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Scattering of Electromagnetic Waves by a Multi-Element System of Pass-through Resonators in a Rectangular Waveguide

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Abstract—The problem of electromagnetic waves diffraction by a system of pass-through resonators in a rectangular waveguide coupling by diaphragms with resonant slots was solved by the generalized method of induced magnetomotive forces (MMFs). A distinctive feature of the solution is characterized by using approximating functions defining magnetic currents in the slots obtained from solutions of current integral equations by the asymptotic averaging method. Multi-parameter studies of electrodynamic characteristics of such structures have been carried out. The comparison of numerical results with experimental data is presented.

1. INTRODUCTION

Single and multi-element diaphragms with slots having pronounced resonant properties are widely used in antenna-waveguide devices operating in centimeter and millimeter wavelength ranges. These devises include: band-pass and band-stop filters, polarizers, shielding structures, phase shifters, etc. Such structures have been attracting the attention of researchers (see, for example, [3–15]) for 70 years after classic works by Southworth and Lewin [1, 2]. A special place is occupied by combined resonator-slot systems allowing to improve characteristics of waveguide devices, for example, significantly increase the Q-factor of entire structures [16–19]. The analysis of these structures can be carried out based on using approximate radio engineering methods (equivalent circuits method [16, 17]), mode matching technique [18–20] or commercial software, requiring large resources and computer time (Ansys HFSS [21, 22]). However, in a number of publications (for example, [16, 17]), the interaction between adjacent diaphragms is not always taken into account, while diaphragms are assumed to be infinitely thin. A burden of the mode matching technique is its relatively slow convergence due to an inadequate modeling of the field singularities at metallic edges [20].

In this paper, an approximate analytical solution (in a strict electrodynamic setting) to a problem concerning a system of resonators (unlike [16–22] resonators of different sizes) sequentially positioned in a rectangular waveguide and coupling through diaphragms with resonant slots of finite thickness is presented. A distinctive feature of the solution is characterized by using approximating functions defining magnetic currents in the slots obtained from solutions of current integral equations by the asymptotic averaging method. The solution takes into account the mutual influence between resonators, and it is built on the basis of generalized method of induced MMF [23] and Green's functions for corresponding volumes. In the plane of waveguide cross-sections, slots in diaphragm can be displaced relative to waveguide wide walls. Unlike [23], where one-cavity structures were considered, the paper presents a new analytical model of a rectangular waveguide with multiple non-periodic discontinuities in the form of resonant irises.

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2. FORMULATION OF THE PROBLEM AND SOLUTION OF INTEGRAL EQUATIONS FOR THE CURRENTS

Consider a chain of resonators in an infinite rectangular waveguide with perfectly conducting walls. The resonators are coupling to each other through diaphragms with resonant slot S_i $(i = 1, 2 \dots N)$ as shown in Fig. 1. The waveguide cross section is $\{a \times b\}$.

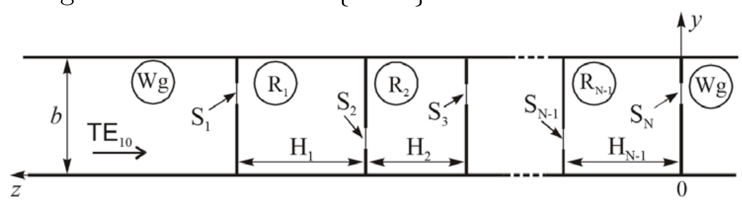

Figure 1. The structure geometry and notations.

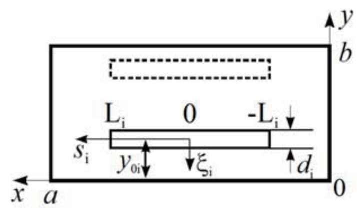

Figure 2. The geometry of slots and notations.

Let geometric slot dimensions satisfy the following relations

$$\frac{d_i}{2L_i} \ll 1, \quad \frac{d_i}{\lambda} \ll 1, \tag{1}$$

where $2L_i$ and d_i are the length and width of the slots (Fig. 2), and λ is the wavelength in free space. The problem consists in finding equivalent magnetic currents in the slots and scattering matrix elements of the entire system. If TE_{10}-wave propagates in the waveguide from the region $z = \infty$, the currents can be represented as

$$\vec{J}_i(s_i, \xi_i) = \vec{e}_{s_i} J_{0i} f_i(s_i) \chi_i(\xi_i), \tag{2}$$

where $\vec{e}_{s_i}$ are the unit vectors; s_i and ξ_i are the local coordinates associated with slots; J_{0i} are unknown current amplitudes. The functions $f_i(s_i)$ must satisfy the boundary condition $f_i(\pm L_i) = 0$, and the functions $\chi_i(\xi_i)$ must satisfy the condition on the slot edges and normalization condition

$$\int\limits_{\xi_i} \chi_i(\xi_i)\, d\xi_i = 1. \tag{3}$$

Since the slots cut in the infinitely thin walls are located symmetrically with respect to the axial line of the waveguide wide wall, it is advisable to choose the functions $f_i(s_i)$ and $\chi_i(\xi_i)$ in the form [23]:

$$\begin{aligned} f_i(s_i) &= \cos k s_i \cos k_c L_i - \cos k L_i \cos k_c s_i, \\ \chi_i(\xi_i) &= \frac{1}{\pi\sqrt{(d_i/2)^2 - \xi_i^2}}, \end{aligned} \tag{4}$$

where $k = 2\pi/\lambda$, $k_c = 2\pi/\lambda_c$, and $\lambda_c = 2a$ is the critical wavelength of the TE_{10}-wave.

It should be noted that the problem of a resonant diaphragm was solved by the variational method by using only the single function $f(s) = \cos(\pi s/2L)$ [2]; therefore, the solution depends only on the slot dimensions and, unlike the proposed method, does not take into account the waveguide parameters.

Let us use the boundary conditions for the continuity of the tangential components of the magnetic fields on the slot surfaces and the generalized method of induced MMF for multi-slot structures. Then, taking into account (3) and (4), the system of algebraic equations for unknown current amplitudes J_{0i} can be represented as

$$\begin{cases} J_{01}\left(Y_1^{Wg}+Y_1^{R_1}\right)+J_{02}Y_{12}^{R_1}=-\dfrac{i\omega}{2k}\displaystyle\int_{-L_1}^{L_1} f_1(s_1)H_{0s_1}(s_1)\,ds_1, \\ J_{02}\left(Y_2^{R_1}+Y_2^{R_2}\right)+J_{01}Y_{12}^{R_1}+J_{03}Y_{23}^{R_2}=0, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, \\ J_{0N}\left(Y_N^{Wg}+Y_N^{R_{N-1}}\right)+J_{0,N-1}Y_{N,N-1}^{R_{N-1}}=0, \end{cases} \tag{5}$$

where $H_{0s_1}(s_1)$ is the projection of the TE_{10}-wave field onto the first slot axis; Y^{Wg} and Y^{R_i} are the matrix coefficients determined by using the Green's functions for a vector potential of a semi-infinite rectangular waveguide and a rectangular resonator [23]:

$$\begin{aligned} Y_{1(N)}^{Wg} &= \frac{4\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n(k^2-k_x^2)}{kk_z}\cos k_y y_{01(N)}\cos k_y\left(y_{01(N)}+\frac{d_{1(N)}}{4}\right)I^2(kL_{1(N)}), \\ Y_i^{R_i} &= \frac{4\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n(k^2-k_x^2)}{kk_z}\coth k_zH_i\cos k_y y_{0i}\cos k_y\left(y_{0i}+\frac{d_i}{4}\right)I^2(kL_i), \\ Y_{ij}^{R_{i,j}} &= \frac{4\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n(k^2-k_x^2)}{kk_z shk_zH_i}\cos k_y y_i\cos k_y\left(y_{0j}+\frac{d_j}{4}\right)I(kL_i)\,I(kL_j)\,. \end{aligned} \tag{6}$$

In formulas (6), the following notations are adopted:

$$\begin{aligned} I_i(kL_i) =& 2\left\{\frac{k\sin kL_i\cos k_xL_i-k_x\cos kL_i\sin k_xL_i}{k^2-k_x^2}\cos k_cL_i\right. \\ &\left.-\frac{k_c\sin k_cL_i\cos k_xL_i-k_x\cos k_cL_i\sin k_xL_i}{k_c^2-k_x^2}\cos kL_i\right\}, \end{aligned}$$

$k_x=(2m-1)\pi/a$; $k_y=n\pi/b$; $k_z=\sqrt{k_x^2+k_y^2-k^2}$; $\varepsilon_n=\begin{cases}1, & n=0,\\ 2, & n\neq 0;\end{cases}$ y_{0i} are the axial line coordinates of the slot; $H_{i,j}$ are the longitudinal dimensions of the resonators; $i,j=1,2\dots N$. The issues of convergence of series in the form (6) are considered in detail in [2, 23, 24]. The number of members of the double series in expressions (6) was chosen so as to ensure the calculation of the values of the conductivity of the slots in each of the jointed regions with an accuracy of 0.1%.

Since the magnetic field $H_{0s_1}(s_1)=2H_0\cos k_cs_1$, the amplitude of the magnetic current in the first slot can be found by solving the equation system (5). The solution can be represented as:

$$J_{01}=-\frac{i\omega}{k^2}H_0\frac{F(kL_1)}{Y_1^{\Sigma}-Y_{\Sigma N}}, \tag{7}$$

where H_0 is the incident wave amplitude,

$$\begin{aligned} Y_1^{\Sigma} &= Y_1^{Wg}+Y_1^{R_1}, \quad Y_i^{\Sigma}=Y_i^{R_i}+Y_i^{R_{i-1}}, \quad Y_N^{\Sigma}=Y_N^{R_{N-1}}+Y_N^{Wg}, \\ Y_{\Sigma N} &= \cfrac{Y_{12}^{R_1}Y_{21}^{R_1}}{+Y_2^{\Sigma}-\cfrac{Y_{23}^{R_2}Y_{32}^{R_2}}{\cfrac{\dots\dots\dots\dots}{+Y_{N-1}^{\Sigma}-\cfrac{Y_{N-1,N}^{R_{N-1}}Y_{N,N-1}^{R_{N-1}}}{Y_N^{\Sigma}}}}}, \\ F(kL_1) &= 2\cos k_cL_1\frac{\sin kL_1\cos k_cL_1-(k_c/k)\cos kL_1\sin k_cL_1}{1-(k_c/k)^2}-\cos kL_1\frac{\sin 2k_cL_1+2k_cL_1}{(2k_c/k)}. \end{aligned}$$

Then the field reflection and power transmission coefficients S_{11} and $|S_{12}|^2$ can be written as

$$S_{11}=\left\{1-\frac{8\pi k_gF^2(kL_1)}{iabk^3(Y_1^{\Sigma}-Y_{\Sigma N})}\right\}e^{-i2k_gz}, \quad |S_{12}|^2=1-|S_{11}|^2, \quad k_g=\sqrt{k^2-k_c^2}. \tag{8}$$

The formula for the power transmission coefficient was derived by using the power balance condition. It can be shown [23] that under the condition $(h_i/\lambda \geq 1)$, where h_i is the resonator wall thickness, the equation system (5) is valid up to terms of order $\{(d_i h_i)/\lambda^2\}$. In this case, the wall thickness d_i in formulas (6) should be replaced by d_{ei} according to the following approximate relations:

$$d_{ei} = d_i \left(1 - \frac{h_i}{\pi d_i} \ln \frac{d_i}{h_i}\right) \quad \text{if} \quad \frac{h_i}{d_i} \ll 1, \tag{9a}$$

$$d_{ei} = d_i \left(\frac{8}{\pi e} \exp\left(-\frac{\pi h_i}{2 d_i}\right)\right) \quad \text{if} \quad \frac{h_i}{d_i} \geq 1, \tag{9b}$$

which were obtained by taking into account that the function $\chi(\xi)$ for a two-dimensional perfectly conducting rectangular wedge can be approximately written as $[(d/2)^2 - \xi^2]^{-1/3}$. In [25], a relation is presented that quite satisfactorily unites formulas (9):

$$d_{ei} \cong d_i \exp\left(-\frac{\pi h_i}{2 d_i}\right) \tag{10}$$

3. NUMERICAL AND EXPERIMENTAL RESULTS

The plots of the power transmission coefficient $|S_{12}|^2$ in the range of fundamental mode of rectangular waveguide with cross-section $\{23 \times 10\}$ mm^2 are shown in Figs. 3–7 for the various structure configurations.

If the lengths of two resonators differ, the amplitude-frequency characteristic of the structure becomes asymmetric, and the bandwidth at a level of 0.5 $|S_{12}|^2$ increases (curve $H_2 = b/2$ in Fig. 4). Even insignificant variation of one slot dimensions (within $\Delta 2L/\lambda_{res} = \pm 0.015$, λ_{res} is a resonant wavelength a single diaphragm) increases the pulsation level in the passband and shifts the resonant wavelength λ_{res}^{Σ} of the entire system (Fig. 5). If the diaphragm thickness increases, the Q-factor of the two-resonator structure also increases, while the resonant wavelength λ_{res}^{Σ} stays almost unchanged (Fig. 6). The displacement of the slots in the waveguide cross-sectional plane to its side wall under condition that the remaining system parameters do not change shifts the resonant wavelength λ_{res}^{Σ} to the long-wave part of the operating range. In this case, the Q-factor of the entire structure markedly increases (Fig. 7).

Comparison of the results of calculations and experimental data confirms the adequacy of the proposed mathematical model for simulating real electrodynamic processes occurring in the system of

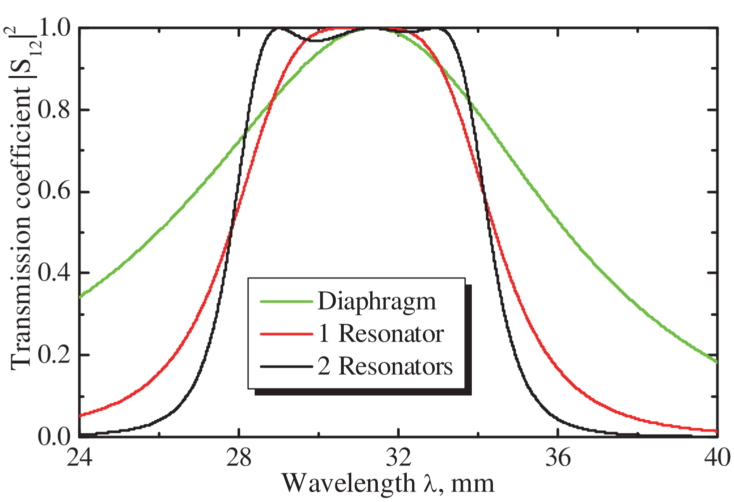

Figure 3. Power transmission coefficient $|S_{12}|^2$ versus wavelength for the system with parameters: $2L_i = 16\,\text{mm}$, $d_i = h_i = 1.6\,\text{mm}$, $y_{0i} = b/2$, $H_i = b$.

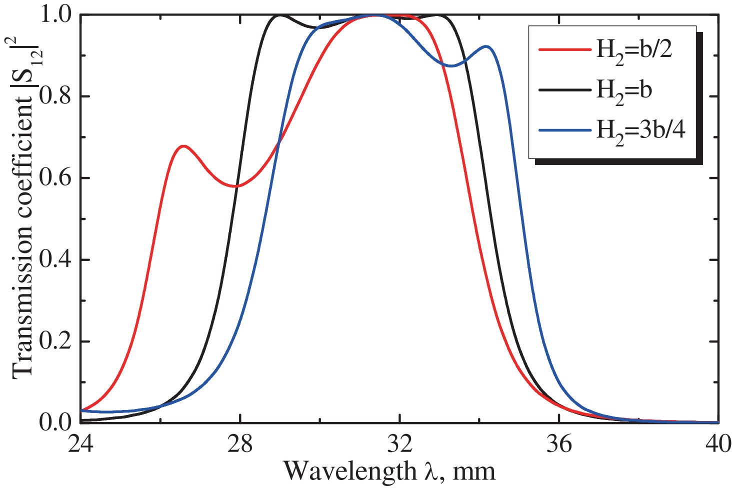

Figure 4. Power transmission coefficient $|S_{12}|^2$ versus wavelength for the system with parameters: $2L_i = 16\,\mathrm{mm}$, $d_i = h_i = 1.6\,\mathrm{mm}$, $y_{0i} = b/2$, $H_1 = b$.

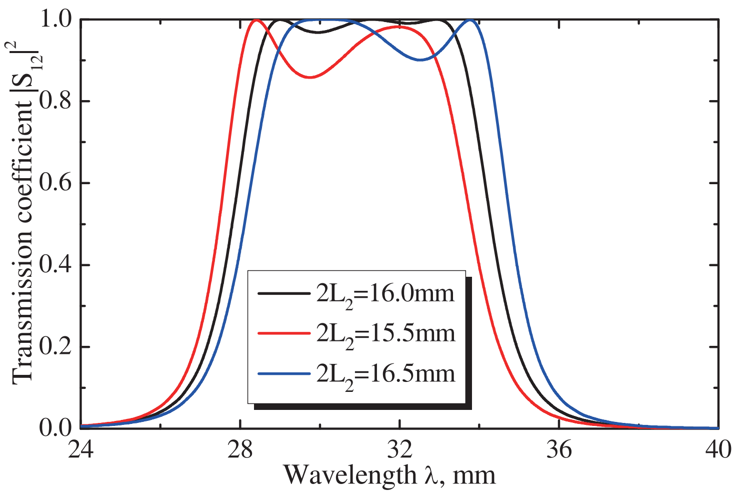

Figure 5. Power transmission coefficient $|S_{12}|^2$ versus of wavelength for the system with parameters: $2L_1 = 2L_3 = 16\,\mathrm{mm}$, $d_i = h_i = 1.6\,\mathrm{mm}$, $y_{0i} = b/2$, $H_i = b$.

feed-through resonators coupling through the slotted diaphragms (Fig. 9). Part of the experimental layout is shown in Fig. 8. The waveguide with the second diaphragm is connected through the holes to the model shown in the photos.

Let us further consider the possibilities of changing the geometry of the structure under study. Let the slot be arbitrarily located in the plane of the diaphragm, as shown in Fig. 10.

Then the distribution function of the magnetic current in the slot, found by the asymptotic averaging method [26], has the form:

$$f(s) = \left\{ \begin{array}{l} \sin\dfrac{\pi x_0}{a} \sin kL(\cos ks \cos \tilde{k}L - \cos kL \cos \tilde{k}s) \\ +\cos\dfrac{\pi x_0}{a} \cos kL(\sin ks \sin \tilde{k}L - \sin kL \sin \tilde{k}s) \end{array} \right\}, \tag{11}$$

where $\tilde{k} = k_c \cos\varphi$. As follows from (11), in this case the current distribution function has both a symmetric component with respect to the center of the slot and an antisymmetric one, and at $x_0 = a/2$ goes over to (4). Note also that the antisymmetric component of the current is equal to zero for the tuned ($kL = \pi/2$) slot. The study of the properties of a rectangular slot, arbitrarily located in the plane

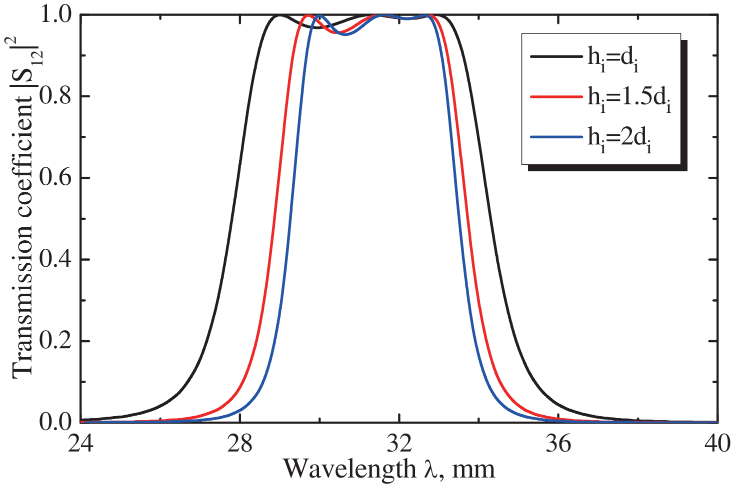

Figure 6. Power transmission coefficient $|S_{12}|^2$ versus wavelength for the system with parameters: $2L_i = 16\,\mathrm{mm}$, $d_i = 1.6\,\mathrm{mm}$, $y_{0i} = b/2$, $H_i = b$.

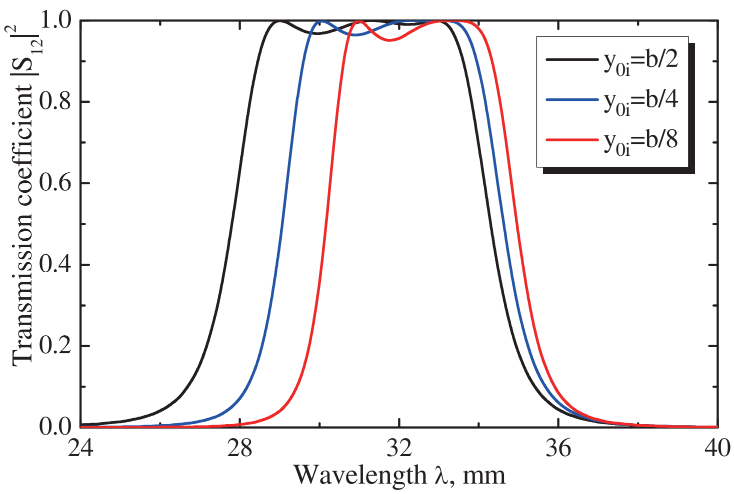

Figure 7. Power transmission coefficient $|S_{12}|^2$ versus wavelength for the system with parameters: $2L_i = 16\,\mathrm{mm}$, $d_i = h_i = 1.6\,\mathrm{mm}$, $H_i = b$.

of the cross section of the waveguide, was carried out in [8] by the method of moments. However, the calculated and experimental data presented in [8] correspond to nonresonant apertures. The exception is the coordinate resonance diaphragms, for which Table 1 shows comparative results ($a = 22.86\,\mathrm{mm}$, $b = 10.16\,\mathrm{mm}$, $h = 0.1\,\mathrm{mm}$).

Table 1. Resonance frequencies [GHz] of symmetrical coordinate diaphragms.

$2L \times d$, mm	Calculation [8]	Experiment [8]	Calculation, $\arg S_{11} = 0$ in (8)
16.9×0.9	8.87	8.84	8.84
14.8×0.5	10.22	10.20	10.13
12.9×0.9	11.62	11.65	11.66

Figure 8. Photos of the experimental layout.

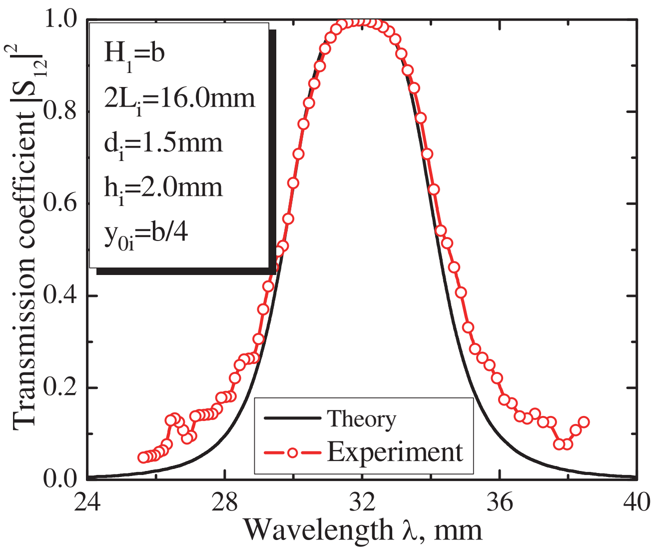

Figure 9. Power transmission coefficient $|S_{12}|^2$ versus wavelength: comparison of calculated results and experimental data.

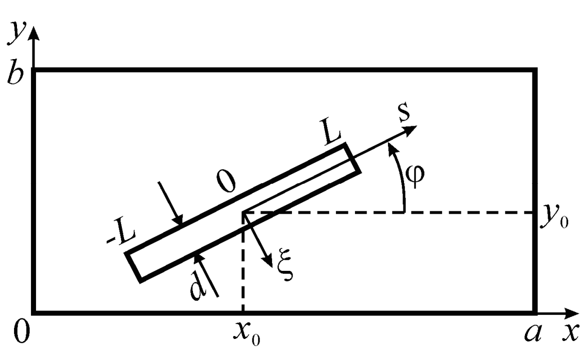

Figure 10. The geometry of the arbitrary located slot and notations.

4. CONCLUSION

In conclusion, we can state that the proposed approach to problem solving based on the generalized method of induced MMF and approximate analytical solutions of integral equations for the magnetic current in single coupling slot can be used without fundamental difficulties for following cases: slots are cut at any angle to waveguide walls, multi-slot diaphragms [23], and diaphragms with impedance surfaces [24].

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