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DOA Estimation of Quasi-Stationary Signals Based on a Separated Generalized Nested Array

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ABSTRACT: This paper proposes a sparse array consisting of two separate generalized nested arrays. The unit element-spacing of each generalized nested array can be adjusted to multiple half-wavelengths of the incident signal. By adjusting the element-spacing, the mutual coupling effect can be greatly reduced. For this array, a direction of arrival (DOA) estimation method of quasi-stationary signals has also been proposed. By using the received signals of the separated generalized nested array, a signal subspace is obtained. Then, this subspace is filled into a higher-order signal subspace to avoid angle ambiguity. Using the higher-order signal subspace, DOAs of all signals can be estimated by spectral peak search. Simulation results show that the proposed separated generalized nested array has better performance than the conventional nested array in DOA estimation.

1. INTRODUCTION

Direction of arrival (DOA) estimation technique for spatial signals is widely used in many fields, such as mobile communication and radar positioning. Many classical algorithms including multiple signal classification (MUSIC) [1, 2] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [3] have been proposed based on stationary signals. In fact, perfectly stationary signals are almost nonexistent. However, some non-stationary signals whose statistical properties can remain stable for a certain period are called quasi-stationary signals. At present, there are many methods to estimate the DOAs for quasi-stationary signals, the most common being Khatri-Rao multiple signal classification (KR-MUSIC) [4], tensor modeling [5], and sparse reconstruction [6].

In recent years, sparse arrays have been widely used in DOA estimation. Nested array [7] is one of the popular sparse arrays, which has a higher degree of freedom than the coprime array [8]. Many DOA estimation algorithms [9–11] for quasi-stationary signals by varying nested arrays have also been proposed. However, the traditional nested array includes a uniform subarray with element-spacing being half-wavelength of the incident signal. In practice, there will be a strong mutual coupling effect in this subarray. In [12], a sparse array consisting of multiple uniform arrays with adjustable element-spacing has been proposed and shows excellent performance in reducing mutual coupling. However, it is difficult for this array to use virtual elements to improve the degree of freedom of the array as the traditional sparse array.

To reduce the mutual coupling of an array, a separated generalized nested array with adjustable element-spacing is proposed. Based on this array, a special DOA estimation algorithm for quasi-stationary signals is proposed. The contributions of the paper are twofold: 1) Compared with conventional nested array, the proposed array has strong robustness to mutual coupling due to the adjustable element-spacing; 2) the algorithm based on the proposed nested array shows higher estimation accuracy than the analogous method based on the conventional nested array.

Symbol description In this paper, we use $(\cdot)^T$, $(\cdot)^*$,$(\cdot)^H$, $(\cdot)^+$, $\otimes$ and $E\{\cdot\}$ to define transpose, conjugate, conjugate transpose, Moore-Penrose generalized inverse, Kronecker product and expectation, respectively. We also use $\mathbf{V}(i:j)$ to choose the ith row to the jth row of the matrix $\mathbf{V}$. In addition, $\mathbf{I}_F$ denotes an F-order unit matrix, and $\mathbf{J}_F$ denotes an F-order matrix with the elements on the back-diagonal being 1 and the other elements being 0.

2. DESCRIPTION OF ALGORITHM

2.1. Array Geometry and Data Model

Consider a separated generalized nested array shown in Fig. 1, where p ($p > 1$) is an integer, $d = \lambda/2$, and λ is the wavelength of the incident signal. Let the first element be the reference element. The distances between all the elements and the reference element are 0, pd, $\ldots$, $(N-1)pd$, Np, $(2N+1)pd$, $\ldots$, $[M(N+1)-1]pd$, $[2M(N+1)p-p+1]d$, $[2M(N+1)p+1]d$, $\ldots$, and $[3M(N+1)p-2p+1]d$ in order.

Suppose that the number of signals is K and that the DOA of the kth signal is θ_k ($k = 1, 2, \ldots, K$). The signal vectors received by the two subarrays are $\mathbf{x}_1(t) = [x_{1,1}(t), x_{1,2}(t), \ldots, x_{1,N}(t), x_{1,N+1}(t), x_{1,N+2}(t), \ldots, x_{1,N+M}(t)]^T \in C^{(M+N)\times 1}$ and $\mathbf{x}_2(t) = [x_{2,1}(t), x_{2,2}(t), \ldots, x_{2,N}(t), x_{2,N+1}(t), x_{2,N+2}(t), \ldots, x_{2,N+M}(t)]^T \in C^{(M+N)\times 1}$.

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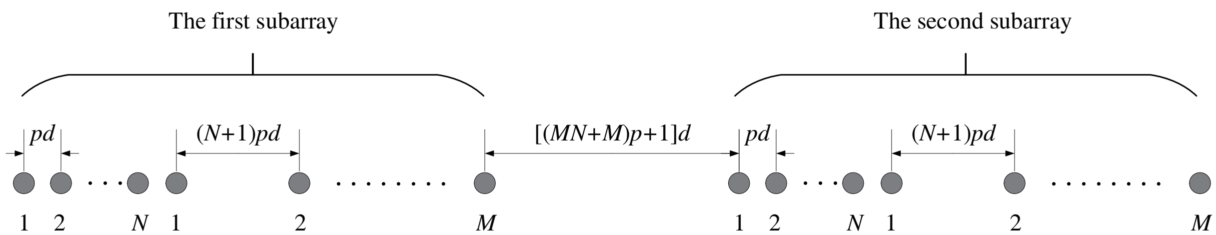

FIGURE 1. Schematic diagram of the proposed separated nested array.

Denote $\mathbf{n}_1 = [n_{1,1}, n_{1,2}, \ldots, n_{1,N+M}]^T \in C^{(M+N)\times 1}$ and $\mathbf{n}_2 = [n_{2,1}, n_{2,2}, \ldots, n_{2,N+M}]^T \in C^{(M+N)\times 1}$ as the uncorrelated noise vectors received by the two subarrays. Then $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ can be expressed by

$$\begin{cases} \mathbf{x}_1(t) = \mathbf{A}_1\mathbf{s}(t) + \mathbf{n}_1(t) \\ \mathbf{x}_2(t) = \mathbf{A}_2\mathbf{s}(t) + \mathbf{n}_2(t) \end{cases} \tag{1}$$

where $\mathbf{A}_1 = [\mathbf{a}_1(\theta_1), \mathbf{a}_1(\theta_2), \ldots, \mathbf{a}_1(\theta_K)] \in C^{(M+N)\times K}$ with

$$\mathbf{a}_1(\theta_k) = [1, \ldots, e^{-i\frac{2\pi}{\lambda}(N-1)pd\sin\theta_k}, e^{-i\frac{2\pi}{\lambda}Npd\sin\theta_k},$$

$$\ldots, e^{-i\frac{2\pi}{\lambda}[M(N+1)-1]pd\sin\theta_k}]^T$$

and$\mathbf{A}_2 = [\mathbf{a}_2(\theta_1), \mathbf{a}_2(\theta_2), \quad \ldots, \mathbf{a}_2(\theta_K)] \in C^{(M+N)\times K}$ with

$$\mathbf{a}_2(\theta_k) = [e^{-i\frac{2\pi}{\lambda}[2M(N+1)p-p+1]d\sin\theta_k},$$

$$\ldots, e^{-i\frac{2\pi}{\lambda}[3M(N+1)p-2p+1]d\sin\theta_k}]^T$$

for $k = 1, 2, \ldots, K$.

Let F be the number of frames and L be the snapshots of each frame. The total number of snapshots is $T = FL$. We denote three covariance vectors as $\mathbf{r}_{f1,1} \in C^{(\frac{D+1}{2})\times 1}$, $\mathbf{r}_{f2,1} \in C^{(\frac{D+1}{2})\times 1}$ and $\mathbf{r}_{f2,2} \in C^{(\frac{D+1}{2})\times 1}$ for the fth frame. The jth components of $\mathbf{r}_{f1,1}$, $\mathbf{r}_{f2,1}$, and $\mathbf{r}_{f2,2}$ are given by

$$\begin{cases} \mathbf{r}_{f1,1}(j) = E\{x_{1,N+1+j_1}(t)x^*_{1,N+2-j_2}(t)\} \\ \mathbf{r}_{f2,1}(j) = E\{x_{2,j_2}(t)x^*_{1,N+M-j_1}(t)\}, \ j = 1, \ldots, \frac{D+1}{2} \\ \mathbf{r}_{f2,2}(j) = E\{x_{2,N+1+j_1}(t)x^*_{1,N+2-j_2}(t)\} \end{cases} \tag{2}$$

where $D = 2M(N+1)-1$, and j is the uniquely decomposed by $j = j_1(N+1) + j_2$ with $1 \le j_2 \le N+1$.

The vectors $\mathbf{r}_{f1,1}$, $\mathbf{r}_{f2,1}$, and $\mathbf{r}_{f2,2}$ can be estimated by the sample data in the fth frame according to (2).

Using $\mathbf{r}_{f1,1}$, we can construct a vector $\mathbf{r}_{f1,2} \in C^{(\frac{D+1}{2})\times 1}$ by

$$\mathbf{r}_{f1,2} = \mathbf{J}_{(\frac{D+1}{2})}\mathbf{r}^*_{f1,1}. \tag{3}$$

We continue to construct a vector $\mathbf{r}_{f1} \in C^{D\times 1}$ as

$$\mathbf{r}_{f1} = \begin{bmatrix} \mathbf{r}_{f1,2}\left(1 : \frac{D-1}{2}\right) \\ \mathbf{r}_{f1,1} \end{bmatrix}. \tag{4}$$

Using $\mathbf{r}_{f2,1}$ and $\mathbf{r}_{f2,2}$, we can construct a vector $\mathbf{r}_{f2} \in C^{D\times 1}$ as

$$\mathbf{r}_{f2} = \begin{bmatrix} \mathbf{r}_{f2,1}\left(1 : \frac{D-1}{2}\right) \\ \mathbf{r}_{f2,2} \end{bmatrix}. \tag{5}$$

Let $\mathbf{r}_{f3} = \mathbf{J}_D\mathbf{r}^*_{f2}$, and we can get a non-redundancy covariance vector $\mathbf{r}_f \in C^{3D\times 1}$ by

$$\mathbf{r}_f = \begin{bmatrix} \mathbf{r}_{f3} \\ \mathbf{r}_{f1} \\ \mathbf{r}_{f2} \end{bmatrix}. \tag{6}$$

Let σ_n^2 be the power of noise and p_{fk} be the power of the kth signal in the fth frame. Then, as [4], a non-redundancy covariance matrix $\mathbf{Y} = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_F] \in C^{3D\times F}$ can be obtained and expressed by

$$\mathbf{Y} = \begin{bmatrix} \mathbf{B}_3 \\ \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \mathbf{\Phi}^T + \begin{bmatrix} 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_n^2 & \sigma_n^2 & \ldots & \sigma_n^2 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 \end{bmatrix}, \tag{7}$$

where

$$\mathbf{\Phi} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{22} & \cdots & p_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ p_{F1} & p_{F1} & \cdots & p_{FK} \end{bmatrix}, \tag{8}$$

$\mathbf{B}_1 = [\mathbf{b}_1(\theta_1), \mathbf{b}_1(\theta_2), \ldots, \mathbf{b}_1(\theta_K)] \in C^{D\times K}$ with

$$\mathbf{b}_1(\theta_k) = [e^{i\frac{2\pi\ [M(N+1)-1]p}{\lambda}d\sin\theta_k}, \ldots, e^{i\frac{2\pi}{\lambda}pd\sin\theta_k}, 1,$$

$$e^{-i\frac{2\pi}{\lambda}pd\sin\theta_k}, \ldots, e^{-i\frac{2\pi[M(N+1)-1]p}{\lambda}d\sin\theta_k}]^T,$$

$\mathbf{B}_2 = [\mathbf{b}_2(\theta_1), \mathbf{b}_2(\theta_2), \ldots, \mathbf{b}_2(\theta_K)] \in C^{D\times K}$ with

$$\mathbf{b}_2(\theta_k) = [e^{-i\frac{2\pi\ [M(N+1)p+1]}{\lambda}d\sin\theta_k}, e^{-i\frac{2\pi\ [(M(N+1)+1)p+1]}{\lambda}d\sin\theta_k},$$

$$\ldots, e^{-i\frac{2\pi\ [(3M(N+1)-2)p+1]}{\lambda}d\sin\theta_k}]^T,$$

and $\mathbf{B}_3 = [\mathbf{b}_3(\theta_1), \mathbf{b}_3(\theta_2), \ldots, \mathbf{b}_3(\theta_K)] \in C^{D\times K}$ with

$$\mathbf{b}_3(\theta_k) = [e^{i\frac{2\pi\ [(3M(N+1)-2)p+1]}{\lambda}d\sin\theta_k},$$

$$\ldots, e^{i\frac{2\pi\ [(M(N+1)+1)p+1]}{\lambda}d\sin\theta_k}, e^{i\frac{2\pi\ [M(N+1)p+1]}{\lambda}d\sin\theta_k}]^T.$$

As in [4], we define an F-order matrix $\mathbf{P}_F^{\perp} = \mathbf{I}_F - (\mathbf{1}_F\mathbf{1}_F^T)/F$, where each component of $\mathbf{1}_F \in C^{F\times 1}$ is equal to 1. Performing eigenvalue decomposition (EVD) of $\mathbf{YP}_F^{\perp}(\mathbf{YP}_F^{\perp})^H$, we can get the eigenvectors of the K maximum eigenvalues to form the signal subspace $\mathbf{U}_s \in C^{3D\times K}$.

Remark: For nested array [7], when the signal subspace is obtained by dealing with the corresponding non-redundancy covariance matrix, all angles can be estimated by using this signal subspace. But for the proposed array, we cannot get unambiguous DOA estimation by using $\mathbf{U}_s$.

2.2. Subspace Repair and DOA Estimation

Firstly, we draw three sub-matrices from $\mathbf{U}_s$ by

$$\begin{cases} \mathbf{U}_1 = \mathbf{U}_s\,(1:D) \\ \mathbf{U}_2 = \mathbf{U}_s\,(D+1:2D) \\ \mathbf{U}_3 = \mathbf{U}_s\,(2D+1:3D) \end{cases}. \tag{9}$$

It is easy to know that we can find an invertible matrix $\mathbf{T} \in C^{K\times K}$ satisfying

$$\begin{cases} \mathbf{U}_1 = \mathbf{B}_3\mathbf{T} \\ \mathbf{U}_2 = \mathbf{B}_1\mathbf{T} \\ \mathbf{U}_3 = \mathbf{B}_2\mathbf{T} \end{cases}. \tag{10}$$

Define six matrices $\mathbf{U}_{11} \in C^{(D-1)\times K}$, $\mathbf{U}_{12} \in C^{(D-1)\times K}$, $\mathbf{U}_{21} \in C^{(D-1)\times K}$, $\mathbf{U}_{22} \in C^{(D-1)\times K}$, $\mathbf{U}_{31} \in C^{(D-1)\times K}$ and $\mathbf{U}_{32} \in C^{(D-1)\times K}$ by

$$\begin{cases} \mathbf{U}_{11} = \mathbf{U}_1(1:D-1) \\ \mathbf{U}_{12} = \mathbf{U}_1(2:D) \\ \mathbf{U}_{21} = \mathbf{U}_2(1:D-1) \\ \mathbf{U}_{22} = \mathbf{U}_2(2:D) \\ \mathbf{U}_{31} = \mathbf{U}_3(1:D-1) \\ \mathbf{U}_{32} = \mathbf{U}_3(2:D) \end{cases}. \tag{11}$$

As [3], we can denote a matrix $\mathbf{\Psi}_1 \in C^{K\times K}$ as

$$\mathbf{\Psi}_1 = \begin{bmatrix} \mathbf{U}_{11} \\ \mathbf{U}_{21} \\ \mathbf{U}_{31} \end{bmatrix}^+ \begin{bmatrix} \mathbf{U}_{12} \\ \mathbf{U}_{22} \\ \mathbf{U}_{32} \end{bmatrix}, \tag{12}$$

and it is easy to know

$$\mathbf{\Psi}_1 = \mathbf{T}^{-1}\mathbf{\Lambda}_1\mathbf{T}, \tag{13}$$

where $\mathbf{\Lambda}_1 = \text{diag}\{e^{-i\frac{2\pi p}{\lambda}d\sin\theta_1}, e^{-i\frac{2\pi p}{\lambda}d\sin\theta_2}, \ldots, e^{-i\frac{2\pi p}{\lambda}d\sin\theta_K}\}$.

Then, we can construct four matrices $\bar{\mathbf{U}}_1 \in C^{D\times K}$, $\bar{\mathbf{U}}_{2-1} \in C^{D\times K}$, $\bar{\mathbf{U}}_{2-2} \in C^{D\times K}$ and $\bar{\mathbf{U}}_3 \in C^{D\times K}$ by

$$\begin{cases} \bar{\mathbf{U}}_1 = \mathbf{U}_1\mathbf{\Psi}_1^D \\ \bar{\mathbf{U}}_{2-1} = \mathbf{U}_2\left(\mathbf{\Psi}_1^{-1}\right)^D \\ \bar{\mathbf{U}}_{2-2} = \mathbf{U}_2\left(\mathbf{\Psi}_1\right)^D \\ \bar{\mathbf{U}}_3 = \mathbf{U}_3\left(\mathbf{\Psi}_1^{-1}\right)^D \end{cases}. \tag{14}$$

Using the four matrices, we can construct a matrix $\mathbf{\Psi}_2 \in C^{K\times K}$ as

$$\mathbf{\Psi}_2 = \begin{bmatrix} \mathbf{U}_1 \\ \bar{\mathbf{U}}_1 \\ \mathbf{U}_2 \\ \bar{\mathbf{U}}_{2-2} \end{bmatrix}^+ \begin{bmatrix} \bar{\mathbf{U}}_{2-1} \\ \mathbf{U}_2 \\ \bar{\mathbf{U}}_3 \\ \mathbf{U}_3 \end{bmatrix}, \tag{15}$$

and it is easy to know

$$\mathbf{\Psi}_2 = \mathbf{T}^{-1}\mathbf{\Lambda}_2\mathbf{T}, \tag{16}$$

where $\mathbf{\Lambda}_2 = \text{diag}\{e^{-i\frac{2\pi d}{\lambda}\sin\theta_1}, e^{-i\frac{2\pi d}{\lambda}\sin\theta_2}, \ldots, e^{-i\frac{2\pi d}{\lambda}\sin\theta_K}\}$.

We introduce p matrices $\mathrm{M}_v = \mathbf{I}_D \otimes \mathbf{e}_v$, $v = 1, 2, \ldots, p$, where $\mathbf{e}_v \in C^{p\times 1}$ is the vector with all elements set to zero except the vth element equal to 1.

Get three matrices $\tilde{\mathbf{U}}_1 \in C^{pD\times K}$, $\tilde{\mathbf{U}}_2 \in C^{pD\times K}$, and $\tilde{\mathbf{U}}_3 \in C^{pD\times K}$ by

$$\begin{cases} \tilde{\mathbf{U}}_1 = \sum\limits_{v=1}^{p} \mathbf{M}_v\mathbf{U}_1\mathbf{\Psi}_2^{v-1} \\ \tilde{\mathbf{U}}_2 = \sum\limits_{v=1}^{p} \mathbf{M}_v\mathbf{U}_2\mathbf{\Psi}_2^{v-2} \\ \tilde{\mathbf{U}}_3 = \sum\limits_{v=1}^{p} \mathbf{M}_v\mathbf{U}_3\mathbf{\Psi}_2^{v-3} \end{cases}. \tag{17}$$

Combining $\tilde{\mathbf{U}}_1$, $\tilde{\mathbf{U}}_2$ and $\tilde{\mathbf{U}}_3$, we can obtain the repaired signal subspace $\tilde{\mathbf{U}} \in C^{3pD\times K}$ by

$$\tilde{\mathbf{U}} = \begin{bmatrix} \tilde{\mathbf{U}}_1 \\ \tilde{\mathbf{U}}_2 \\ \tilde{\mathbf{U}}_3 \end{bmatrix}. \tag{18}$$

Synthesizing (9), (16), and (17), it is easy to know

$$\tilde{\mathbf{U}} = \mathbf{CT}, \tag{19}$$

where $\mathbf{C} = [\mathbf{c}(\theta_1), \mathbf{c}(\theta_2), \ldots, \mathbf{c}(\theta_K)] \in C^{3pD\times K}$ with

$$\mathbf{c}(\theta_k) = [e^{i\frac{2\pi}{\lambda}[\frac{(3D-1)p}{2}+1]d\sin\theta_k}, e^{i\frac{2\pi}{\lambda}[\frac{(3D-1)p}{2}]d\sin\theta_k}, \ldots, e^{-i\frac{2\pi}{\lambda}[\frac{(3D+1)p}{2}-2]d\sin\theta_k}]^T. \tag{20}$$

Performing Schmidt orthogonalization of the columns of $\tilde{\mathbf{U}}$, we can obtain $\tilde{\mathbf{U}}^{\mathbf{o}} \in C^{3pD\times K}$. Similar to [1, 2, 11], we denote a cost function as

$$f(\theta) = \frac{1}{\mathbf{c}^H(\theta)(\mathbf{I}_{3pD} - \tilde{\mathbf{U}}^{\mathbf{o}}(\tilde{\mathbf{U}}^{\mathbf{o}})^H)\mathbf{c}(\theta)}, \tag{21}$$

where $\mathbf{c}(\theta) = [e^{i\frac{2\pi}{\lambda}[\frac{(3D-1)p}{2}+1]d\sin\theta}, e^{i\frac{2\pi}{\lambda}[\frac{(3D-1)p}{2}]d\sin\theta}, \ldots, e^{-i\frac{2\pi}{\lambda}[\frac{(3D+1)p}{2}-2]d\sin\theta}]^T$. We can get the DOAs of all signals by searching the spectral peaks of $f(\theta)$.

3. SIMULATION

In this section, we compare the performance of the proposed separated generalized nested array with the nested array [7] in DOA estimation. For the nested array [7], we suppose that the signal subspace is also obtained by dealing with the non-redundancy covariance matrix. The number of elements for the

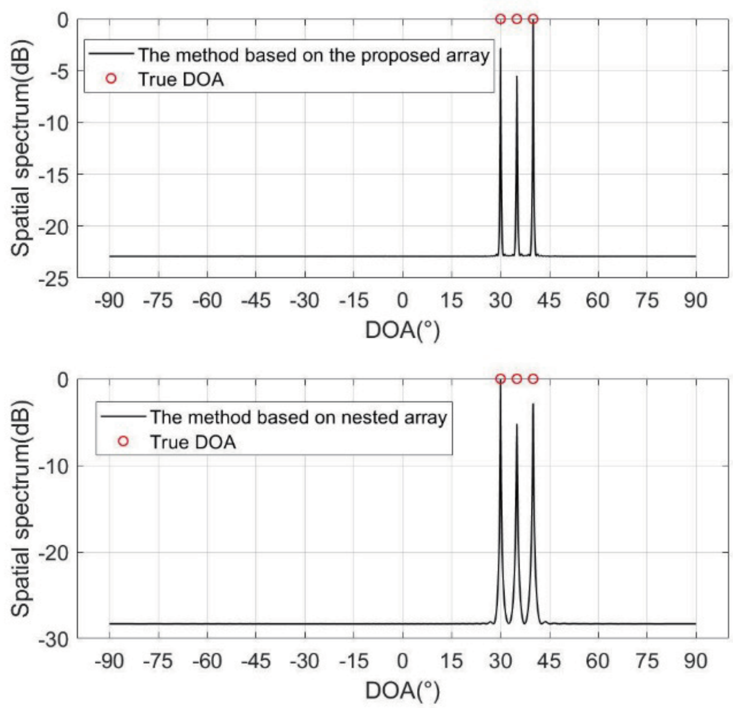

FIGURE 2. Comparison of spatial spectrums with mutual coupling being ignored.

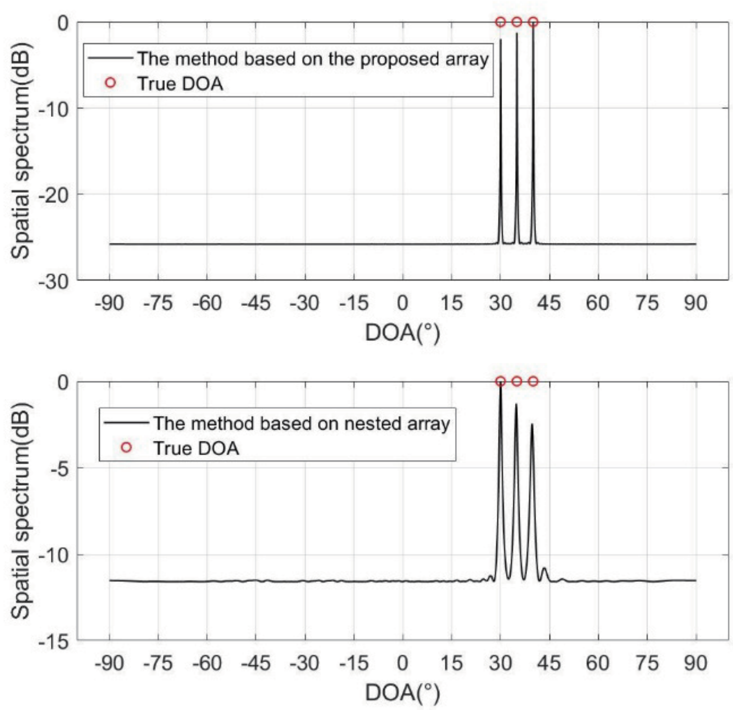

FIGURE 3. Comparison of spatial spectrums with mutual coupling being considered.

two kinds of arrays is fixed at 10. Suppose that the separated generalized nested array with $M = 3$, $N = 2$, and $p = 3$ is used for the proposed algorithm. The directions of the three quasi-stationary signals are 30°, 35°, and 40°, respectively. The total number of frames is fixed at 50. The simulation conditions include considering mutual coupling and ignoring mutual cou-

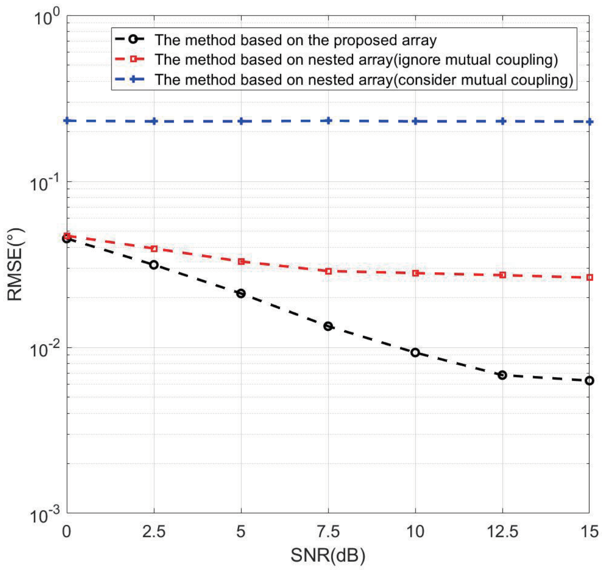

FIGURE 4. RMSE versus SNR.

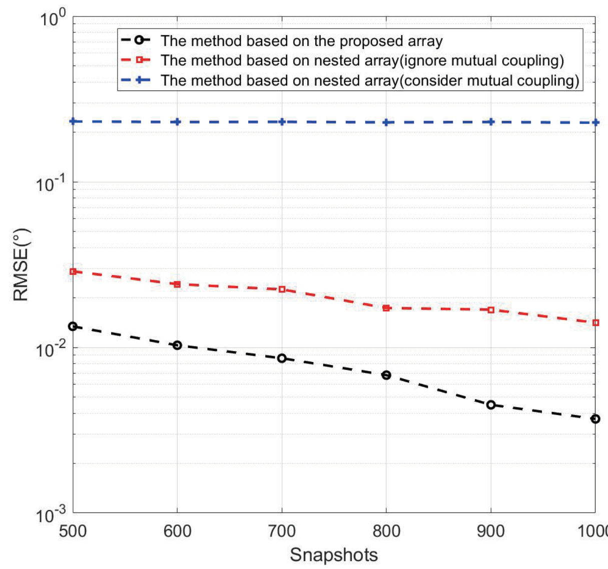

FIGURE 5. RMSE versus snapshots.

pling. When the mutual coupling effect is considered, it is assumed that the mutual coupling only exists between two sensors spaced no more than the half-wavelength of the incident signal. The mutual coupling coefficient between two sensors with half-wavelength spacing is $0.4e^{\pi i/4}$. Root mean square error (RMSE) is obtained through 500 independent repeated experiments and defined as

$$\sqrt{\frac{1}{500K}\sum_{w=1}^{500}\sum_{k=1}^{K}(\hat{\theta}_{kw}-\theta_k)^2}, \tag{22}$$

where $\hat{\theta}_{kw}$ is the estimation of θ_k in the wth experiment.

Figure 2 shows the comparative result of spectrum peak search without considering the effect of mutual coupling, where the signal-to-noise ratio (SNR) is 10 dB and $L = 500$. According to the result in the figure, we can find that the two methods can resolve the three signals clearly in the case of ignoring mutual coupling.

Figure 3 shows the comparative result of spectrum peak search as mutual coupling is presented, where the SNR is 10 dB and $L = 500$. According to the result in the figure, we can find that the estimation performance of the nested array [7] has significantly decreased when the mutual coupling is considered.

Figure 4 shows the RMSE comparison under different SNRs with $L = 500$. Fig. 5 shows the RMSE comparison under different snapshots with SNR $= 7.5$ dB. The comparison results can prove that the proposed method based on the proposed array has better performance than the similar method based on the nested array [7]. It is easy to find that the dimensions of the final signal subspace obtained by the proposed array are higher than that of signal subspace obtained by the nested array [7]. Hence, the proposed method has higher estimation accuracy than the similar method based on nested array [7].

4. CONCLUSION

In this paper, a separated generalized nested array is proposed. Being different from the conventional nested array, this array has adjustable element spacing, so the mutual coupling effect between sensors can be reduced effectively. By using this sparse array, a DOA estimation algorithm for quasi-stationary signal has been proposed and shows much higher estimation accuracy than the similar method based on nested array.

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