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Tensor-Based Robust Adaptive Beamforming for Multiple-Input Multiple-Output Radar under Random Mismatch Scenarios

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ABSTRACT: Adaptive beamforming for multiple-input multiple-output (MIMO) radar systems suffers from performance deterioration under the scenarios with multiple random mismatches. This paper explores the theory of tensor algebra and exploits the inherent multidimensional structure of data matrix received by MIMO radar systems. For dealing with the beamforming problem induced by multiple random mismatches including steering vector error, mutual coupling, sensor position error, and coherent local scattering, we develop a robust method based on a third-order tensor in conjunction with a gradient-based optimization process. The proposed method captures the multidimensional structure information embedded in the data matrix received by a MIMO radar and produces appropriate estimates for transmit and receive signal direction vectors required for beamforming. Using a third-order tensor helps to alleviate the effect of the multiple random mismatches in the tensor data domain. The gradient-based optimization process further enhances the capabilities of the third-order tensor in estimating transmit and receive signal direction vectors for adaptive beamforming of a MIMO radar. The main computational complexity of the proposed method is dominated by a trilinear alternating least squares algorithm and the well-known gradient-based algorithm. The proposed method provides better performance than the existing robust methods. Simulation results are presented to confirm the effectiveness of the proposed method.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) radar deploys multiple antennas at both transmit and receive sides for emitting signal waveforms and receiving the echoes reflected by the targets. At the transmit side, each antenna can transmit orthogonal or uncorrelated signal waveforms. This capability allows MIMO radar systems to possess the main merit of providing the waveform diversity which can be exploited to increase the degrees of freedom (DOFs). As a result, the performance of MIMO radars in resolution, detection, and parameter identifiability can be significantly enhanced [1–3]. It has been shown that most of the conventional adaptive beamforming methods can be easily applied to MIMO radar systems for extracting the desired target signal while rejecting interference and noise in many applications [4–8]. Moreover, a MIMO radar possesses another advantage, namely, virtual array property (VAP). The VAP leads to a favorable characteristic that the aperture of the virtual array can be up to $N_T \times N_R$ and much larger than the physical receiving array, where N_T and N_R are the numbers of transmit antennas and receive antennas, respectively. Therefore, the DOFs of MIMO radar systems can be considerably increased. From the beamforming view point, MIMO radar systems are very capable of improving the angular resolution and increasing the number of resolvable targets due to the VAP [9]. However, applying this VAP for MIMO radar beamforming makes the performance of MIMO radar beamformers more sensitive to various kinds of imperfections as compared to the conventional phased-array radar beamformers. Consequently, adaptive beamforming with robustness required for alleviating the performance deterioration of MIMO radar beamformers due to the problem of mismatches becomes even more important.

Many well-known robust techniques have been developed to deal with traditional beamforming of adaptive antenna arrays. Some notable among them are presented by [10–14]. Some of these traditional robust methods have also been applied to perform MIMO radar beamforming [15–18]. However, directly applying the traditional robust beamforming methods may suffer from some difficulties. One of them is due to the virtual steering vector (VSV) of MIMO radars using uniform linear array (ULA). The VSV comes from the Kronecker product of transmit and receive steering vectors. In general, the VSV does not have the Vandermonde vector structure and hence, becomes a major obstacle for employing the well-known conventional robust methods to tackle multiple random mismatches without sacrificing the full DOFs of MIMO radars. Recently, several robust adaptive beamforming methods taking the full DOFs of MIMO radar into account have been presented [18, 19]. In [18], the authors consider the case of collocated MIMO radar beamforming under nonrandom steering angle mismatch. In contrast, the authors of [16, 17, 19–21] consider MIMO radar beamforming under the problem of steering vector mismatch. Nevertheless, to the best knowledge of the authors, it is worth considering the problem of MIMO radar beamforming in severe

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environments, such as the scenarios with multiple random mismatches including steering vector error, antenna sensors with random position errors, mutual coupling (MC) effect between antenna sensors, and coherent local scattering induced by the spatial distribution of the multipath propagation [22].

In this paper, we present a novel sensor array beamforming method to deal with the performance degradation due to multiple random mismatches for MIMO radar beamforming. The proposed method fully utilizes the VAP to enhance its robust capabilities against the considered mismatch problem. First, the received data after being matched-filtered and rearranged at the receive side is adopted to create the so-called virtual data matrix. Then, we formulate a third-order measurement tensor signal model to fully exploit the multidimensional structure inherent in the virtual data matrix. Based on the third-order tensor signal model, the virtual data matrix can be factorized into a sum of component rank-one tensors with the factor matrices related to transmit and receive signal direction vectors. In order to find the estimates for transmit and receive direction vectors required for beamforming, we construct the slices of the three dimensional data along transmit and receive array directions, i.e., the mode-1 and mode-2 matrix unfoldings of the third-order tensor. Through a trilinear alternating least square (TALS) algorithm which is most commonly used to compute many tensor decompositions, we can find appropriate estimates for transmit and receive direction vectors, respectively. Due to taking the advantages of the multidimensional structure inherent in the virtual data matrix, the tensor-based method helps us to obtain favorable estimates of transmit and receive direction vectors for achieving robustness against multiple random mismatches. However, using the TALS algorithm may induce the influence of error accumulation and is quite sensitive to local minima during the estimation process. To alleviate these drawbacks, we present an optimization process to further refine the estimates of transmit and receive direction vectors. The objective function of the optimization process is the projection of the estimate of VSV onto the noise subspace associated with the virtual data matrix. Hence, the well-known gradient-based iterative algorithm can be employed to solve the optimization problem. The refined estimates of transmit and receive direction vectors are adopted for achieving robust adaptive beamforming of MIMO radars. As to the issue of computational complexity, the main computational complexity of the proposed method is dominated by the TALS algorithm which is the most efficient gradient-based algorithm and is also easy to implement. It is comparable to the existing robust methods, such as [19]. The superiority of the proposed method over the existing robust methods is confirmed through simulation results.

This paper is organized as follows. Section 2 briefly reviews some basic definitions of tensor algebra and the signal model of MIMO radar systems. We also briefly describe the principle of transmit/receive beamforming for MIMO radar systems based on minimum variance distortionless response (MVDR) with and without mismatches. The proposed method for dealing with transmit/receive beamforming for MIMO radar systems in the presence of multiple random mismatches is presented in Section 3. We introduce the principle of tensor-based signal model of a bistatic MIMO radar. The theoretical analysis and formulation of a third-order tensor model in conjunction with an optimization process for estimating the transmit and receive direction vectors are established in this section. The main computational complexity regarding the proposed method is evaluated in Section 4. Simulation results for illustration and comparison are presented in Section 5. Finally, we conclude this paper in Section 6.

2. FUNDAMENTALS AND SIGNAL MODEL OF MIMO RADARS

2.1. Mathematical Fundamentals

Here, we briefly describe some basics regarding tensor algebra applied to the research work. More details can be reviewed in the literature [23–25].

Definition 1 (**Matrix Unfoldings**): The mode-n matrix unfolding of an $(I_1 \times I_2 \times I_3 \times \ldots \times I_N)$-dimensional tensor $\boldsymbol{\Upsilon}$ with N indices is denoted by $[\boldsymbol{\Upsilon}]_{(n)}$, where the $(i_1 \times i_2 \times i_3 \times \ldots \times i_N)$-entry of $\boldsymbol{\Upsilon}$ maps to the (i_n, j)-th entry of $[\boldsymbol{\Upsilon}]_{(n)}$, where $j = 1 + \Sigma_{k=1,k\neq n}^{N}(i_k - 1)J_k$ and $J_k = \Pi_{m=1,m\neq n}^{k-1} I_m$.

Definition 2 (**Trilinear Decompoition**): The trilinear decomposition of a third-order tensor $\boldsymbol{\Upsilon}$ factorizes $\boldsymbol{\Upsilon}$ into a sum of component rank-one tensors

$$\boldsymbol{\Upsilon} = \sum_{k=1}^{K} \mathbf{A}_k \circ \mathbf{B}_k \circ \mathbf{C}_k, \tag{1}$$

where $\circ$ denotes the outer product, and K is a positive integer associated with the rank of $\boldsymbol{\Upsilon}$. $\mathbf{A}_k, \mathbf{B}_k$, and $\mathbf{C}_k$ are rank-one tensors.

2.2. Signal Model of MIMO Radar

Consider a bistatic MIMO radar system with uniform linear N_T antennas and N_R antennas deployed at transmit and receive sides, respectively. Let θ and ϕ be the the different spatial angles for a far-field target or signal. Then, the corresponding signal direction vectors associated with the transmit and receive antennas can be expressed by

$$\mathbf{a}_T(\theta) = \left[1\ e^{j2\pi\frac{d}{\lambda}\sin\theta}\ e^{j4\pi\frac{d}{\lambda}\sin\theta} \ldots e^{j(N_T-1)2\pi\frac{d}{\lambda}\sin\theta}\right]^T, \tag{2}$$

and

$$\mathbf{a}_R(\phi) = \left[1\ e^{j2\pi\frac{d}{\lambda}\sin\phi}\ e^{j4\pi\frac{d}{\lambda}\sin\phi} \ldots e^{j(N_R-1)2\pi\frac{d}{\lambda}\sin\phi}\right]^T, \tag{3}$$

respectively, where d and λ denote the distance between two adjacent antenna sensors and the wavelength of the signal sources, respectively. The superscript T denotes the transpose operation. Hence, the array data vector received by the receive antennas is given by

$$\mathbf{x}(t) = [x_1(t)\ x_2(t) \ldots x_{N_R}(t)]^T = \alpha_d(t)\mathbf{a}_R(\phi_d)\mathbf{a}_T^T(\theta_d)\mathbf{s}_d(t) + \sum_{q=1}^{Q} \alpha_q(t)\mathbf{a}_R(\phi_q)\mathbf{a}_T^T(\theta_q)\mathbf{s}_q(t) + \mathbf{n}(t), \tag{4}$$

where $\mathbf{s}(t) = [s_1(t)\ s_2(t) \ldots s_{N_T}(t)]^T$ is the signal vector transmitted by the transmit antennas. $s_i(t)$, $i = 1, 2, \ldots, N_T$ are mutually orthogonal with unit energy during the radar pulse width P_W. Each $s_i(t)$ is the complex waveform radiated by the ith transmit antenna. $\mathbf{n}(t)$ is the additive white Gaussian noise vector. Q is the number of interference signals. (θ_d, ϕ_d) and (θ_q, ϕ_q) denote the direction angles of the desired signal and the qth interferer, respectively. $\alpha_d(t)$ and $\alpha_q(t)$ denote the complex-valued reflection coefficients associated with the desired signal and the qth interferer, respectively. By performing matched filtering on $\mathbf{x}(t)$ and vectorizing the matched-filtered output, we have the received data vector given as follows

$$\begin{aligned} \mathbf{y}(t) &= [\mathbf{y}_1(t)^T\ \mathbf{y}_2(t)^T \ldots \mathbf{y}_{N_T}(t)^T]^T \\ &= \alpha_d(t)\mathbf{a}_R(\phi_d) \otimes \mathbf{a}_T(\theta_d) \\ &\quad + \sum_{q=1}^{Q} \alpha_q(t)\mathbf{a}_R(\phi_q) \otimes \mathbf{a}_T(\theta_q) + \mathbf{z}(t), \end{aligned} \tag{5}$$

where $\otimes$ denotes the Kronecker product, and the mth element $\mathbf{y}_m(t)$ of $\mathbf{y}(t)$, $m = 1, 2, \ldots, N_T$, represents the result of matched filtering the received data vector to the mth transmitted waveform at the receiver and is an $N_R \times 1$ vector given by

$$\mathbf{y}_m(t) = \int_0^{P_W} \mathbf{x}(t)s_m(t)^* dt, \quad m = 1, 2, \ldots, N_T, \tag{6}$$

where the superscript $*$ is the complex conjugation. $\mathbf{z}(t)$ is the corresponding $N_T N_R \times 1$ virtual noise vector and is assumed to have independent and identically distributed (iid) Gaussian entries with zero mean and covariance matrix $\sigma^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix with size $N_T N_R \times N_T N_R$. Moreover, the vector $\mathbf{a}(\theta, \phi) = \mathbf{a}_R(\phi) \otimes \mathbf{a}_T(\theta)$ represents the $N_T N_R \times 1$ transmit-receive steering vector or virtual steering vector. Assume that the received signals and noise are uncorrelated with each other. Accordingly, we have the data covariance matrix of the MIMO radar given by

$$\begin{aligned} \mathbf{R}_{MIMO} &= E[\mathbf{y}(t)\mathbf{y}^H(t)] = |\alpha_d(t)|^2\mathbf{a}(\theta_d, \phi_d)\mathbf{a}^H(\theta_d, \phi_d) \\ &\quad + \sum_{q=1}^{Q} |\alpha_q(t)|^2\mathbf{a}(\theta_q, \phi_q)\mathbf{a}^H(\theta_q, \phi_q) + \sigma_n^2\mathbf{I}, \end{aligned} \tag{7}$$

where $|\alpha_d(t)|^2$ and $|\alpha_q(t)|^2$ represent the desired signal power and the qth interferer's power, respectively. For practical situations, we only have the following sample covariance matrix instead of $\mathbf{R}_{MIMO}$

$$\hat{\mathbf{R}}_{MIMO} = \frac{1}{L}\sum_{l=1}^{L} \mathbf{y}(t_l)\mathbf{y}^H(t_l), \tag{8}$$

where t_l and L denote the lth time instant for taking the data snapshot $\mathbf{y}(t_l)$ from $\mathbf{y}(t)$ and the total number of data snapshots used for computing $\hat{\mathbf{R}}_{MIMO}$, respectively. The expression of the output signal-to-interference-plus-noise ratio (SINR) of a MIMO radar beamformer based on the L data samples can be expressed by

$$SINR = \frac{|\alpha_d(t)|^2|\mathbf{w}^H\mathbf{a}(\theta_d, \phi_d)|^2}{\mathbf{w}^H\hat{\mathbf{R}}_{jn}\mathbf{w}}, \tag{9}$$

where $\hat{\mathbf{R}}_{jn}$ represents the sample covariance matrix associated with the interference-plus-noise. $\mathbf{w}$ is the weight vector used for beamforming.

2.3. Conventional Adaptive MIMO Radar Beamforming without Mismatches

According to the theory regarding the adaptive beamforming of phased-array systems [26], the authors of [16] extend the principle of adaptive array beamforming to MIMO radar beamformers. The basic concept is maximizing the output signal to interference plus noise power ratio (SINR) of a MIMO radar beamformer while constraining the power gain of the desired signal to be one for finding the optimal complex weighting vector $\mathbf{w}_o$ [16]. Accordingly, the corresponding optimization problem can be formulated as follows

$$\begin{aligned} &Minimize_{\mathbf{w}}\ \ \mathbf{w}^H\hat{\mathbf{R}}_{MIMO}\mathbf{w} \\ &Subject\ to\ \ \ \mathbf{a}(\theta_d, \phi_d)^H\mathbf{w} = 1. \end{aligned} \tag{10}$$

The optimal solution $\mathbf{w}_o$ of (10) can be easily obtained by applying the Lagrange multiplier method and is given as follows

$$\mathbf{w}_o = \frac{\hat{\mathbf{R}}_{MIMO}^{-1}\mathbf{a}(\theta_d, \phi_d)}{\mathbf{a}^H(\theta_d, \phi_d)\hat{\mathbf{R}}_{MIMO}^{-1}\mathbf{a}(\theta_d, \phi_d)}. \tag{11}$$

This leads to the so-called minimum variance distortionless response (MVDR) MIMO radar beamformer in the literature [16]. Apparently, severe performance deterioration of the MVDR MIMO radar beamformer will occur even if a small mismatch like steering vector mismatch exists in the beamforming environments [19].

2.4. Adaptive MIMO Radar Beamforming with Mismatches

For the situation with mismatches, the steering vector used for beamforming is set to the presumed direction vector of the desired signal. However, the actual direction vector of the desired signal is no longer equal to the steering vector. As a result, the MVDR MIMO radar beamformer suffers from severe performance degradation because the beamformer treats the desired signal as an interferer. Several mismatches considered for MIMO radar beamforming in this paper are described as follows. The first one is the steering angle mismatch. Let (θ_d, ϕ_d) and (θ_a, ϕ_a) be the presumed and actual spatial angles for the desired signal, respectively. For fixed steering angle errors, the angle errors $\delta\theta = \theta_d - \theta_a$ and $\delta\phi = \phi_d - \phi_a$ are nonrandom. The second mismatch is due to antenna elements with random position errors. In this case, the actual position vector of the ith antenna element of an N-element linear array is expressed by

$$\hat{\mathbf{d}}_i = \mathbf{d}_i + \delta\mathbf{d}_i, \tag{12}$$

where $\mathbf{d}_i = [x_i, 0]$ denotes the presumed position vector and $\delta\mathbf{d}_i = [\delta x_i, \delta y_i]$ the random position error vector. δx_i and δy_i

represent the position deviations along the X and Y axes, respectively. Then, the actual direction vector of a signal with spatial angle θ can be expressed by

$$\hat{\mathbf{a}}(\theta) = \Big[e^{j2\pi \frac{\hat{\mathbf{d}}_1}{\lambda}[\sin(\theta)\cos(\theta)]^T} e^{j2\pi \frac{\hat{\mathbf{d}}_2}{\lambda}[\sin(\theta)\cos(\theta)]^T} \dots e^{j2\pi \frac{\hat{\mathbf{d}}_N}{\lambda}[\sin(\theta)\cos(\theta)]^T} \Big]^T. \quad (13)$$

From (13), we note that the actual direction vector is significantly different from that of the steering vector as shown by (2). The third mismatch is the issue regarding the mutual coupling effect (MCE) for arrays deployed in limited spacing. The MCE on the system model is widely considered by taking a mutual coupling matrix $\mathbf{C}$ into account and replacing the direction vector $\mathbf{a}(\theta)$ of a signal with a distorted direction vector $\tilde{\mathbf{a}}(\theta) = \mathbf{C}\mathbf{a}(\theta)$. For the case with unknown mutual coupling (UMC), the most widely used UMC model for MIMO radar systems adopts two banded symmetric Toeplitz matrices $\mathbf{C}_T$ and $\mathbf{C}_R$ representing the mutual coupling matrices associated with the transmit and receive antenna arrays, respectively [27]. In general, they can be expressed as $\mathbf{C} = Toeplitz[\mathbf{c}^T\ \mathbf{0}_{1\times(N-P)}]$, where $\mathbf{c} = [1, c_1, c_2, \dots, c_{P-1}]^T$ denotes the vector containing the nonzero mutual coupling coefficients between antenna elements at the least P inter-element spacings and $Toeplitz[\mathbf{x}]$ a symmetric Toeplitz matrix with the bracketed vector $\mathbf{x}$ as its first row. Finally, the fourth mismatch is the one due to coherent local scattering (CLS) induced by the spatial distribution of the multipath propagation [22]. Under the CLS effect considered in this paper, the signal direction vectors can be modeled as follows [22]

$$\mathbf{a}_{TCLS}(\theta) = \sqrt{N_T}\frac{\mathbf{a}_T(\theta_a) + \sum_{i=1}^{NT_i} e^{j\psi_{Ti}}\mathbf{a}_T(\theta_a + \delta_{Ti})}{||\mathbf{a}_T(\theta_a) + \sum_{i=1}^{NT_i} e^{j\psi_{Ti}}\mathbf{a}_T(\theta_a + \delta_{Ti})||}, \quad (14)$$

and

$$\mathbf{a}_{RCLS}(\phi) = \sqrt{N_R}\frac{\mathbf{a}_R(\phi_a) + \sum_{i=1}^{NR_i} e^{j\psi_{Ri}}\mathbf{a}_R(\phi_a + \delta_{Ri})}{||\mathbf{a}_R(\phi_a) + \sum_{i=1}^{NR_i} e^{j\psi_{Ri}}\mathbf{a}_R(\phi_a + \delta_{Ri})||}, \quad (15)$$

at the transmit and receive sides, respectively, where $\mathbf{a}_T(\theta_a)$ and $\mathbf{a}_R(\phi_a)$ denote the signal direction vectors of the direct pathes with nominal direction angles θ_a and ϕ_a at the transmit and receive sides, respectively. $\mathbf{a}_T(\theta_a+\delta_{Ti})$ and $\mathbf{a}_R(\phi_a+\delta_{Ri})$ represent the direction vectors of the ith scattered signals at the transmit and receive sides, respectively. ψ_{Ti} and ψ_{Ri} represent the random phase delays of the ith scattered signals at the transmit and receive sides, respectively. δ_{Ti} and δ_{Ri} are the random angular spreads associated with the ith scattered signals at the transmit and receive sides, respectively. NT_i and NR_i are the total numbers of local scatterers associated with the desired signal at the transmit and receive sides, respectively. Moreover, $||\mathbf{x}||$ designates the norm of the vector $\mathbf{x}$. In the presence of the above four mismatches, the received data vector $\mathbf{y}(t)$ can be expressed as follows

$$\mathbf{y}(t) = \alpha_d(t)\mathbf{C}_{aR}\hat{\mathbf{a}}_R(\phi_d) \otimes \mathbf{C}_{aT}\hat{\mathbf{a}}_T(\theta_d) + \sum_{q=1}^{Q} \alpha_k(t)\mathbf{C}_{aR}\hat{\mathbf{a}}_R(\phi_q) \otimes \mathbf{C}_{aT}\hat{\mathbf{a}}_T(\theta_q) + \mathbf{z}(t), \quad (16)$$

where $\mathbf{C}_{aR}$ and $\mathbf{C}_{aT}$ denote two banded complex symmetric Toeplitz matrices conveying the actual mutual coupling effects associated with the receive and transmit antenna arrays, respectively. The signal direction vector with hat represents the direction vector conveys the influences coming from the steering angle error, random sensor position errors, and coherent local scattering. As we can see from the above formulation, the four mismatches significantly change the eigenstructure of the MIMO data covariance matrix given by (7). Consequently, the optimal weight vector obtained by (11) will drive the MVDR MIMO radar beamformer to suppress the desired signal and hence, deteriorate the beamforming capabilities.

3. PROPOSED ROBUST METHOD

In this section, a method with robustness against the considered multiple random mismatches is presented for adaptive beamforming of MIMO radars.

3.1. The Tensor-Based Signal Model

First, we can express $\mathbf{y}(t)$ of (16) as follows

$$\mathbf{y}(t) = \mathbf{A}\mathbf{g}(t) + \mathbf{z}(t), \quad (17)$$

where

$$\mathbf{A} = \left[\mathbf{a}(\theta_d, \phi_d)\ \mathbf{a}(\theta_1, \phi_1)\ \dots \ \mathbf{a}(\theta_{(Q-1)}, \phi_{(Q-1)})\right], \quad (18)$$

$$\mathbf{a}(\theta_k, \phi_k) = \mathbf{C}_{aR}\hat{\mathbf{a}}_R(\phi_k) \otimes \mathbf{C}_{aT}\hat{\mathbf{a}}_T(\theta_k), \quad (19)$$

$k = d, 1, 2, \dots, Q-1$ and

$$\mathbf{g}(t) = \left[\alpha_d(t)\ \alpha_1(t)\ \dots\ \alpha_{(Q-1)}(t))\right]^T. \quad (20)$$

Next, we take L data snapshots from $\mathbf{y}(t)$ of (16) to obtain the sample data vectors $\mathbf{y}(t_l)$, $l = 1, 2, \dots, L$. Using these $\mathbf{y}(t_l)$, $l = 1, 2, \dots, L$, we then construct a sample data matrix $\mathbf{Y}$ with size $N_T N_R \times L$ as follows

$$\mathbf{Y} = [\mathbf{y}(t_1)\ \mathbf{y}(t_2) \dots \mathbf{y}(t_L)] = \mathbf{A}\mathbf{G} + \mathbf{Z}, \quad (21)$$

where $\mathbf{G} = [\mathbf{g}(t_1)\ \mathbf{g}(t_2)\ \dots\ \mathbf{g}(t_L)]$ is a $Q\times L$ signal matrix containing the L complex-valued reflection coefficients of the Q signal sources, and $\mathbf{Z} = [\mathbf{z}(t_1)\ \mathbf{z}(t_2)\ \dots\ \mathbf{z}(t_L)]$ is an $N_T N_R \times L$ matrix containing the corresponding noise values. To formulate a third-order tensor model based on (21), we rewrite (21) as follows [28]

$$\mathbf{Y} = [\mathbf{y}(t_1)\ \mathbf{y}(t_2)\ \dots\ \mathbf{y}(t_L)] = [\mathbf{A}_R| \otimes |\mathbf{A}_T]\,\mathbf{G} + \mathbf{Z}, \quad (22)$$

where $|\otimes|$ represents the columnwise Kronecker product. $\mathbf{A}_R$ denotes the $N_R \times Q$ receive direction matrix and $\mathbf{A}_T$ the $N_T \times Q$ transmit direction matrix, respectively. They are given as follows

$$\mathbf{A}_R = \left[\hat{\mathbf{a}}_R(\phi_d)\ \hat{\mathbf{a}}_R(\phi_1) \dots \hat{\mathbf{a}}_R(\phi_{(Q-1)})\right], \quad (23)$$

and

$$\mathbf{A}_T = \left[\hat{\mathbf{a}}_T(\theta_d)\ \hat{\mathbf{a}}_T(\theta_1) \dots \hat{\mathbf{a}}_T(\theta_{(Q-1)})\right], \quad (24)$$

where $\hat{\mathbf{a}}_R(\phi_i)$ and $\hat{\mathbf{a}}_T(\theta_i)$, $i = d, 1, 2, \dots, (Q-1)$ represent the signal direction vectors conveying the considered multiple random mismatches at the receive and transmit sides, respectively. From (22), we observe that each entry of $\mathbf{Y}$ is obtained from the product of the unique entity in $\mathbf{A}_R$, $\mathbf{A}_T$, and $\mathbf{G}$. This reveals an inherent multidimensional structure associated with the data matrix $\mathbf{Y}$, i.e., the data matrix $\mathbf{Y}$ demonstrates three diversities. Conventional matrix-based beamforming techniques do not consider these diversities. However, the inherent multidimensional structure can be exploited through a third-order tensor model. First, we rearrange the entries of $\mathbf{Y}$ into a third-order tensor $\vec{\mathbf{Y}}$ with the (n_R, n_T, l)th entry given by

$$\vec{\mathbf{Y}}(n_R, n_T, l) = \sum_{q=1}^{Q} \mathbf{A}_R(n_R, q) \circ \mathbf{A}_T(n_T, q) \circ \mathbf{G}(l, q) + \vec{\mathbf{Z}}(n_R, n_T, l), \tag{25}$$

where $\vec{\mathbf{Z}}(n_R, n_T, l)$ denotes the (n_R, n_T, l)th entry of the rearranged noise tensor $\vec{\mathbf{Z}}$, $\mathbf{A}_R(n_R, q)$ the (n_R, q)th entry of $\mathbf{A}_R$, $\mathbf{A}_T(n_T, q)$ the (n_T, q)th entry of $\mathbf{A}_T$, and $\mathbf{G}(l, q)$ the (l, q)th entry of $\mathbf{G}$. $n_R = 1, 2, \ldots, N_R$, $n_T = 1, 2, \ldots, N_T$, and $l = 1, 2, \ldots, L$.

According to the mode-n matrix unfolding of Definition 1 and the trilinear decomposition of Definition 2, the third-order tensor $\vec{\mathbf{Y}}$ given by (25) can be decomposed into three slices in three different directions as follows

$$\vec{\mathbf{Y}}_{(3)} = \mathbf{Y} = [\mathbf{A}_R | \otimes | \mathbf{A}_T]\, \mathbf{G} + \mathbf{Z}, \tag{26}$$

$$\vec{\mathbf{Y}}_{(2)} = [\mathbf{A}_T | \otimes | \mathbf{G}]\, \mathbf{A}_R + \mathbf{Z}_{(2)}, \tag{27}$$

and

$$\vec{\mathbf{Y}}_{(1)} = [\mathbf{G} | \otimes | \mathbf{A}_R]\, \mathbf{A}_T + \mathbf{Z}_{(1)}, \tag{28}$$

where $\vec{\mathbf{Y}}_{(3)}$ corresponds to the received data matrix and represents the three-dimensional slice of the three-order tensor data along the spatial direction; $\vec{\mathbf{Y}}_{(2)}$ represents the three-dimensional slice of the three-order tensor data along the receive array direction; and $\vec{\mathbf{Y}}_{(1)}$ represents the three-dimensional slice of the three-order tensor data along the transmit array direction. Moreover, $\mathbf{Z}_{(2)}$ and $\mathbf{Z}_{(1)}$ denote the rearranged noise slices along the receive and transmit array directions, respectively.

3.2. Joint Estimation of Direction and Signal Matrices

After obtaining the three slices $\vec{\mathbf{Y}}_{(3)}$, $\vec{\mathbf{Y}}_{(2)}$, and $\vec{\mathbf{Y}}_{(1)}$, we utilize the concept of trilinear alternating least square (TALS) presented by [29] to find the estimates for direction and signal matrices. The TALS algorithm is easy to understand and implement. Moreover, Its convergence is guaranteed as mentioned by [30].

3.2.1. The Essential of the TALS

The TALS is a well-known approach for solving the trilinear decomposition problem for multi-way data. It can be shown that TALS is globally monotonically convergent as mentioned by [30]. To implement the TALS algorithm for estimating the direction and signal matrices from $\vec{\mathbf{Y}}_{(3)}$, $\vec{\mathbf{Y}}_{(2)}$, and $\vec{\mathbf{Y}}_{(1)}$, the first step is to fit one of $\vec{\mathbf{Y}}_{(3)}$, $\vec{\mathbf{Y}}_{(2)}$, and $\vec{\mathbf{Y}}_{(1)}$ matrices using least squares (LS) algorithm with the other two matrices fixed. The second step is to fit the remaining two matrices in a similar manner to the first step. Then, we repeat the first and second steps until a preset stoping criterion is satisfied. The additional benefits of using TALS approach during the iteration process are as follows: (1) Each update is a standard least squares problem. (2) It is unnecessary to tune parameters. (3) It is easy to incorporate the necessary constraints on some or all of the matrices.

3.2.2. The Details of Estimation with TALS Algorithm

Here, we describe the joint estimation of direction and signal matrices according to the TALS algorithm. The goal of the joint estimation is to find a trilinear decomposition $\hat{\vec{\mathbf{Y}}}$ with Q components that is the solution of the following least square (LS) fitting of $\vec{\mathbf{Y}}$

$$Minimize \quad ||\vec{\mathbf{Y}} - \hat{\vec{\mathbf{Y}}}||_F \quad Subject\ to$$

$$\hat{\vec{\mathbf{Y}}}(n_R, n_T, l) = \sum_{q=1}^{Q} \hat{\mathbf{A}}_R(n_R, q) \circ \hat{\mathbf{A}}_T(n_T, q) \circ \hat{\mathbf{G}}(l, q), \tag{29}$$

where $|| \bullet ||_F$ denotes the Frobenius norm. Accordingly, the procedure of the proposed joint estimation is described in detail as follows:

Step 1: Generate the initial guesses of $\mathbf{A}_R$, $\mathbf{A}_T$, and $\mathbf{G}$ randomly and designate them as $\hat{\mathbf{A}}_R^{(0)}$, $\hat{\mathbf{A}}_T^{(0)}$, and $\hat{\mathbf{G}}^{(0)}$, respectively. Determine the value of threshold η.

Step 2: At the kth iteration, perform the LS fitting of $\mathbf{Y}$ by solving the following linear LS problem

$$Minimize_{\hat{\mathbf{G}}(k)} ||\mathbf{Y} - [\hat{\mathbf{A}}_R(k-1) | \otimes | \hat{\mathbf{A}}_T(k-1)] \hat{\mathbf{G}}(k)^T ||_F, \tag{30}$$

to find the solution

$$\hat{\mathbf{G}}(k)^T = [\hat{\mathbf{A}}_R(k-1) | \otimes | \hat{\mathbf{A}}_T(k-1)]^{\dagger} \mathbf{Y}, \tag{31}$$

where † denotes the Moore-Penrose pseudoinverse.

Step 3: Perform the LS fitting of $\mathbf{A}_R$ by solving the following linear LS problem

$$Mimimize_{\hat{\mathbf{A}}_R(k)} ||\vec{\mathbf{Y}}_{(2)} - [\hat{\mathbf{A}}_T(k-1) | \otimes | \hat{\mathbf{G}}(k)] \hat{\mathbf{A}}_R(k)^T ||_F, \tag{32}$$

to find the solution

$$\hat{\mathbf{A}}_R(k)^T = [\hat{\mathbf{A}}_T(k-1) | \otimes | \hat{\mathbf{G}}(k)]^{\dagger} \vec{\mathbf{Y}}_{(2)}. \tag{33}$$

Step 4: Perform the LS fitting of $\mathbf{A}_T$ by solving the following linear LS problem

$$Minimize_{\hat{\mathbf{A}}_T(k)} ||\vec{\mathbf{Y}}_{(1)} - [\hat{\mathbf{G}}(k) | \otimes | \hat{\mathbf{A}}_R(k)] \hat{\mathbf{A}}_T(k)^T ||_F, \tag{34}$$

to find the solution

$$\hat{\mathbf{A}}_T(k)^T = [\hat{\mathbf{G}}(k) | \otimes | \hat{\mathbf{A}}_R(k)]^{\dagger} \vec{\mathbf{Y}}_{(1)}. \tag{35}$$

Step 5: Compute $E = ||\mathbf{Y} - [\hat{\mathbf{A}}_R(k) | \otimes | \hat{\mathbf{A}}_T(k)] \hat{\mathbf{G}}(k)^T ||_F$. If $E \leq \eta$, then terminate the iterative process and go to *Step* 6. Otherwise, set k to $k+1$ and go to *Step* 2.

Step 6: Perform the normalization for each column vector of $\hat{\mathbf{A}}_R(k)$ and $\hat{\mathbf{A}}_T(k)$ as follows

$$\tilde{\mathbf{F}} = \frac{\mathbf{F}}{||\mathbf{F}||}, \tag{36}$$

where $\tilde{\mathbf{F}}$ denotes the normalization of the vector $\mathbf{F}$ and $||\mathbf{F}||$ the norm of $\mathbf{F}$.

Step 7: Designate the qth column vectors of $\hat{\mathbf{A}}_R(k)$ and $\hat{\mathbf{A}}_T(k)$ after normalization as the estimates $\tilde{\mathbf{a}}_R(\phi_q)$ and $\tilde{\mathbf{a}}_T(\theta_q)$ of $\hat{\mathbf{a}}_R(\phi_q)$ and $\hat{\mathbf{a}}_T(\theta_q)$, respectively, $q = 1, 2, \ldots, Q$.

Step 8: Construct the estimates of the direction matrices as follows

$$\tilde{\mathbf{A}}_R = [\sqrt{N_R}\tilde{\mathbf{a}}_R(\phi_1)\ \sqrt{N_R}\tilde{\mathbf{a}}_R(\phi_2)\ldots\sqrt{N_R}\tilde{\mathbf{a}}_R(\phi_Q)] \quad (37)$$

and

$$\tilde{\mathbf{A}}_T = [\sqrt{N_T}\tilde{\mathbf{a}}_T(\theta_1)\ \sqrt{N_T}\tilde{\mathbf{a}}_T(\theta_2)\ldots\sqrt{N_T}\tilde{\mathbf{a}}_T(\theta_Q)] \quad (38)$$

Step 9: Construct the estimated virtual direction matrix as follows

$$\begin{aligned}[\tilde{\mathbf{A}}_R \otimes \tilde{\mathbf{A}}_T] &= [\tilde{\mathbf{a}}_R(\phi_1)\otimes\tilde{\mathbf{a}}_T(\theta_1)\ \ \tilde{\mathbf{a}}_R(\phi_2)\otimes\tilde{\mathbf{a}}_T(\theta_2) \\ &\quad \ldots\ \tilde{\mathbf{a}}_R(\phi_Q)\otimes\tilde{\mathbf{a}}_T(\theta_Q)]. \end{aligned} \quad (39)$$

To enhance the accuracy of the estimated direction vector corresponding to the desired signal, we propose an eigen-based scheme as follows. First, performing the eigen-value decomposition (EVD) of $\hat{\mathbf{R}}_{MIMO}$ given by (8) provides

$$\hat{\mathbf{R}}_{MIMO} = \sum_{i=1}^{N_T N_R} \lambda_i \mathbf{e}_i \mathbf{e}_i^H, \quad (40)$$

where $\mathbf{e}_i$ denotes the ith eigenvector associated with the ith eigenvalue λ_i. As shown in [26], the eigenvalues of the sample covariance matrix $\hat{\mathbf{R}}_{MIMO}$ shown by (40) can be catalogued as the following three groups

$$\lambda_i = \lambda_i^s + \lambda_{mim}, \quad i = 1, 2, \ldots, Q, \quad (41)$$

$$\lambda_i = \lambda_i^u + \lambda_{mim}, \quad i = Q+1, 2, \ldots, N_T N_R - 1, \quad (42)$$

and

$$\lambda_i = \lambda_{mim}, \quad i = N_T N_R, \quad (43)$$

where the first group contains the most significant eigenvalues due to the signal sources while the second and third groups contain the $N_T N_R - Q$ least significant eigenvalues due to noise. λ_i^u represent the differences between the other $N_T N_R - Q - 1$ least significant eigenvalues due to noise and λ_{mim}. Accordingly, we can designate the number of the most significant eigenvalues as the estimate $\hat{Q}$ of Q. Next, taking the average of the most significant eigenvalues due to the signal sources gives

$$\bar{\lambda} = \frac{1}{\hat{Q}} \sum_{q=1}^{\hat{Q}} \lambda_q. \quad (44)$$

Then, we proceed with performing the inner product of the presumed virtual steering vector $\mathbf{a}(\theta_a, \phi_a)$ and each column vector of $[\tilde{\mathbf{A}}_R \otimes \tilde{\mathbf{A}}_T]$ and designating the column vector of $[\tilde{\mathbf{A}}_R \otimes \tilde{\mathbf{A}}_T]$ with the maximal inner product as the estimated virtual steering vector $\mathbf{a}_{de}$.

Finally, we implement an optimization process to find an appropriate estimate for the virtual steering vector to perform robust adaptive MIMO beamforming in the presence of multiple mismatches. Motivated by the RMVB concepts of [16, 31], the optimization problem is formulated as follows

$$Minimize_{\mathbf{u}}\ \ \mathbf{u}^H \mathbf{E}_N \mathbf{E}_N^H \mathbf{u}$$

$$Subject\ to\ \ ||\mathbf{u}|| = \sqrt{N_T N_R} \quad (45)$$

with the initial guess of $\mathbf{u}$ set to $\mathbf{a}_{de}$. Compared to the optimisation problem stated in [16], the proposed optimization of (45) is easier to solve because no additional uncertainty constraints like [16, 31] should be satisfied during the optimization process. Accordingly, the solution of (45) can be easily found by using the the well-known stochastic gradient iterative algorithm. At the nth iteration, the update formula is given by

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \beta \nabla_{\mathbf{u}} J(\mathbf{u})|_{\mathbf{u}=\mathbf{u}(n)} \quad (46)$$

where the step size β is set to $\frac{1}{trace(\mathbf{E}_N \mathbf{E}_N^H)}$ for facilitating convergence and obtaining satisfactory results. During the iteration, the norm of $\mathbf{u}(n)$ should be scaled back to $\sqrt{N_T N_R}$ according to

$$\mathbf{u}(n) = \sqrt{N_T N_R} \frac{\mathbf{u}(n)}{||\mathbf{u}(n)||}, \quad (47)$$

for satisfying the constraint of (45). Moreover, we stop the iteration if $||\mathbf{u}(n+1) - \mathbf{u}(n)|| \leq \kappa$, where κ denotes the preset threshold value to guarantee that the variations of the obtained estimates are negligible such that the convergence of this iterative process is achieved. The solution is designated as $\mathbf{a}_{di}$ after the iteration is terminated.

After obtaining the estimate $\mathbf{a}_{di}$ for the virtual steering vector, we proceed with finding the optimal weight vector for beamforming. It has been shown in [32] that avoiding the phenomenon of signal cancellation under mismatches is in favor of using the interference-plus-noise covariance (IPNC) matrix instead of $\hat{\mathbf{R}}_{MIMO}$. To construct an appropriate IPNC matrix, we first perform the inner product of $\mathbf{a}_{di}$ and each column vector of $[\tilde{\mathbf{A}}_R \otimes \tilde{\mathbf{A}}_T]$ and then rearrange the column vectors of $[\tilde{\mathbf{A}}_R \otimes \tilde{\mathbf{A}}_T]$ from the leftmost column vector with the largest inner product to rightmost column vector with the least inner product to produce a new virtual direction matrix $[\breve{\mathbf{A}}_R \otimes \breve{\mathbf{A}}_T] = [\breve{\mathbf{a}}_R(\phi_1) \otimes \breve{\mathbf{a}}_T(\theta_1)\ \breve{\mathbf{a}}_R(\phi_2) \otimes \breve{\mathbf{a}}_T(\theta_2) \ldots \breve{\mathbf{a}}_R(\phi_{\hat{Q}}) \otimes \breve{\mathbf{a}}_T(\theta_{\hat{Q}})]$. Then, we construct an IPNC matrix as follows

$$\begin{aligned}\hat{\mathbf{R}}_{I+N} &= \sum\nolimits_{i=2}^{\hat{Q}} \bar{\lambda} [\breve{\mathbf{a}}_R(\phi_i) \otimes \breve{\mathbf{a}}_T(\theta_i)][\breve{\mathbf{a}}_R(\phi_i) \otimes \breve{\mathbf{a}}_T(\theta_i)]^H \\ &\quad + \frac{1}{N_T N_R - \hat{Q}} \sum_{j=\hat{Q}+1}^{N_T N_R} \lambda_j \mathbf{I}_{N_T N_R}, \end{aligned} \quad (48)$$

where $\mathbf{I}_{N_T N_R}$ denotes the identity matrix with size $N_T N_R \times N_T N_R$. Accordingly, the weight vector required for performing robust adaptive MIMO beamforming is computed as follows

$$\mathbf{w} = \frac{\hat{\mathbf{R}}_{I+N}^{-1} \mathbf{a}_{di}}{\mathbf{a}_{di}^H \hat{\mathbf{R}}_{I+N}^{-1} \mathbf{a}_{di}}. \quad (49)$$

4. COMPUTATIONAL COMPLEXITY

Here, we evaluate the main computational complexity required for utilizing the proposed method. From *Step* 2 of the iteration, obtaining $[\hat{\mathbf{A}}_R(k-1)|\otimes|\hat{\mathbf{A}}_T(k-1)]$, $[\hat{\mathbf{A}}_R(k-1)|\otimes|\hat{\mathbf{A}}_T(k-1)]^\dagger$, and $\hat{\mathbf{G}}(k)^T = [\hat{\mathbf{A}}_R(k-1)| \otimes |\hat{\mathbf{A}}_T(k-1)]^\dagger \mathbf{Y}$ requires

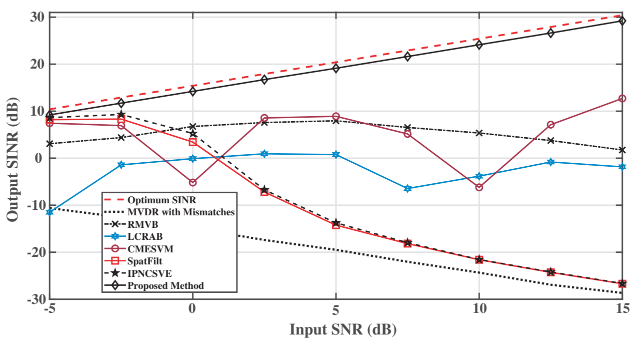

FIGURE 1. The output SINR versus the input SNR for Example 1.

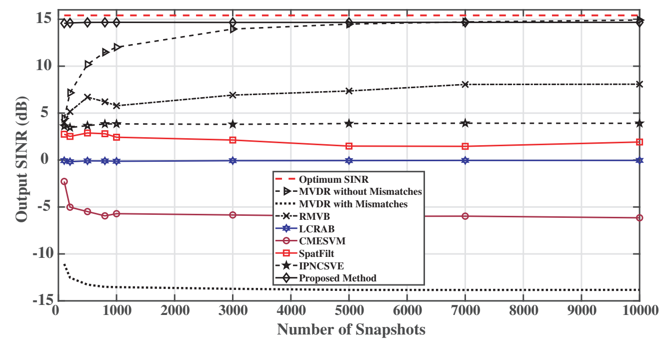

FIGURE 2. The output SINR versus the number of snapshots for Example 1.

$\hat{Q}N_TN_R$ complex multiplications (CMs), $\hat{Q}(N_TN_R)^2 + (N_TN_R)\hat{Q}^2$ CMs, and $L\hat{Q}(N_TN_R)^2$ CMs, respectively. Hence, the total CMs for finding the solution $\hat{\mathbf{G}}(k)^T = [\hat{\mathbf{A}}_R(k-1)| \otimes |\hat{\mathbf{A}}_T(k-1)]^\dagger \mathbf{Y}$ are about $O(\hat{Q}N_TN_R + 2\hat{Q}(N_TN_R)^2 + 2(N_TN_R)\hat{Q}^2 + \hat{Q} + \hat{Q}L(N_TN_R)^2)$. From *Step* 3 of the iteration, obtaining $[\hat{\mathbf{A}}_T(k-1)| \otimes |\hat{\mathbf{G}}(k)]$, $[\hat{\mathbf{A}}_T(k-1)| \otimes |\hat{\mathbf{G}}(k)]^\dagger$, and $\hat{\mathbf{A}}_R(k)^T = [\hat{\mathbf{A}}_T(k-1)| \otimes |\hat{\mathbf{G}}(k)]^\dagger \vec{\mathbf{Y}}_{(2)}$ requires $\hat{Q}N_TL$ complex multiplications (CMs), $\hat{Q}(N_TL)^2 + (N_TL)\hat{Q}^2$ CMs, and $N_R\hat{Q}(N_TL)^2$ CMs, respectively. Hence, the total CMs for finding the solution $\hat{\mathbf{A}}_R(k)^T = [\hat{\mathbf{A}}_T(k-1)| \otimes |\hat{\mathbf{G}}(k)]^\dagger \vec{\mathbf{Y}}_{(2)}$ are about $O(\hat{Q}N_TL + 2\hat{Q}(N_TL)^2 + 2(N_TL)\hat{Q}^2 + \hat{Q} + \hat{Q}N_R(N_TL)^2)$. From *Step* 4 of the iteration, obtaining $[\hat{\mathbf{G}}^{(k)}| \otimes |\hat{\mathbf{A}}_R^{(k)}]$, $[\hat{\mathbf{G}}(k)| \otimes |\hat{\mathbf{A}}_R(k)]^\dagger$, and $\hat{\mathbf{A}}_T(k)^T = [\hat{\mathbf{G}}(k)| \otimes |\hat{\mathbf{A}}_R^{(k)}]^\dagger \vec{\mathbf{Y}}_{(1)}$ requires $\hat{Q}N_RL$ complex multiplications (CMs), $\hat{Q}(N_RL)^2 + (N_RL)\hat{Q}^2$ CMs, and $N_T\hat{Q}(N_RL)^2$ CMs, respectively. Hence, the total CMs for finding the solution $\hat{\mathbf{A}}_T(k)^T = [\hat{\mathbf{G}}(k)| \otimes |\hat{\mathbf{A}}_R(k)]^\dagger \vec{\mathbf{Y}}_{(1)}$ are about $O(\hat{Q}N_RL + 2\hat{Q}(N_RL)^2 + 2(N_RL)\hat{Q}^2 + \hat{Q} + \hat{Q}N_T(N_RL)^2)$. From *Step* 5 of the iteration, computing $E = ||\mathbf{Y} - [\hat{\mathbf{A}}_R(k)| \otimes |\hat{\mathbf{A}}_T(k)]\hat{\mathbf{G}}(k)^T||_F^2$ requires about $O(QN_TN_R + QLN_TN_R + LN_TN_R)$. Accordingly, the main computational complexity for finishing the TALS procedure is about $O(k_t[\hat{Q}N_TN_RL + 2N_TN_R\hat{Q} + N_TN_RL + N_T\hat{Q}L + N_R\hat{Q}L + 3\hat{Q} + 2\hat{Q}(N_TN_R)^2 + 2\hat{Q}(LN_T)^2 + 2\hat{Q}(LN_R)^2 + 2\hat{Q}^2N_TN_R + 2\hat{Q}^2LN_T + 2\hat{Q}^2LN_R + \hat{Q}L(N_TN_R)^2 + \hat{Q}N_R(N_TL)^2 + \hat{Q}N_T(LN_R)^2])$, where k_t denotes the number of iterations.

Next, the normalization of *Step* 6 needs about $2\hat{Q}(N_T+N_R)$ CMs. Performing *Step* 8 and *Step* 9 asks about $\hat{Q}(N_T+N_R)$ and $\hat{Q}N_TN_R$ CMs, respectively. Solving the optimization problem, we have to carry out the EVD of $\hat{R}_{MIMO}$ and calculate $E_NE_N^H$. The CMs required are about $O(2\hat{Q}(N_T + N_R) + 2\hat{Q}N_TN_R + 2(N_TN_R)^3 + (N_TN_R - \hat{Q})(N_TN_R)^2 + k_o[(N_TN_R)^2 + 2N_TN_R])$, where k_o denotes the number of iterations for solving the optimization problem. Moreover, constructing the IPNC matrix needs about $O((\hat{Q}-1)[N_TN_R + (N_TN_R)^2])$. Finally, $O((N_TN_R)^2 + 2N_TN_R)$ is required to compute the optimal weight vector for robust adaptive MIMO radar beamforming.

5. SIMULATION RESULTS

In this section, we consider a bistatic MIMO radar with $N_T = N_R = 6$ for its transmit/receive linear antennas uniformly spaced half the wavelength λ of the desired signal. All of the complex-valued reflection coefficients are Gaussian with mean $= 0$ and variance $= 1$. Moreover, $\kappa = 10^{-6}$ and $\eta = 10^{-5}$. For comparison and confirmation, we present the simulation results of using the proposed method and some notable existing robust techniques including [16, 19–21, 32,]. Moreover, the simulation results for the MVDR radar beamformer without mismatches are also provided. For the convenience of presenting the simulation results, the following abbreviations are adopted for representing the simulation results of using the existing robust methods: (1) RMVB for [16], (2) IPNCSVE for [32], (3) LCRAB for [20], (4) SpatFilt for [19], and (5) CMESVM for [21]. The theoretical optimum results (termed Optimum SINR) can be achieved for all cases which are also provided for illustration and comparison.

5.1. Example 1

For the desired signal, the spatial angle is set to $(\theta_a, \phi_a) = (-5°, 0°)$. Two interferers with interference-to-noise power ratio (INR) = 0 and 10 dB have spatial angles $(\theta_1, \phi_1) = (25°, 15°)$ and $(\theta_2, \phi_2) = (-20°, -10°)$, respectively. The number of data snapshots taken for computing the sample covariance matrix is 5000, and the number of Monte Carlo runs is 50. The steering angle is set to $(\theta_d, \phi_d) = (0°, 5°)$. The steering angle error is 5° for θ and ϕ. The random position errors δx_i and δy_i, $i = 1, 2, \ldots, N_t(N_r)$ are Gaussian with mean $= 0$ and standard deviation $= 0.1\lambda$. Both of ψ_{Ti} and ψ_{Ri} are random variables uniformly distributed in $[0, 2\pi]$. Both of δ_{Ti} and δ_{Ri} are uniform random variables with mean $= 0$ and standard deviation $= 5$. NT_i and NR_i are set to 4. The mutual coupling coefficients are fixed and set to $\mathbf{c}_T = \mathbf{c}_R = [1, 0.9\exp -j(\pi/3), 0.75j\pi/4]^T$ for both of the transmit and receive antennas. Fig. 1 plots the output signal-to-interference plus noise power ratio (SINR) of the bistatic MIMO radar versus the signal-to-noise power ratio (SNR) of the desired signal with the steering angle = $(\theta_d, \phi_d) = (0°, 5°)$. Fig. 2 depicts the output SINR versus the number of data snapshots with the steering angle = $(\theta_d, \phi_d) = (0°, 5°)$ and SNR = 0 dB. Moreover, the simulation results for the MVDR radar beamformer

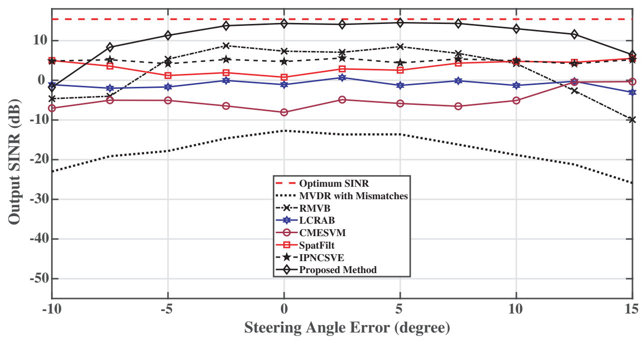

FIGURE 3. The output SINR versus the steering angle error for Example 1.

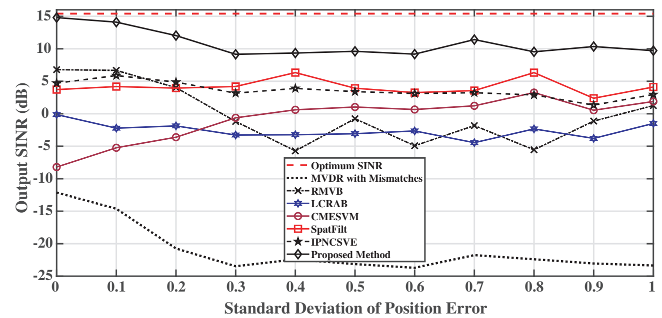

FIGURE 4. The output SINR versus the standard deviation of position error for Example 1.

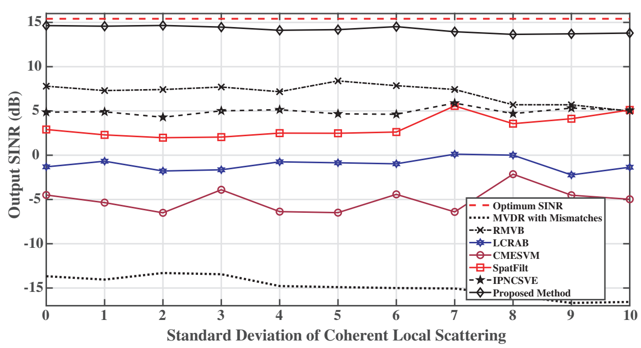

FIGURE 5. The output SINR versus the standard deviation of coherent local scattering for Example 1.

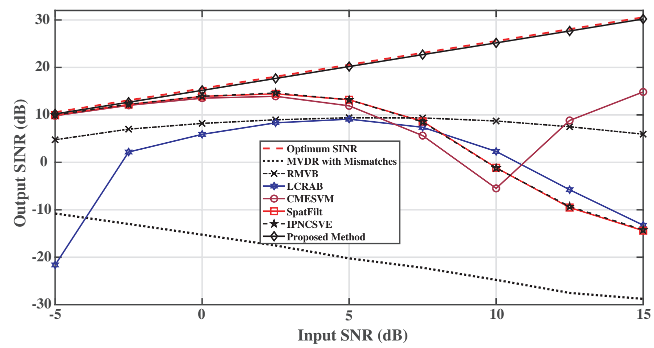

FIGURE 6. The output SINR versus the input SNR for Example 2.

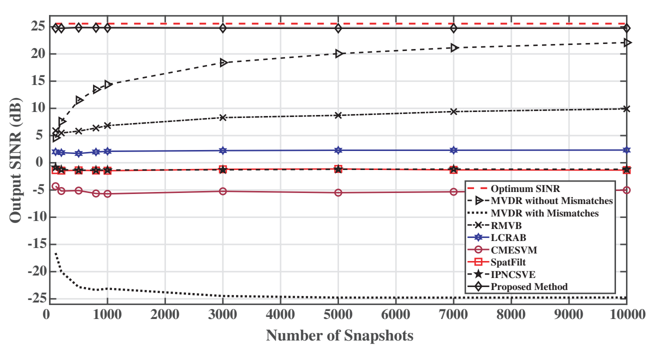

FIGURE 7. The output SINR versus the number of snapshots for Example 2.

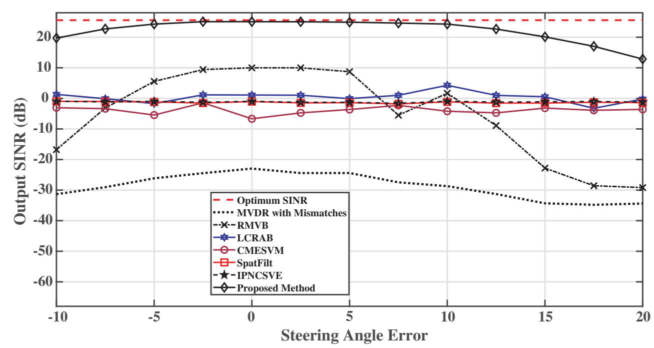

FIGURE 8. The output SINR versus the steering angle error for Example 2.

without mismatches are also provided for illustration and comparison. Fig. 3 shows the output SINR versus the steering angle error with the steering angle errors $\delta\theta = \delta\phi$ varying from -10° to 15° under SNR = 0 dB, standard deviation of position error = 0.1λ, and standard deviation of coherent local scattering = 5. The output SINR versus the standard deviation of position errors under SNR = 0 dB and standard deviation of coherent local scattering = 5 is shown in Fig. 4. The output SINR versus the standard deviation of CLS under SNR = 0 dB and standard deviation of position error = 0.1λ is shown in Fig. 5. As we can observe from these simulation results, the proposed method provides better performance than the existing robust methods in dealing with the problem of performance degradation due to the considered multiple mismatches for bistatic MIMO radar beamforming.

5.2. Example 2

For the desired signal, the spatial angle is set to $(\theta_a, \phi_a) = (-5^\circ, 0^\circ)$. Two interferers with interference-to-noise power ratio (INR) = 10 dB have spatial angles $(\theta_1, \phi_1) = (30^\circ, 20^\circ)$ and $(\theta_2, \phi_2) = (-25^\circ, -15^\circ)$, respectively. The number of data snapshots taken for computing the sample covariance matrix is 5000, and the number of Monte Carlo runs is 50. The steering angle is set to $(\theta_d, \phi_d) = (0^\circ, 5^\circ)$. The steering angle error is 5° for θ and ϕ. The random position errors δx_i and δy_i, $i = 1, 2, \dots, N_t(N_r)$ are Gaussian with mean $= 0$ and standard deviation $= 0.1\lambda$. Both of ψ_{Ti} and ψ_{Ri} are random variables uniformly distributed in $[0, 2\pi]$. Both of δ_{Ti} and δ_{Ri} are uniform random variables with mean $= 0$ and standard deviation $= 5$. NT_i and NR_i are set to 4. The mu-

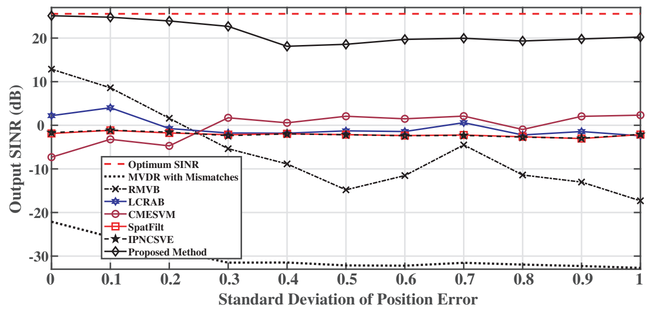

FIGURE 9. The output SINR versus the standard deviation of position error for Example 2.

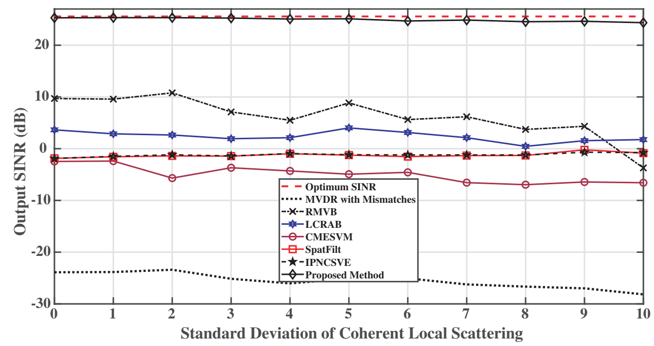

FIGURE 10. The output SINR versus the standard deviation of CLS for Example 2.

tual coupling coefficients are fixed and set to $\mathbf{c}_T = \mathbf{c}_R = [1, 0.9 \exp -j(\pi/3), 0.75j\pi/4]^T$ for both of the transmit and receive antennas. Fig. 6 plots the output signal-to-interference plus noise power ratio (SINR) of the bistatic MIMO radar versus the signal-to-noise power ratio (SNR) of the desired signal with the steering angle set to $(\theta_d, \phi_d) = (0°, 5°)$. Fig. 7 depicts the output SINR versus the number of data snapshots with the steering angle set to $(\theta_d, \phi_d) = (0°, 5°)$ and SNR = 10 dB. Moreover, the simulation results for the MVDR radar beamformer without mismatches are also provided for illustration and comparison. Fig. 8 shows the output SINR versus the steering angle error with the steering angle errors $\delta\theta = \delta\phi$ varying from $-10°$ to $20°$ under SNR = 10 dB, standard deviation of position error = 0.1λ, and standard deviation of coherent local scattering = 5. The output SINR versus the standard deviation of position errors under SNR = 10 dB and standard deviation of coherent local scattering = 5 is shown in Fig. 9. The output SINR versus the standard deviation of CLS under SNR = 10 dB and standard deviation of position error = 0.1λ is shown in Fig. 10. Again, we can observe from the simulation results that the proposed method is superior to the existing robust methods in dealing with the problem of performance degradation due to the considered multiple mismatches for bistatic MIMO radar beamforming.

6. CONCLUSION

In this paper, we have developed a robust adaptive beamforming technique with full degrees of freedom for MIMO radar systems under random mismatches including steering vector error, mutual coupling, sensor position error, and coherent local scattering. The novelty of the proposed technique is that estimating the required virtual steering vector is formulated by exploiting the algebra of a third-order tensor and capturing the multidimensional structure information embedded in the data vector received by a bistatic MIMO radar. Moreover, a gradient-based optimization process is proposed to further enhance the accuracy of the resulting estimate of the virtual steering vector required for robust adaptive MIMO beamforming. As a result, the well-known stochastic gradient iterative algorithm can be employed to find the optimal estimate of the virtual steering vector efficiently. The convergence property and the computational complexity regarding the proposed technique have also been evaluated. Simulation results have been provided for confirming the effectiveness of the proposed technique in dealing with multiple random mismatches. For the further research, it may be interesting to investigate the effectiveness of monostatic MIMO radar and the effects of some important parameters such as the signal frequency, bandwidth, and antenna gain by applying the proposed method.

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