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A Generalized Solution for H-Polarized Scattering from Shallow Cavities with an Arbitrary Profile

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ABSTRACT: In this paper, a generalized manner is developed for the problem of the scattering of H-polarized electromagnetic waves from a shallow cavity with an arbitrary profile. Considering a proper auxiliary border and employing the region-matching technique, some close-form expressions are derived to compute the fields inside and outside the cavity. Next, we apply this approach to two cavities with different shapes and verify it by the Method of Moments (MoM).

1. INTRODUCTION

Because of the complexity of scattering wave from cavities, apertures, and discontinuities, many researchers are working on various methods to solve the problem of scattering in such structures [1–21]. There are numerous numerical methods presented to compute scattered waves from open cavities with arbitrary shapes [6, 7, 10, 11, 14]. However, the analytic and semi-analytic solutions are still valuable due to their high computational efficiency and also help us to understanding the wave propagation and scattering characteristics, physically [1–5, 8, 9, 12, 13, 15–21]. The present method is a semi-analytic treatment that uses wavefunction expansion and region matching techniques to provide a closed-form of solution for a shallow cavity with a specified and continuous profile in a Perfect Electric Conductor (PEC) plane. Some studies have been done on shallow circular, elliptical, and triangular cavities [13, 19, 20]. However, this study focuses on a more general manner to apply more practical and different problems and cavities. Here, to expand the electromagnetic fields by proper wave functions, an auxiliary border is considered, and thus the analyzed area is divided into two subregions. Then the tangential fields for two subregions are expressed in terms of an infinite series of proper wave functions. To compute the unknown series coefficients, the continuity of the boundary conditions is applied on the cavity wall and auxiliary border to construct a system of linear equations. Finally, the proposed manner is successfully applied to two different open cavities.

2. FORMULATION

The 2-D model treated in this research is shown in Fig. 1. This figure represents a shallow cavity created by a PEC wall with a known profile $l: y = f(x)$. The excitation of the model is the magnetic field H_z^i (H-polarized cylindric a plane wave) and its reflected field H_z^r as

$$H_z^i = e^{ik_0 r\cos(\varphi-\varphi^i)}$$

$$\text{and}\quad H_z^r = e^{ik_0 r\cos(\varphi+\varphi^i)} \quad \varphi^i \in (0,\pi) \tag{1}$$

where k_0 and φ^i are the free space propagation constant and incident angle. The semi-circular auxiliary border Ω_1 with radius a is considered to divide the analyzed region into two subregions (Region 1 and Region 2). This auxiliary interface that the cavity is placed inside allows us to employ a proper cylindrical wavefunction to expand the fields inside Region 2 that satisfy the Helmholtz equation and boundary condition at the cavity wall, simultaneously.

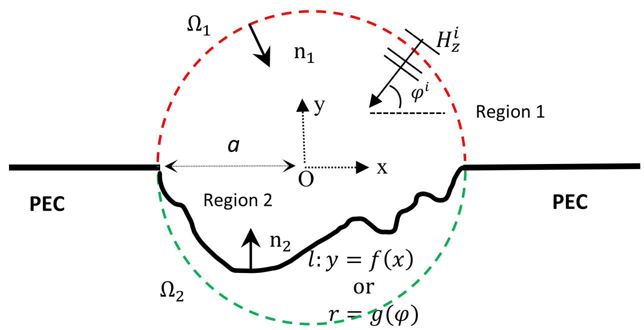

FIGURE 1. Geometry of a shallow open cavity with an arbitrary profile.

Here, we use a classification criterion for the term "shallow cavity" as the cavity walls should be inside the auxiliary border Ω_2. The scattered field H_z^s and the total magnetic field H_z^1 in Region 1 (which is the sum of H_z^i and H_z^r) and the total magnetic field H_z^2 in Region 2 expressed in the cylindrical coordinate system (r,φ) can be written as

$$H_z^s = \sum_{m=0}^{+\infty} A_m H_m^{(2)}(k_0 r)\cos(m\varphi) \tag{2}$$

$$H_z^1 = H_z^i + H_z^r + H_z^s = \sum_{m=0}^{+\infty}\Big[4i^m J_m(k_0 r)\cos(m\varphi^i) + A_m H_m^{(2)}(k_0 r)\Big]\cos(m\varphi) \tag{3}$$

$$H_z^2 = \sum_{m=0}^{+\infty}\left[B_m\sin(m\varphi) + C_m\cos(m\varphi)\right]J_m(k_0 r) \tag{4}$$

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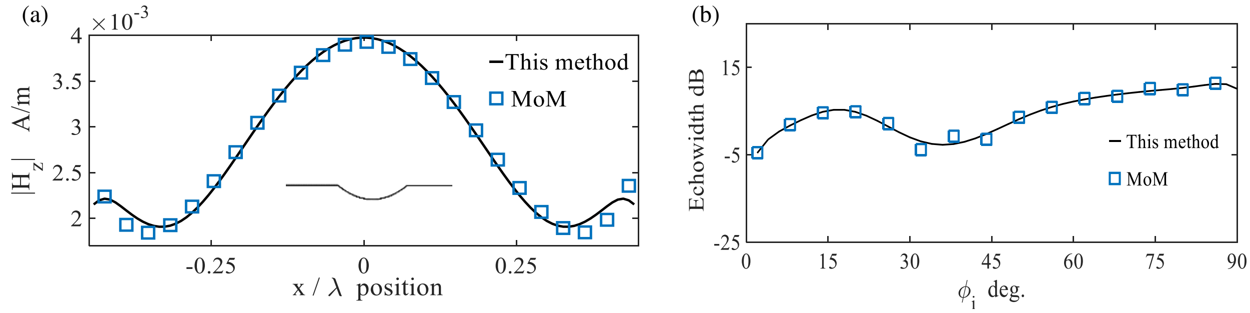

FIGURE 2. The results for a cavity with $l: y = x^2 - 0.2$. (a) $|H_z|$ at $y = 0$ for the incidence angle $\varphi^i = 90°$. (b) The echowidth as the function of the incidence angle φ^i.

where $J_m(\cdot)$ and $H_m^{(2)}(\cdot)$ are the mth order Bessel function of the first kind and the Hankel function of the second kind, respectively. The complex expansion coefficients A_m, B_m and C_m are to be determined. The boundary conditions on the auxiliary border Ω_1 and the cavity surface l are

$$H_z^1 = H_z^2 \text{ on } \Omega_1, \quad \frac{\partial H_z^1}{\partial n_1} = \frac{\partial H_z^2}{\partial n_1} \text{ on } \Omega_1$$

$$\text{and} \quad \frac{\partial H_z^2}{\partial n_2} = 0 \text{ on } l \tag{5}$$

Matching boundary conditions across the auxiliary interface Ω_1, multiplying the matching conditions by cosine functions and integrating over the range $[0, \pi]$ yields the following two independent linear algebraic equations,

$$C_n + A_n Q_n/P_n + \sum_{\substack{m=1 \\ m+n=\text{odd}}}^{+\infty} B_m \frac{P_m}{P_n} \frac{2m\delta_n}{\pi(m^2 - n^2)} = 2\delta_n i^n \sin\left(n\varphi^i\right) \tag{6}$$

where P_n and Q_n for the first independent equation are $J_n(k_0 a)$ and $H_n^{(2)}(k_0 a)$, respectively, and for the second equation, these parameters are $J'_n(k_0 a)$ and $H_n^{'(2)}(k_0 a)$, respectively. In addition, the transfer magnetic field H_z^2 inside the cavity should satisfy Neumann boundary condition $\partial H_z^2/\partial n_2 = 0$ where $\vec{n}_2$ is the normal vector to the curve l. In a similar fashion, by multiplying the boundary condition on the cavity wall by cosine function and integrating over the range $[\pi, 2\pi]$, we have

$$\int_\pi^{2\pi} \frac{\partial H_z^2}{\partial n_2} \cos(n\varphi)\, d\varphi = \int_\pi^{2\pi} \nabla H_z^2 \cdot \vec{n}_2 \cos(n\varphi)\, d\varphi = 0 \tag{7}$$

To construct the third independent equation for B_m and C_m from integral (7), we attempt to obtain expressions for two terms ∇H_z^2 and $\vec{n}_2$ in terms of r and φ, analytically. Thus, we have

$$\nabla H_z^2 = \left(\frac{\partial H_z^2}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial H_z^2}{\partial \varphi}\vec{\varphi}\right) = \sum_{m=0}^{+\infty} k_0 [B_m \sin(m\varphi) + C_m \cos(m\varphi)] J'_m(k_0 r)\, \vec{r} + \sum_{m=0}^{+\infty} [m B_m \cos(m\varphi) - m C_m \sin(m\varphi)] \frac{J_m(k_0 r)}{r}\, \vec{\varphi} \tag{8}$$

To derive the vector $\vec{n}_2$ that is normal to the cavity wall, we assume that the curve l is also defined by a polar equation $r = g(\varphi)$ which expresses the dependence of the length of the radius r on the polar angle φ. We have $x = r\cos(\varphi) = g(\varphi)\cos(\varphi)$ and $y = r\sin(\varphi) = g(\varphi)\sin(\varphi)$. Consequently, the slope of the normal line to the cavity wall is $\frac{\partial x}{\partial \varphi} \Big/ \frac{\partial y}{\partial \varphi}$, and thus the normal vector $\vec{n}_2$ is given by

$$\vec{n}_2 = \left(\frac{\partial g(\varphi)}{\partial \varphi}\cos(\varphi) - g(\varphi)\sin(\varphi),\ \frac{\partial g(\varphi)}{\partial \varphi}\sin(\varphi) + g(\varphi)\cos(\varphi)\right) \tag{9}$$

Now, by substituting (8) and (9) into (7), we can obtain the integral in (7) numerically to determine the coefficient of third linear equation constructed for B_m and C_m. The series in (2)–(4) can be truncated at $m = M$. The linear algebraic equations (6) to (7) make a system of linear equations and can be solved by the matrix methods for the coefficients A_m, B_m, and C_m.

3. RESULTS

To verify the solution, we apply the proposed method to two different cases and compare the results by the MoM used in FEKO software. First, we considered a cavity with $l: y = x^2 - 0.2$ and incidence angle $\varphi^i = 90°$. Fig. 2(a) illustrates the amplitude of H_z on the cavity opening versus the aperture position x. Also, comparisons of the echo width versus the incidence angle with MoM are given in Fig. 2(b). As demonstrated in Fig. 2, the results obtained from this method are in good agreement with those generated by MoM. To examine the efficiency of this method, we measured the simulation time for the cavity introduced in the first example. This time for this manner MoM is 9.12 sec. and 25 min., respectively, which demonstrates that this method is efficient. However, as the size of

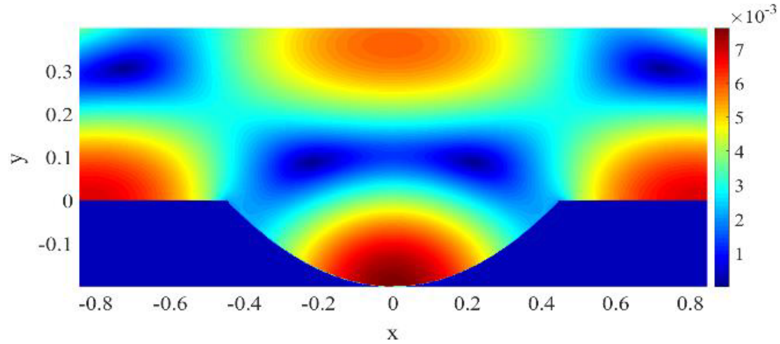

FIGURE 3. The distribution of $|H_z|$ around the cavity with $l: y = x^2 - 0.2$ at $\varphi^i = 90°$.

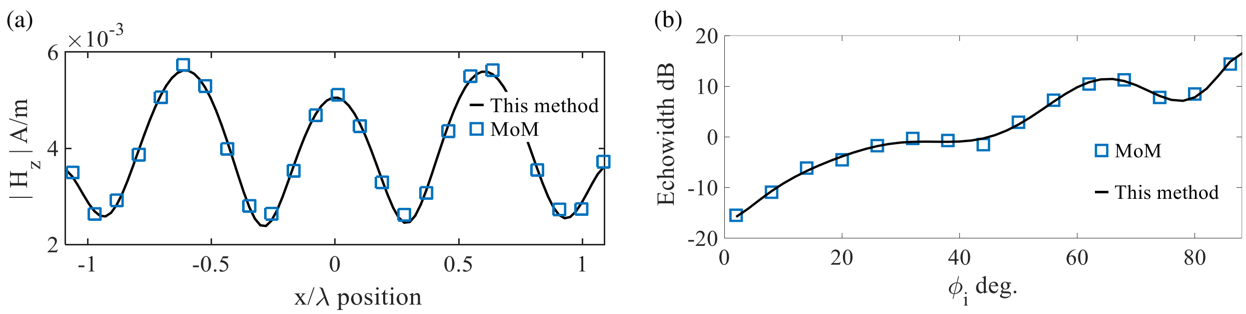

FIGURE 4. The results for a cavity with $l: y = \sqrt[3]{0.1x^2} - 0.5$ (Neile's parabola). (a) The amplitude of H_z at $y = 0$ for $\varphi^i = 90°$. (b) The echowidth as the function of the incidence angle φ^i.

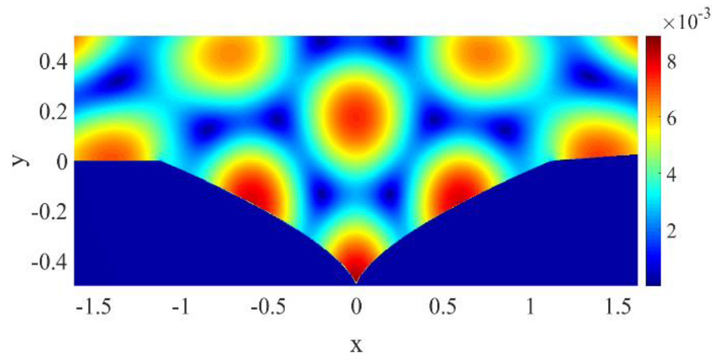

FIGURE 5. The distribution of $|H_z|$ around the cavity (near field) described at the caption of Fig. 4.

cavity increases, the truncation number M should also increase, and consequently, computation time increases. A rule of thumb that we can use to find the value of the truncation number M is that it should be greater than k_0W.

To show the capability of this method in near field analysis, we also computed the $|H_z|$ in the neighborhood of the cavity described in the caption of Fig. 2 and displayed the results in Fig. 3. The turbulence in the magnetic field around the cavity causes the scattering of waves in the far zone. Next, we changed the cavity shape to a semi-cubical parabola (Neile's parabola) with $l: y = \sqrt[3]{0.1x^2} - 0.5$. Fig. 4 shows the amplitude of H_z and the echowidths obtained by this method and MoM for the semi-cubical cavity at $\varphi^i = 90°$. Similar to the first example, we also presented the distribution of $|H_z|$ around this cavity in Fig. 5.

4. CONCLUSION

An efficient method for H-polarized wave scattering by a shallow cavity with an arbitrary shape was developed. The technique of wavefunction expansions and region matching method are used to solve this problem. We examine its validity and computational efficiency by comparing with the MoM and show its applicability for the determination of fields at far and near zones.

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