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Low Complexity Irregular Clusters Tiling Through Quarter Region Rotational Symmetry

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Abstract—In order to reduce the complexity and cost of an $N \times M$ large planar array from a practical point of view, firstly, the array matrix is divided into four equal $\frac{N}{4} \times \frac{M}{4}$ quarter regions, and then only one quarter is selected to be optimized. After that, this selected quarter region is tiled with a few irregular polyomino clusters (IPCs) and then rotating it to the other three-quarter regions. This method is called Quarter Region Rotational Symmetry (QRRS). The copy from the selected region is rotated by three angles 90, 180, and 270 degrees respectively until the main planar array is filled. Two methods of feeding clusters based on amplitude only and phase only were used to reduce the complexity further. In addition, the complexity can be reduced more by applying the thinning technique with clusters or building clusters for a part of the planar array. A genetic algorithm (GA) is used to implement these ideas until a radiation pattern (RP) useful for modern applications. An additional constraint is included in the optimization process represented by a mask to cover the pattern according to the desired shape. The simulation results showed that the RP can be fully controlled by applying the QRRS technique successfully while reducing the complexity of the feeding network to only 2.25% in the amplitude-only and phase-only cases, and 1.75% and 1.5% in the thinning and partially tiling cases, respectively. Moreover, a detailed design of the feeding network circuit of the main planar array based on IPC is given for practical implementation.

1. INTRODUCTION

Modern large planar array antenna (MLPAA) provides robust techniques for high coverage radiation performance, low sidelobes, and high directivity. But it still suffers from high complexity in practice. It is known that approximately half of the complexity of the array system is due to the transceiver modules (T/R units) that control the amplitude and time delay of each radiating element to perform the desired radiation beamforming [1]. To reduce complexity while still providing satisfactory radiometric features, several untraditional architectures such as clusters [2–4], thinning [5], sparse arrays [6], amplitude-only [7], and phase-only [8] have been proposed instead of optimization using all-element solutions. Such proposals are gaining great attention from researchers to form the required patterns in the fifth-generation communication applications and modern radar systems [9]. In the case of using the cluster methods to reduce the complexity, structures of low complexity are implemented by forming an aperture planar array with primary tiles consisting of two or more radiating elements. Each tile is excited by a single T/R unit with the condition of maintaining the efficiency of the desired RP [10]. Unfortunately, in this case, despite the positive features, a major problem appears, which is represented by the appearance of high periodic sidelobes [11]. In fact, when equidistant rectangular or squareshaped clusters are used to reduce the complexity of the array system, the final RP (FRP) will suffer by generating grating lobes of different undesirable levels, and their numbers increase unavoidably, leading to a widening of the scan angle and operational bandwidth [1].

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To address this defect, arranging clusters of different sizes [12–14] or tiling polyomino shapes within the antenna aperture [4, 15, 16] has been successfully constructed. Actually, using such proposing breaks the periodic quantization, and this leads to a reduction in the level of undesirable sidelobes. Despite the availability of effective clustering methods, on the other hand, there are no perfect design methods for slabbing arrays, especially large arrays, where the mathematical problem for them is very complex [1]. Moreover, optimal tiling theories [4, 17] and efficient algorithms for domino-shaped [18] were reached, which were profitably exploited to build large arrays.

Several algorithms have been proposed by researchers to tile efficient cluster architectures to find an accurate cover solution by performing the RP in terms of controlling the sidelobes and beam scanning. Some researchers used biological algorithms such as GA [19], ant colony [20], particle swarm [21], artificial bee colony (ABC) [22], and others relied on enumerative methods such as X algorithm [23], H-ICRP [24], MT-BCS [24], and compressed sensing [25].

Despite the success of these approaches in the building of clusters, some practical obstacles prevent them from being applied to MLPAA. But if the array has the symmetry property, each tile solution may correspond to a new arrangement after flipping or rotating by a certain angle. Thus, the number of clusters needed to fill the array grows rapidly with the size of the array. For example, when tiling clusters are based on different forms of T-shaped clusters [4], the number of possible clusters allowed for an array of 20×20 is only 62 clusters. In [23], when the array is tiled with L-shaped clusters of 4 elements, the number of possible clusters allowed for an 8×8 array is 4 and rounded at angles of 90, 180, and 270 degrees. In these cases, the computational efficiency of the currently proposed methods remains an area for efficient improvement. Besides, the discussion of mutual coupling between elements or between clusters is not fully considered in these methods. This may lead to a deterioration in sidelobes control performance as well as scanning performance. Reducing the complexity of array systems leads to many advantages, both from an economic point of view and in terms of reducing mutual coupling. The use of the non-traditional cluster principle with the thinning approach provides these advantages, in addition to that, portioning the array into a few rotated clusters so that the same feeding network is used for each rotated cluster.

This paper aims at nonuniform cluster tiling within the $N \times M$ planar array with the suggestion of a QRRS property through which the complexity of the feeding network is reduce,d and good performance of sidelobe control is obtained with good controlling shape performance. These clusters' structures can be designed simply in practice and at a lower cost. The property of the region's rotation is without a geometrical change and with a set of clockwise rotations at angles of 90, 180, and 270 degrees. Two methods of exciting clusters based on amplitude only and phase only were used to reduce the complexity more. In addition, the complexity can be reduced further by applying the thinning technique with clusters or synthesizing a partially clustered planar array. Regarding the tiling of irregular clusters as well as the rotation of these clusters regionally, a genetic algorithm with fast convergence was used. The final array includes a set of clusters in the form of similarly rotated irregular polyominos.

2. MATHEMATICAL MODEL OF PROPOSED APPROACH

2.1. Model of IPC Tiling

Let us consider a planar array consisting of $N \times M$ space placed on a square grid with spacing between the elements of dx and dy along the axes of the surface. This array is filled with radiating elements in the form of IPC. Firstly, this main large array is divided into four quarter regions (four main subarrays). Each quarter region is composed of $\frac{N}{4} \times \frac{M}{4}$ space. Only one region is taken in the optimization process to tile small irregular clusters. Figure 1 shows the initial architecture of the $N \times M$ array space.

To build a mathematical model of the first quarter region of the $N \times M$ array, $\frac{N}{4} \times \frac{M}{4}$ is expressed as an empty matrix filled with zeros:

$$Z_M = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}_{\frac{N}{4} \times \frac{M}{4}} \tag{1}$$

Then, this sub-matrix is filled with different shapes of clusters in the form of different unequal and

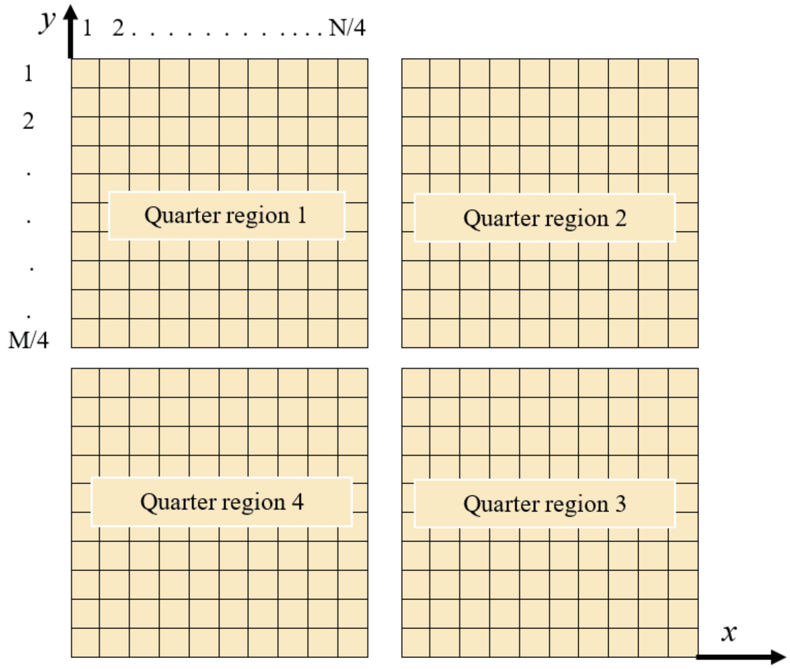


Figure 1. General structure of large planar array with four symmetrical quarter regions.

irregular polyominos. These polyominos are arranged in ascending order in terms of the number of elements, starting from the first cluster at the corner of the array and reaching the center as mentioned in Figure 1. As a result, there will be a tiling of polyominos in different directions. The tiling of the clusters begins by filling the Z_M structure with ones representing the proposed shapes, with the possibility of rotating this shape, for example:

$$P_1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Then, the Z_M structure is filled with other forms of polyominos through P_n matrices containing the suggested shapes:

$$P_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad P_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}, \quad \ldots$$

This is done until the Z_M structure is completely filled. By having the quarter structure matrix with the P_n matrices, it is possible to create and tile the first quarter region $\frac{N}{4} \times \frac{M}{4}$ of the large planar array with IPC at the x and y positions. It is important to consider that it is not possible to tile the same shape of the proposed cluster sequentially inside the structure so that no occupied radiating element remains outside the polyomino forms. Also, it is forbidden to cross any form of cluster outside the boundaries of the main array structure, i.e., all clusters must be arranged within the main structure. After that, the full structure with its contents of the clusters can be written as:

$$S_\gamma(x, y) = Z_{M_{x,y}} + \sum_{i=1}^{k-1} Z_{M_{xi,yi}} \tag{2}$$

where γ and k represent the direction and number of elements in the proposed cluster shape, respectively. If $S = 0$, it means that the option to put the proposed cluster shape in the current position within the structure is possible. Let's try to put the first irregular cluster P_1 inside the empty structure of an array

$\frac{N}{4} \times \frac{M}{4} = 10 \times 10$ as:

$$A = \begin{bmatrix} \mathbf{11}00000000 \\ \mathbf{1}000000000 \\ 0000000000 \\ 0000000000 \\ 0000000000 \\ 0000000000 \\ 0000000000 \\ 0000000000 \\ 0000000000 \\ 0000000000 \end{bmatrix} \tag{3}$$

Now, we have the matrix of two models, the first quarter structure matrix and the proposed IPC shapes matrix. This criterion enables us to describe the geometric relationship between the elements in the array system. In addition, the electromagnetic relationship between the elements must be considered in order to calculate the desired RP.

To control the RP, we start with the planar array factor (PAF), which includes several control parameters such as amplitude and phase excitations, as follows [4]:

$$\text{PAF}(\theta, \emptyset) = \sum_{n=1}^{N/4} \sum_{m=1}^{M/4} w_{nm} e^{-jk[ndx(u-u_o)+mdy(v-v_o)]} \tag{4}$$

where $w_{nm} = a_{nm}e^{jp_{nm}}$ represents the complex weights; a_{nm} is amplitude excitation; p_{nm} is phase excitation; k is the wave number equal to $2\pi/\lambda$; λ is the wavelength; dx and dy are the distance between elements in the x and y axes ($dx = dy = 0.5$); and (u_o and v_o) express the directions of the main beam on the two axes. Each radiating element in Eq. (4) is associated with an individual RF (attenuator and phase shifter), but using the cluster technique all elements in a single cluster are associated with a single radio frequency (RF), so Eq. (4) must be rewritten. If we consider regular rectangular or square clusters consisting of $N_o \times M_o$ elements with amplitude and phase excitations, the clustering factor (CF) can be written as:

$$\text{CF}(\theta, \emptyset) = \sum_{n=1}^{N_o} \sum_{m=1}^{M_o} a_{nm} e^{jp_{nm}} e^{-jk[ndx(u-u_o)+mdy(v-v_o)]} \tag{5}$$

However, using irregular and unequal polyominos will complicate the issue of tiling and differ from regular rectangular or square clusters. To take this complication into account, the axes and direction of each polyomino must be taken in the CF calculations, so the CF can be written as:

$$\text{CF}(\theta, \emptyset) = \sum_{i=1}^{k} a_i e^{jp_i} e^{-jk[P_{i,1}^{\gamma} dx(u-u_o)+P_{i,2}^{\gamma} dy(v-v_o)]} \tag{6}$$

As a result, the final PAF containing the proposed IPC is calculated:

$$\text{PAF}(\theta, \emptyset) = \sum_{c=1}^{C} CF(\theta, \emptyset)\, e^{-j2\pi f T_c} \tag{7}$$

where C is the number of polyomino clusters in $\frac{N}{4} \times \frac{M}{4}$, and T_c is the time delay and can be computed from [26]:

$$T_c = \frac{1}{c_L}[x_i dx](u-u_o) + \frac{1}{c_L}[y_i dy](v-v_o) \tag{8}$$

where c_L is the velocity in free space. In order to greatly reduce the complexity of the planar array system as well as reduce the mutual coupling between the clusters, a thinning approach can be applied to the optimization process. Here, some polyominos are turned off by the GA. The cancellation process depends on searching for the least effective clusters on the FRP. To do this, an additional parameter is added to Eq. (7) as follows:

$$\mathrm{PAF}\,(\theta, \emptyset) = \sum_{c=1}^{C} Z_c \mathrm{CF}\,(\theta, \emptyset)\, e^{-j2\pi f T_c} \tag{9}$$

where Z_c is a sub-matrix through which it is decided which cluster is in the ON or OFF state. The values of this matrix should contain ones and zeros depending on the optimization process to choose the appropriate clusters in the construction of the FRP.

2.2. The Fully Planar Array Construction with QRRS Model

The subarray $\frac{N}{4} \times \frac{M}{4}$ is shown in Figure 1, which is tiled with IPC that will be rotated in a series of clockwise rotations by angles of 90, 180, and 270 degrees so that the main array is completely filled as shown in Figure 2. In this case, the main array will consist of four quarter regions. The first quarter region (reference region) is represented by the main polyomino clusters, and the rest of the quarters are rotated copies of the first quarter. It is worth noting that there is no geometric change in the process of tiling the other three-quarter subarrays. The use of the QRRS technique leads to a significant reduction in the complexity of the feeding network, then the complexity percentage (CP) can be calculated through:

$$\mathrm{CP} = \frac{\textit{number of P clusters in first quarter}}{N \times M} \times 100\% \tag{10}$$

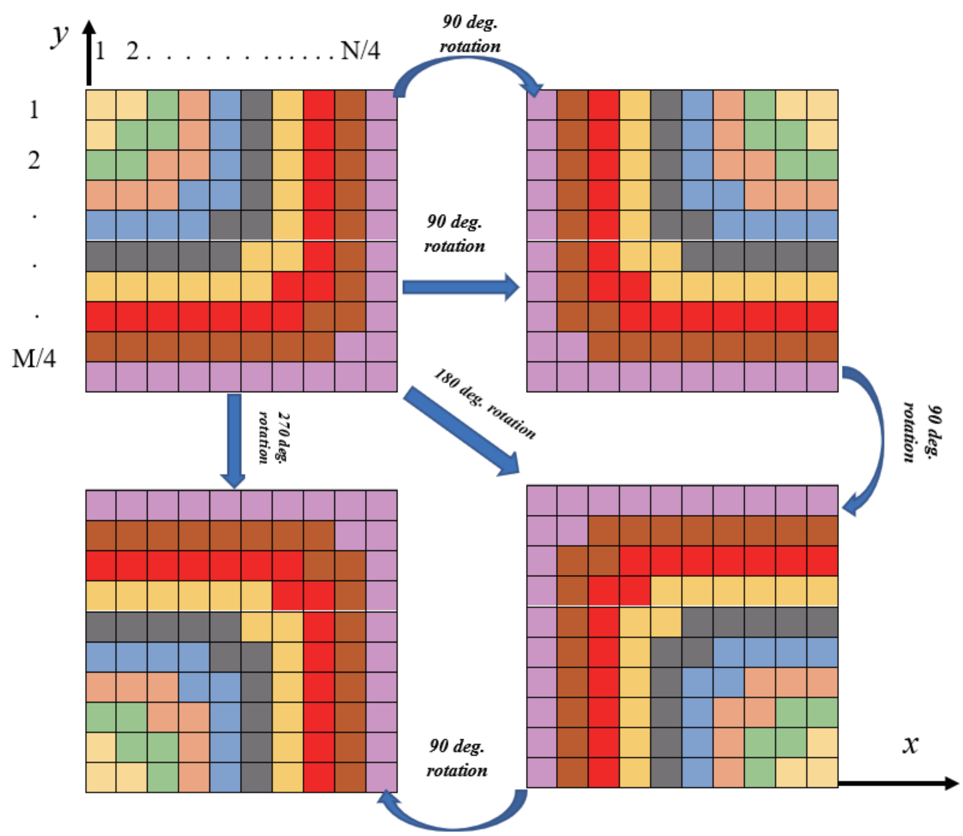


Figure 2. Configuration of irregular clusters slabbing with QRRS.

2.3. Valuation of Sidelobe Reduction and Desired Shaping Rendering

To demonstrate the effectiveness of applying the proposed QRRS idea to construct the MLPAA pattern, the FRP must be appropriately controlled in terms of reducing the sidelobes and designing desired shape. To do this, an extra constraint condition is applied to the optimization process. This condition is represented in determining the level of the upper lobes to a certain extent that is determined by the designer. At the same time the desired shape of the RP is formed. This condition can be expressed as:

$$Extra\ constraint_{for2D\text{-}pattern} = \sum_{x} abs[PAF\,(\theta, \emptyset) - upper\ lobes\ border]^2 \quad (11)$$

$$Extra\ constraint_{for3D\text{-}pattern} = \sum_{xy} abs[PAF\,(\theta, \emptyset) - upper\ surface\ lobes\ grid]^2 \quad (12)$$

3. COMPUTER VALIDATION AND PERFORMANCE EVALUATION

The proposed framework discusses an analysis of the designing and tiling of IPC blocks with full aperture coverage that radiates a minimal RP with low sidelobes. Besides achieving this performance, the complexity of the array system is reduced by illustrating a set of representative examples. In all examples, a genetic algorithm is used with the following characteristics: population size of 21; selection is roulette; crossover is the single point; mutation rate is 0.155.

3.1. Low Complexity with Amplitude only IPC Tiling

In this section, a planar array with $N \times M = 20 \times 20$ elements has been selected to be tiled with IPC. To reduce the complexity of the array system, amplitude-only weights are used instead of complex weights in the optimization process. This leads to a reduction in the total complexity percentage (CP) by half initially. To do this, the parameter p_{nm} is set to zero in Eq. (5), and the amplitude a_{nm} weights are chosen between 0 and 1 in the optimization process.

Firstly, only $\frac{N}{4} \times \frac{M}{4} = 10 \times 10$ of the main array is tiled with IPC, then this quarter region is tripled and rotated by angles 90, 180, and 270 degrees respectively to fill the main array. It is worth mentioning here that the feeding networks used in the first quarter will be used in other quarter regions. Therefore, the CP according to Eq. (10) is 2.25%, i.e., only $P = 9$ clusters are used.

Figure 3(a) shows the top view of the first quarter of the amplitude-only distribution of the tiled IPC. Figure 3(b) shows the top view of the amplitude-only distribution of the main array after rotating the first quarter region three times. Figure 3(c) shows the top view of the phase-only ($p_{nm} = 0$) distribution for all elements in all clusters. Figure 3(d) is the v-cut RP covering with 2D-upper lobes border mask, and Figure 3(e) is the 3D-RP covering with 3D-upper lobes border mask grid. It is noted through these figures that the obtained results confirm the presented analysis. Also, the desired RP for the tiled irregular cluster arrays does not exceed the covered constraint mask. Moreover, reducing the number of clusters leads to a decrease in the mutual coupling between clusters.

3.2. Low Complexity with Phase-Only IPC Tiling

In this section, phase-only weights distribution of $\frac{N}{4} \times \frac{M}{4}$ region instead of amplitude only weights is employed. In other words, the amplitude values are fixed to 1, and the phase values are confined between $-\pi/2$ and $\pi/2$ in the optimization process. Also, the use of phase only weights reduces the total complexity of the system by half initially.

Figure 4(a) shows the top view of the amplitude-only ($a_{nm} = 1$) distribution for all elements in all clusters. Figure 4(b) shows the phase excitation distribution using the QRRS technique, and Figures 4(c) and (d) show the 2D and 3D radiation patterns, respectively. Again, the desired RP is achieved using the QRRS technique with phase-only excitation. It is observed from Figure 4 that the tiling of IPC with phase-only excitation is somewhat difficult. Several runs of the algorithm were made to obtain the desired RP. Thus, through Figures 4(c) and (d), the constrained mask level has been reduced in order to obtain a pattern similar to the pattern in the amplitude-only case.

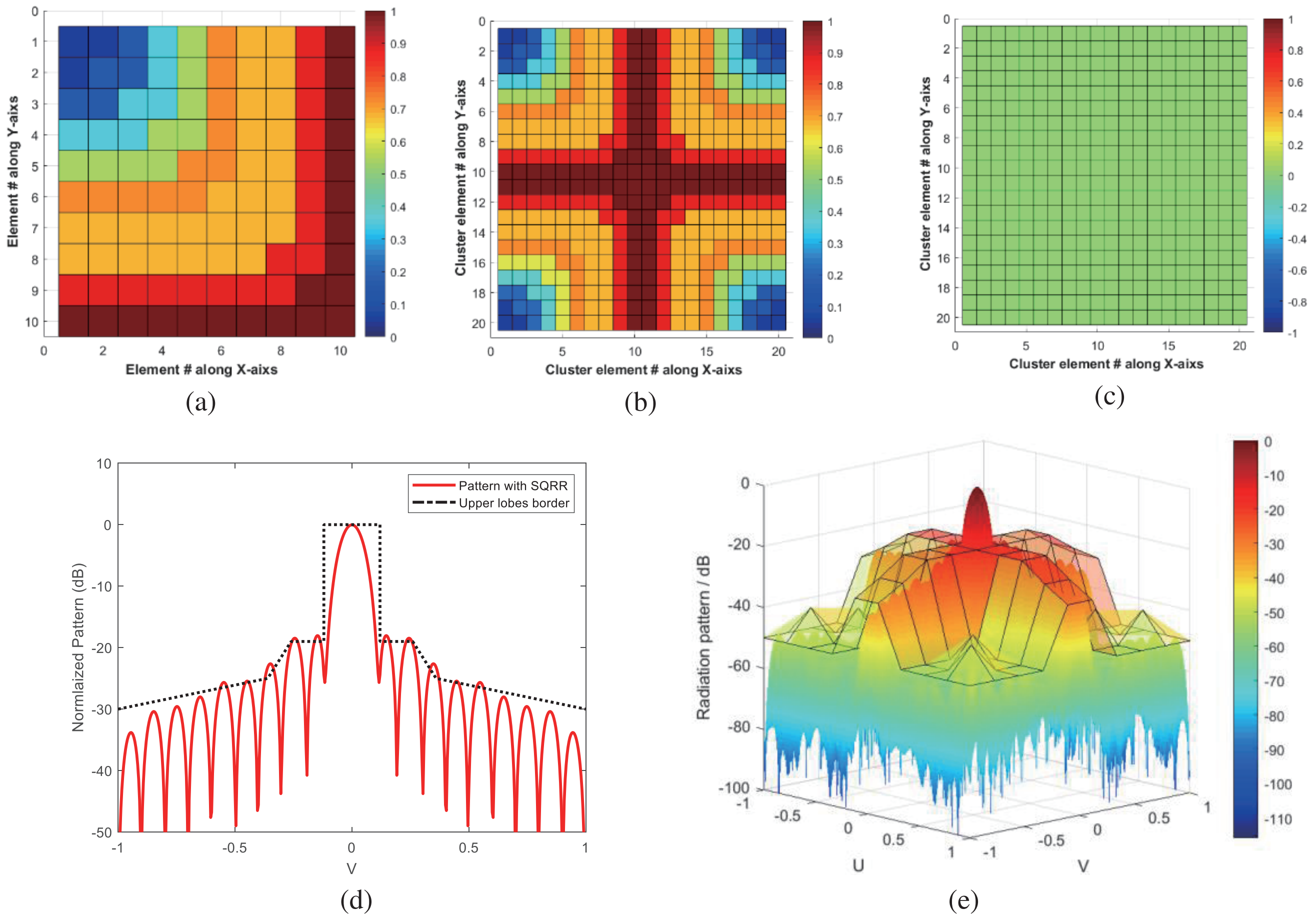


Figure 3. Layout of amplitude only IPC tiling. (a) The top view of first quarter region. (b) Top view of the fully amplitude excitation cluster planar tiling with SQRR. (c) Top view of the phase excitation of the fully cluster planar tiling. (d) The azimuth cuts of the RP with upper lobes border. (e) Final 3D-RP with upper surface lobes grid.

3.3. Ultra-Low Complexity with IPC Thinning

Reducing the number of polyominos leads to a significant reduction in the complexity of the feeding network, and thus the cost is low. The number of elements in the cluster increases as it approaches the center of the array and has higher weights than the clusters located on the periphery. Based on this concept, the complexity can be further reduced by turning off some clusters that have low weight elements. Or approximate the values of the amplitude weights of the elements in the clusters close to the center to ones so that it does not need to use attenuators in practice. To do this, the GA performs this task through the proposed coercive constraint.

Figure 5 shows the v-cut of the RP for IPC tiling with thinned clusters and its corresponding amplitude-only distribution. It is seen through this figure that three clusters have been turned off with the obtaining RP completely similar to Figure 3. Here, after removing three clusters, the CP is 1.5%. This rate is considered very low and has not been obtained previously by other researchers. Moreover, turning off non-sequential clusters leads to greatly reducing mutual coupling.

Figure 6 shows the use of the thinning technique with a phase-only IPC array. Here, the constraints added to the optimization process are reduced to obtain a practically acceptable RP. Figure 7 shows the results of installing partially IPC tiling using the QRRS technique, where the weights of the clusters located at the center are fixed with ones (i.e., uniform excitation) while optimize the rest of the clusters to build the desired pattern. Here, only 7 clusters are constructed, meaning that the CP is 1.75%. By looking at Figures 5 and 6, the desired RP is achieved under an imposed constraint with very few

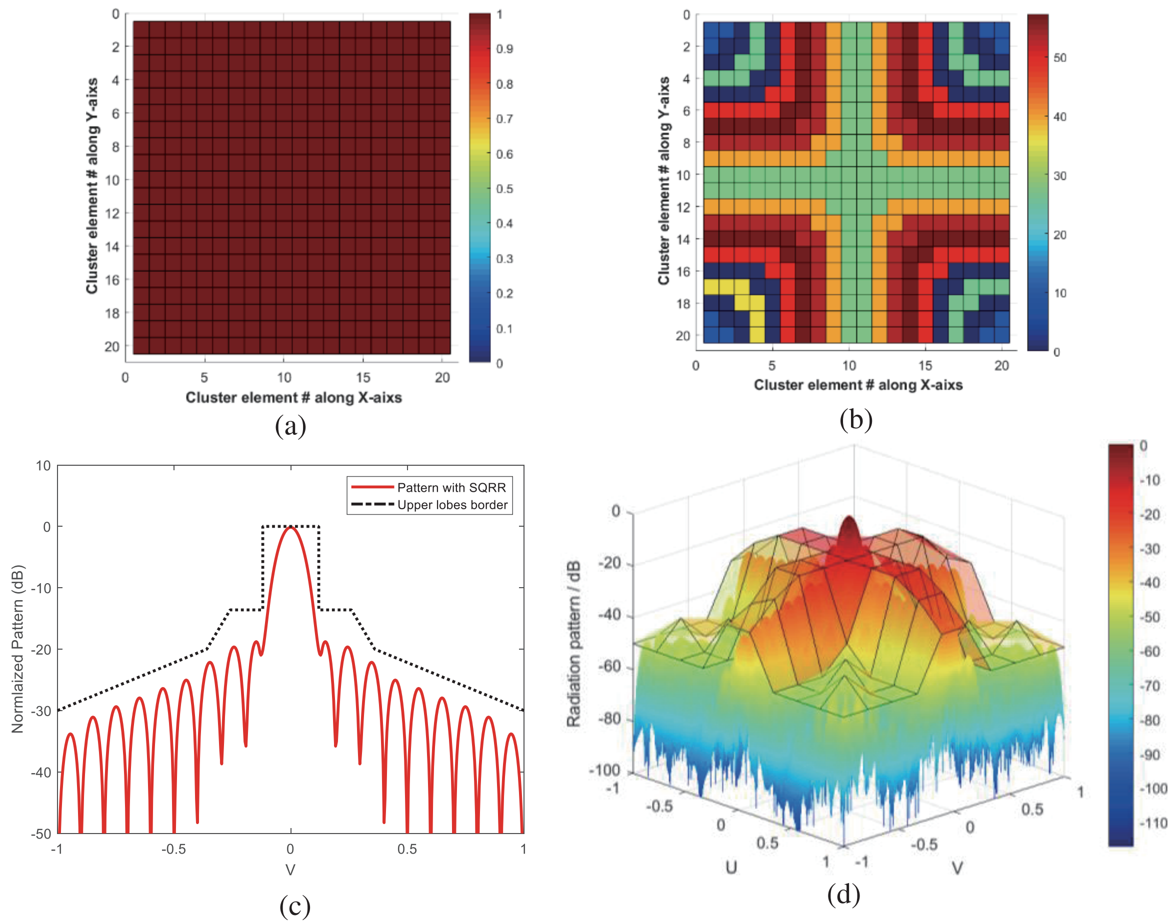


Figure 4. Layout of phase only IPC tiling. (a) The top view of amplitude excitaion. (b) Top view of the fully phase excitation cluster planar tiling with SQRR. (c) The azimuth cuts of the radiation pattern with upper lobes border. (d) Final 3D-radiation pattern with upper surface lobes grid.

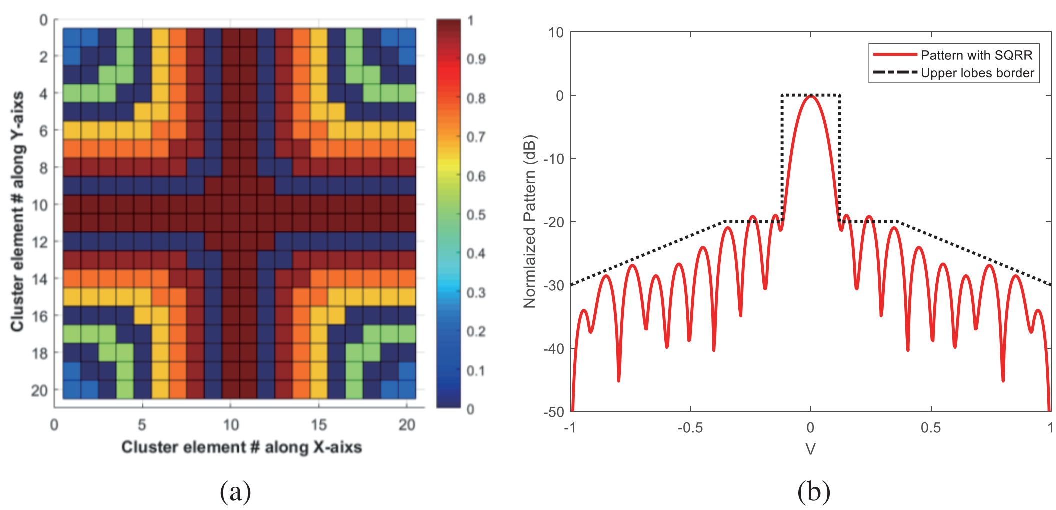
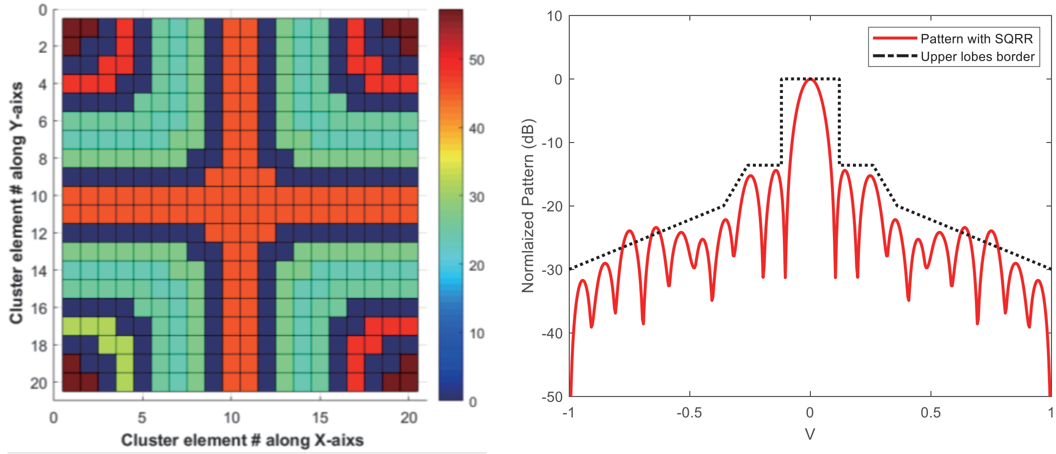
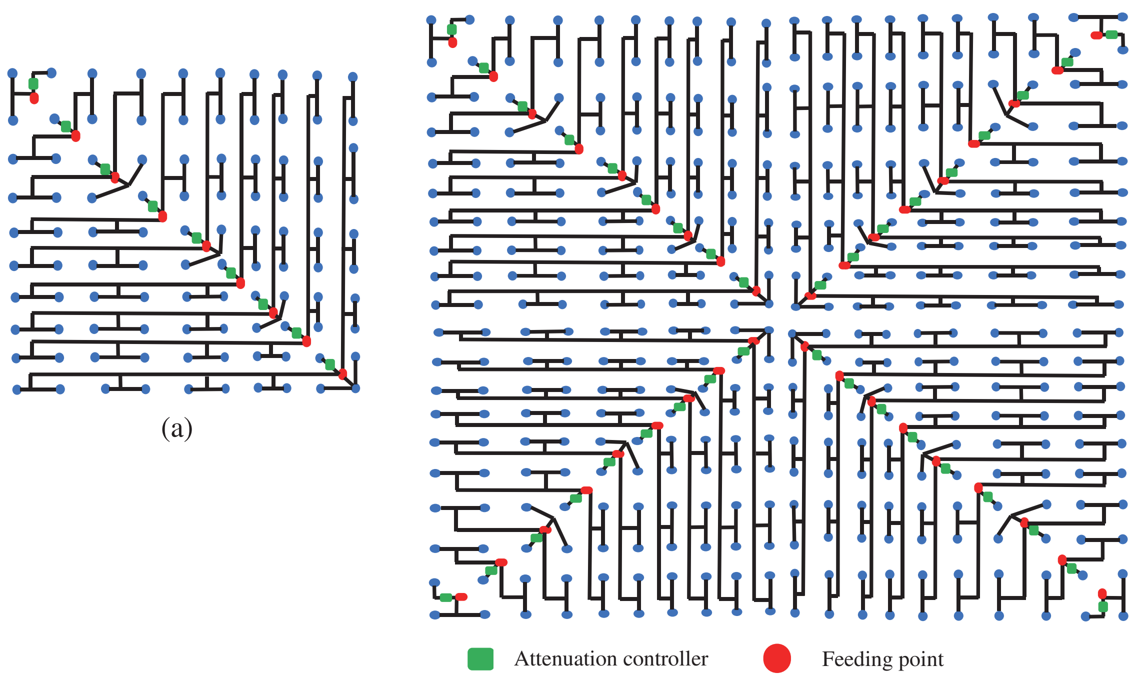


Figure 5. Layout of amplitude only IPC tiling with thinning approach. (a) The top view of amplitude excitaion, (b) the azimuth cuts of the RP with upper lobes border.



(a) (b)

Figure 6. Layout of phase only IPC tiling with thinning approach. (a) The top view of phase excitaion, (b) the azimuth cuts of the RP with upper lobes border.



(a) (b)

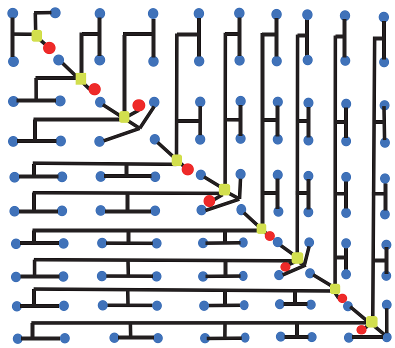
Figure 7. Layout of amplitude only IPC tiling feeding network. (a) The top view of amplitude only feeding network of the first quarter region, (b) the top view of amplitude only feeding network of the full planar regions.

clusters. Of course, there is a clear superiority of these methods (thinned clusters and partially IPC tiling) in controlling the pattern with a few degrees of freedom in the optimization process.

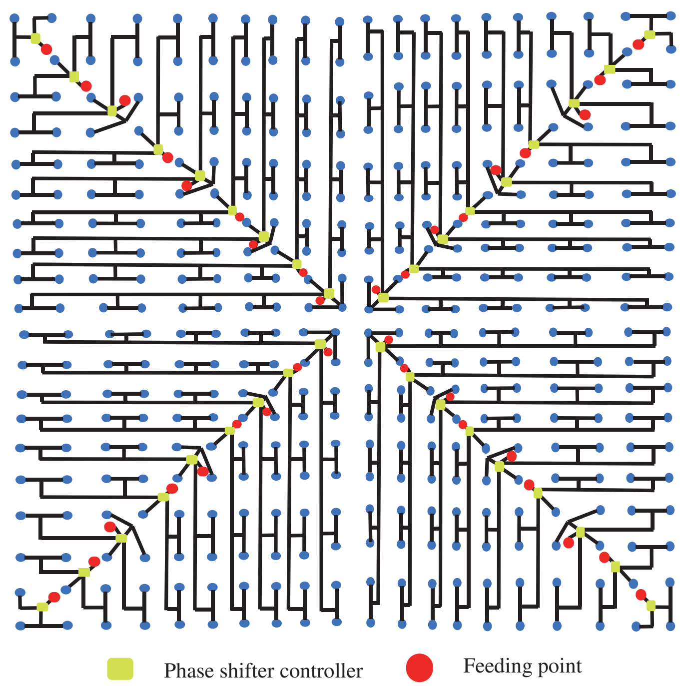
3.4. IPC Power Divider Network Designing

The main benefit of designing a clustered array is to reduce the complexity of the exciting network practically and thus reduce the cost. Anyway, proposing irregular clusters with their rotation and construction in other parts of the main array leads to the required reduction in complexity. In addition to that, with proposing and designing only one-quarter region of the main planar array and rotating it in axes to other regions, there will be little loss in the power dividers. Figure 7(a) shows the amplitude-only feeding network for the first quarter region of the main planar array, and Figure 7(b) shows the amplitude-only feeding network for the main planar array after the rotation of the first region three times. It is observed through this figure that the power dividers for all the clusters in any direction are symmetrical and regular in distribution. Also, all power dividers have the same power flow for each element within the cluster. Due to the different sizes of the clusters in terms of the number of elements, it is necessary to use attenuators within the cluster in order to unify the power flow for all the elements. The attenuator is designed in each cluster independently to attenuate the power to the desired value to feed the required element.

Figure 8(a) shows the phase-only feeding network for the first quarter region of the main planar array, and Figure 8(b) shows the phase-only feeding network for the main planar array. One phase shifter is used for each cluster to give the same phase value to each element.



(a)



(b)

Figure 8. Layout of phase only IPC tiling feeding network. (a) The top view of phase only feeding network of the first quarter region, (b) the top view of phase only feeding network of the full planar regions.

4. CONCLUSION

Low complexity and low-cost efficient IPC tiling method is investigated based on the QRRS technique. It is clear from the current investigations that the desired RP can be achieved by tiling one quarter region of the main planar array with irregular clusters and then rotating it three times to fill the entire array. To fully control this pattern, an additional constraint is imposed on the optimization process represented by a covering mask to define the boundaries of the sidelobes. In addition, the complexity of the feeding network is greatly reduced through the use of amplitude-only or phase-only excitation with thinning technique or partial array tiling. Also, the use of these methods in constructing patterns reduces the effect of mutual coupling between clusters. The simulation results showed the feasibility of the proposed methods in building large planar arrays in modern applications such as multiple-input multiple-output (MIMO) in the fifth generation.

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