alpha\tau_M(\theta_k)}]^T$. In the above, $A(\alpha, \theta)$ can be regarded as the steering vector of the antenna array to the incident signal in the cyclic frequency domain, and $R_X^\alpha(\tau)$ is the pseudo-received data matrix of the antenna array in the cyclic frequency domain.

3. CONSTRUCTION METHOD OF VIRTUAL NESTED ARRAY BASED ON CYCLIC SPECTRAL COMPONENT

The communication signal is usually realized by the signal to be transmitted to modulate a parameter of the periodic signal. For example, the amplitude modulation, frequency modulation, and phase modulation of the sinusoidal carrier, as well as the pulse amplitude, pulse width, and pulse position modulation of the periodic pulse, will produce a signal with cyclostationarity. Autocorrelation function of periodic cyclostationary signal:

$$R_x(t, \tau) = E[x(t)x^*(t - \tau)] = E[x(t + mT_0)x^*(t + mT_0 - \tau)] \quad (7)$$

It can be seen from the above formula that the periodic cyclostationary signal has multiple proportional cyclic frequencies. Assume that the incident signal has a periodic stationary characteristic with a cyclic frequency of α_1, α_2 and has the following relationship

$$\alpha_2 = P\alpha_1, \quad (8)$$

where $P \leq M$. From the direction finding model, the cyclic autocorrelation receiving matrix based on the cyclic frequency α_1 is

$$\mathbf{R}_X^{\alpha_1}(\tau) = \mathbf{A}(\alpha_1, \theta) \mathbf{R}_S^{\alpha_1}(\tau) \quad (9)$$

where

$$\mathbf{A}(\alpha_1, \theta) = [\mathbf{a}(\alpha_1, \theta_1), \dots, \mathbf{a}(\alpha_1, \theta_k), \dots, \mathbf{a}(\alpha_1, \theta_K)] \quad (10)$$

$$a(\alpha_1, \theta_k) = [e^{j2\pi\alpha_1\tau_1(\theta_k)}, e^{j2\pi\alpha_1\tau_m(\theta_k)}, \dots, e^{j2\pi\alpha_1\tau_M(\theta_k)}]^T \quad (11)$$

The cyclic autocorrelation receiving matrix based on the cyclic frequency α_2 is

$$\mathbf{R}_X^{\alpha_2}(\tau) = \mathbf{A}(\alpha_2, \theta) \mathbf{R}_S^{\alpha_2}(\tau) \quad (12)$$

where

$$\mathbf{A}(\alpha_2, \theta) = [\mathbf{a}(\alpha_2, \theta_1), \dots, \mathbf{a}(\alpha_2, \theta_k), \dots, \mathbf{a}(\alpha_2, \theta_K)] \quad (13)$$

$$a(\alpha_2, \theta_k) = [e^{j2\pi\alpha_2\tau_1(\theta_k)}, e^{j2\pi\alpha_2\tau_m(\theta_k)}, \dots, e^{j2\pi\alpha_2\tau_M(\theta_k)}]^T \quad (14)$$

Substitute Equation (8) into Equation (14)

$$a(\alpha_2, \theta_k) = [e^{j2\pi P\alpha_1\tau_1(\theta_k)}, e^{j2\pi P\alpha_1\tau_m(\theta_k)}, \dots, e^{j2\pi P\alpha_1\tau_M(\theta_k)}]^T \quad (15)$$

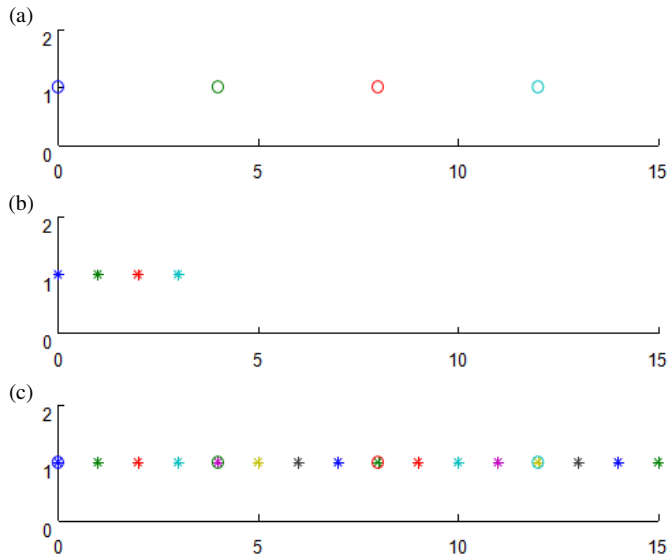


FIGURE 1. The virtual array, (a) array with $d_1 = c/(2\alpha_1)$; (b) array with $d_2 = c/(2\alpha_2)$; (c) extended array with α_1, α_2 .

Assuming a uniform linear array consisting of M -array elements as shown in Figure 1(a), the array element spacing d is equal to half the wavelength of signal with the corresponding frequency α_1 . Then $a(\alpha_2, \theta_k)$ is equivalent to the steering vector of the signal incident on a uniform linear array with an array element spacing of d/P as shown in Figure 1(b). For the convenience of discussion, we assume that $P = M$. For the case of $P < M$, we only need to select P array elements. Set

$$\mathbf{R}_X^\alpha(\tau) = \mathbf{R}_X^{\alpha_1}(\tau) \otimes \mathbf{R}_X^{\alpha_2}(\tau) \quad (16)$$

Based on the properties of Kronecker product $\mathbf{R}_X^\alpha(\tau)$ can be obtained

$$\begin{aligned} \mathbf{R}_X^\alpha(\tau) &= (\mathbf{A}(\alpha_1, \theta) \otimes \mathbf{A}(\alpha_2, \theta))(\mathbf{R}_S^{\alpha_1}(\tau) \otimes \mathbf{R}_S^{\alpha_2}(\tau)), \\ &= \mathbf{A}(\alpha, \theta)\mathbf{R}_S^\alpha(\tau), \end{aligned} \quad (17)$$

where

$$\mathbf{A}(\alpha, \theta) = \mathbf{A}(\alpha_1, \theta) \otimes \mathbf{A}(\alpha_2, \theta) \quad (18)$$

$$\mathbf{R}_S^\alpha(\tau) = \mathbf{R}_S^{\alpha_1}(\tau) \otimes \mathbf{R}_S^{\alpha_2}(\tau) \quad (19)$$

In the above formula (17), $\mathbf{A}(\alpha, \theta)$ can be equivalent to the steering vector of the virtual array, and the virtual array is shown in Figure 1(c).

4. DOA ESTIMATION ALGORITHM BASED ON COMPRESSED SENSING

Assuming that the number of snapshots of array receiving data is N , and the array receiving data matrix can be constructed by using the virtual array receiving model as

$$\begin{aligned} \mathbf{R}_X(\tau) &= [\mathbf{R}_X^\alpha(T_s), \mathbf{R}_X^\alpha(2T_s), \dots, \mathbf{R}_X^\alpha(NT_s)], \\ &= \mathbf{A}(\theta) [\mathbf{R}_S^\alpha(T_s), \mathbf{R}_S^\alpha(2T_s), \dots, \mathbf{R}_S^\alpha(NT_s)] \end{aligned} \quad (20)$$

According to the compressed sensing theory, the DOA of the signal is divided into Q grids, that is

$$\Theta = [\theta_1, \theta_2, \dots, \theta_Q] \quad (21)$$

Thus, the array model $\hat{A}(\alpha, \Theta)$ of the corresponding grid is obtained, and its column vector corresponds to the steering vector of each grid point.

For Equation (20) multi-delay parameter model, θ can be solved by sparse recovery algorithm such as ℓ_1 -SVD algorithm. Singular value decomposition (SVD) of $R_x(\tau)$ can be attained

$$R_x(\tau) = ULV^H \quad (22)$$

For K incident signals, construct an $M^2 \times K$ matrix Y_{SV}

$$Y_{SV} = ULD_K = R_X(\tau)VD_K, \quad (23)$$

Y_{SV} contains most of the signal energy of $R_X(\tau)$, where $D_K = [I_K \mathbf{0}]$, I_K is a unitary matrix of $K \times K$, and $\mathbf{0}$ is a unitary matrix of $K \times N - K$. Set

$$P_{SV} = PVD_K \quad (24)$$

$$N_{SV} = NVD_K \quad (25)$$

We can get

$$Y_{SV} = \mathbf{A}(\theta)P_{SV} + N_{SV} \quad (26)$$

So the spatial spectrum \hat{P}_k can be attained by solving the following equation

$$\hat{P}_k = \arg \min \left\| Y_{SV} - \hat{A}(\alpha, \theta)\hat{P}_k \right\|_f^2 + \eta \left\| \hat{P}_k \right\|_1 \quad (27)$$

The above ℓ_1 -SVD algorithm can be solved quickly by CVX toolbox. The method of using compressed sensing algorithm to get the DOA estimation of the signal for the virtual array is called Virtual Array-ACS-CS (VA-ACS-CS) algorithm.

The steps of VA-ACS-CS algorithm are as follows:

Step 1: For selected cyclic frequencies α_1, α_2 (assuming known or estimated by an existing algorithm), for the received signal of each array element, calculate the cyclic autocorrelation function $R_{x_m}^{\alpha_1}, R_{x_m}^{\alpha_2}$ corresponding to the cyclic frequency.

Step 2: Use $R_{x_m}^{\alpha_1}, R_{x_m}^{\alpha_2}$ to construct new array receiving data matrix $R_X(\alpha_1), R_X(\alpha_2)$ and construct the cyclic autocorrelation matrix $R_X^\alpha(\tau)$ of the virtual array receiving data according to Equation (17).

Step 3: Calculate the pseudo-spectral \hat{P}_k , for the multi-delay model by solving Equation (27).

Step 4: Then search the pseudo-spectrum to attain K maximum values and their corresponding angles, which are the DOAs of the signals.

5. ALGORITHM PERFORMANCE SIMULATION

In this section, the performance of VA-ACS-CS algorithm under different numbers of sources is simulated, and the root mean square error of the algorithm is compared with the existing virtual array expansion algorithm based on quasi-cyclic stationary characteristics, such as DCA-CyclicMUSIC algorithm. Assuming that the antenna array is a uniform linear array with 4-array elements and that the incident signal code rate is 4 Mb/s BPSK modulation signal, the carrier frequency is 10 Mb/s; the cyclic frequency α_1 is 4 MHz; and the cyclic frequency α_2 is 16 MHz. The array element spacing is half of the wavelength corresponding to the cyclic frequency α_1 . Sample the received signal of the array element with a 32 MHz sampling clock. The

snapshots N of each trial is 8192. The root mean square error of the algorithm is attained by 100 Monte-carlo simulations.

5.1. Simulation of DOA Estimation Results When the Number of Sources Is Less Than the Number of Array Elements

Suppose that 2 BPSK modulation signals are incident from -14.48° and 14.48° to a 4-element uniform linear array. Figure 2 shows that the space spectral of algorithm when SNR is 5 dB. It can be seen from Figure 2 that the algorithm can accurately estimate the signal.

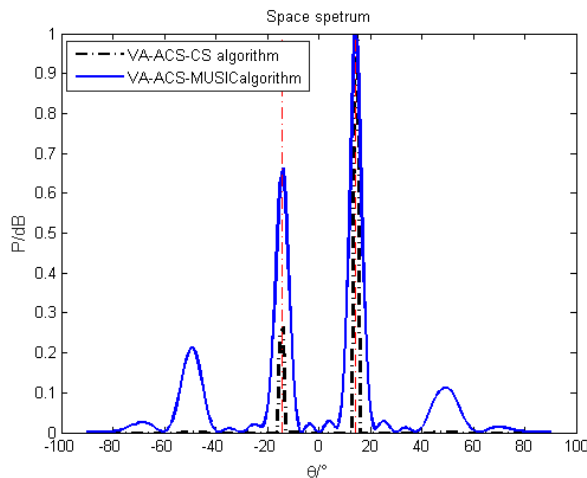


FIGURE 2. The space spectral of algorithm when the number of sources $K = 2$, SNR = 5 dB.

5.2. Simulation of DOA Estimation Results When the Number of Sources Is Greater Than the Number of Array Elements

Assuming that 12 BPSK modulation signals are incident from -61.05° , -48.59° , -38.68° , -26.74° , -14.48° , -7.18° , 7.18° , 14.48° , 26.74° , 38.68° , 48.59° , and 61.05° to a 4-element uniform linear array with a symbol rate of 4 Mb/s. Figure 3 shows the space spectra of algorithm when SNR is 5 dB.

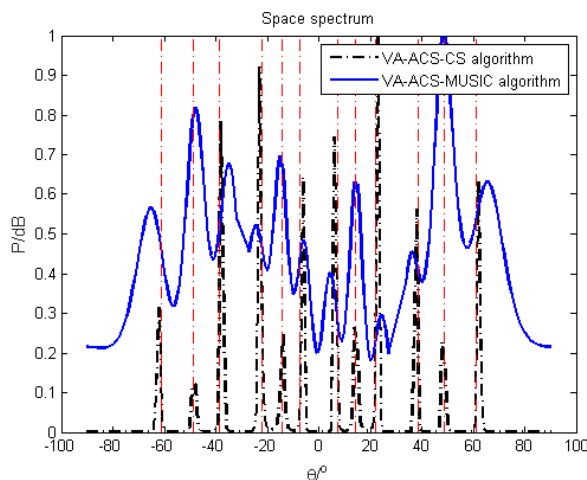


FIGURE 3. The space spectral of algorithm when the number of sources $K = 12$, SNR = 5 dB.

It can be seen from Figure 3 that the virtual array constructed by the cyclic frequency component effectively expands the array degree of freedom, and the algorithm can still accurately estimate the signal when the number of signals is greater than or equal to the number of arrays.

5.3. Simulation of Root Mean Square Error of Different Algorithms

Assume that the 4-BPSK modulation signals are incident from -26.74° , -14.48° , 14.48° , and 26.74° to a 4-element uniform linear array, respectively, and the symbol rate is 4 Mb/s. Figure 4 shows the root-mean-square error (RMSE) of DOA estimation for different algorithms. As shown in Figure 4, the virtual array extended direction finding method proposed in this paper can correctly estimate the DOAs of the signals, and the performance is close to the existing algorithms.

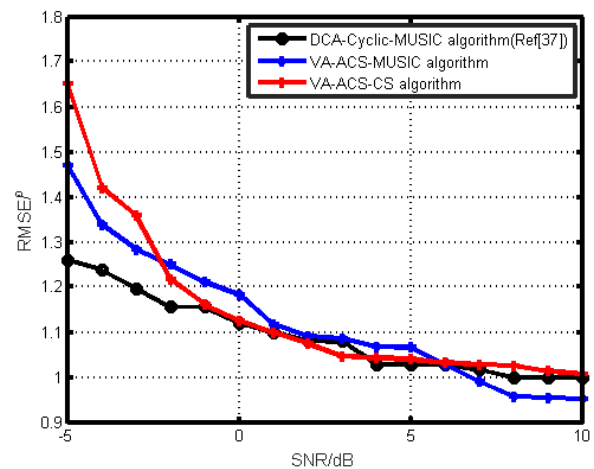


FIGURE 4. The RMSE of DOA estimation for different algorithms.

6. CONCLUSION

In this paper, an array receiving data matrix of the real and virtual uniform linear arrays are constructed by the cyclic autocorrelation function of two different cyclic spectral components of the periodic stationary signal. Then, the virtual nested received data matrices are constructed by using the Kronecker product of two uniform linear received data matrices, and the direction of the incoming wave is estimated by the compressed sensing algorithm. In the case that only a single uniform linear array is needed, the method realizes the expansion of the array elements, thus breaking through the limitation of the array degree of freedom, and realizes the correct estimation of the direction of the incoming wave when the number of array elements is less than the number of sources. For M -uniform linear arrays, the direction of the incoming wave of $M^2 - 1$ signals can be realized. The computer simulation shows that the virtual array extended direction finding method proposed in this paper can correctly estimate the DOA of the signal, and the performance is close to the existing algorithm.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the support from the Key Research and Development Programs of Jiangxi Province, China (No. 20203BBF63043).

REFERENCES

- [1] Gao, J., "Research on virtual array antenna beamforming algorithm," Master Thesis, Heilongjiang University, Harbin, Heilongjiang, China, 2022.
- [2] Lee, H. and J. Chun, "Virtual array response vector for angle estimation of MIMO radar with a wide-band interleaved OFDM signal," *IEEE Communications Letters*, Vol. 25, No. 5, 1539–1543, 2021.
- [3] Imai, S., K. Taguchi, T. Kashiwa, and S. Komatsu, "Estimation of the incoming wave characteristics by MUSIC method using virtual array antenna," *SAE International Journal of Passenger Cars — Electronic and Electrical Systems*, Vol. 8, No. 1, 146–155, 2015.
- [4] Ding, Y., S. Ren, W. Wang, and C. Xue, "DOA estimation based on sum-difference coarray with virtual array interpolation concept," *EURASIP Journal on Advances in Signal Processing*, Vol. 2021, 1–13, 2021.
- [5] Zhou, L., "Research on virtual array beamforming technology based on interpolation transform," Master Thesis, Tianjin University of Technology, Tianjin, China, 2018.
- [6] Li, Y., X. Zhang, F. Gao, W. Wng, and C. Duan, "Application method of adaptive virtual array in deep space exploration," *Space Electronic Technology*, Vol. 17, No. 1, 77–82, 2020.
- [7] Xue, L. and J. Zhang, "Robust beam-forming method based on conjugate virtual array," in *2020 International Conference on Microwave and Millimeter Wave Technology (ICMMT)*, 1–3, Shanghai, China, 2020.
- [8] Sharma, U. and M. Agrawal, " $2q$ th-order cumulants based virtual array of a single acoustic vector sensor," *Digital Signal Processing*, Vol. 123, 103438, 2022.
- [9] Ni, S.-Y., N.-P. Cheng, and Z.-Z. Ni, "Conjugate virtual array beamforming method," *Acta Electronica Sinica*, Vol. 39, No. 9, 2120–2124, 2011.
- [10] Hiroyoshi, Y., I. Hiroyuki, H. Keizo, R. Takuya, and Y. Yoshio, "Experimental study on 2-D surface current velocity estimation of ocean surface current radar using virtual array," *B-Abstracts of IEICE Transactions on Communications (Japanese Edition)*, Vol. 98, No. 9, 1–12, 2015.
- [11] Zhu, J., L. Wang, and Y. Meng, "DOA estimation method based on Khatri-Rao product of virtual array and subspace joint sparse representation," *Computer Measure and Control*, Vol. 25, No. 5, 147–149, 2017.
- [12] Wang, N., "Virtual array construction and DOA estimation of cyclostationary signals based on Khatri-Rao subspace," Master Thesis, Chang'an University, Xi'an, Shaanxi, China, 2016.
- [13] Liu, J., Y. Lu, Y. Zhang, and W. Wang, "DOA estimation with enhanced DOFs by exploiting cyclostationarity," *IEEE Transactions on Signal Processing*, Vol. 65, No. 6, 1486–1496, 2017.
- [14] Li, L., Y. Yu, and H. Han, "A low complexity two-dimensional DOA joint estimation algorithm based on parallel coprime virtual array," *Journal of Electronics & Information Technology*, Vol. 43, No. 6, 1653–1658, 2021.
- [15] Aimin, "Research on DOA estimation algorithm based on virtual hole filling of coprime array," Master Thesis, Nanchang University, Nanchang, Jiangxi, China, 2020.
- [16] Song, H., T. Tang, and J. Qin, "Compressed beamforming direction estimation method based on virtual array," *Journal of Heilongjiang Institute of Engineering*, Vol. 32, No. 2, 32–36, 2018.