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Antenna Reconfiguration Based DOA Estimation for AWGN Channel in MIMO Applications

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Abstract—This paper proposes an underdetermined direction of arrival (DOA) estimation for multiple input and multiple output (MIMO) sparse additive white Gaussian noise (AWGN) channels. Accurate DOA estimation helps in better signal analysis and noise cancellation in the channel. A novel multiplicative multi-kernel basis vector-based non-negative sparse Bayesian learning (NNSBL) algorithm is implemented over a predefined grid. Simultaneously stochastic cuckoo search algorithm (CSA) is exploited virtually to improve the DOA approximation for a nonuniform linear array (NULA) geometry by an optimized antenna reconfiguration model. The simulated and experimental results show that the proposed algorithm yields an optimized root mean square error (RMSE) for different optimized wavelengths of the randomly generated signals. The RMSE convergence graphs demonstrate the effectiveness of the new method for different signal-to-noise (SNR) values.

1. INTRODUCTION

In recent years, most researchers are focused on far-field signal direction estimation by exploiting array signal processing and its optimization methods in their works. DOA estimation determines the direction of multiple superimposed signals in the presence of noise at sensor arrays. It is significant in most of the practical and recent engineering applications such as radar, sonar, wireless communications, radio astronomy, medical imaging, and geophysical exploration. Operating MIMO technology in fifth-generation (5G) mobile communications provides high data rates, increased reliability, improves immunity to interference, and supports the massive number of connected devices. In 5G MIMO sparse channel, DOA estimation can be performed in two ways, by optimal parametric methods and approximation methods. A well-known sparse signal reconstruction (SSR) method called compressive sensing is one of the approximation methods. Compressive sensing in its basic form, has received signals represented on a sparse overcomplete dictionary with less number of weighted vectors as non-zero rows indicating the genuine DOA.

In order to solve the DOA estimation problems, performance evaluation between multiple signal classification (MUSIC) algorithm and estimation of signal properties using the rotational in-variance techniques (ESPRIT) method proves that ESPRIT has less computation time complexity [1]. DOA estimation is a challenge when being applied to MIMO wireless communication systems. Both direction of departure (DOD) and DOA estimation using the MIMO MUSIC algorithm are described by applying Gibbs sampling on the signal [2]. An analysis of basic array signal processing methods over decades is discussed briefly [3]. Paper [4] adopts a dimension reduction algorithm that converts 4D to 2D signals for finding an arriving angle in a linear tripole array. Spatial spectrum calculation with co-prime arrays is developed by generating the Toeplitz matrix that converts the unknown number of coherent signals to non-coherent ones [5]. Using a symmetric MIMO array efficient joint parameters are

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determined [6]. DOA estimation considered with basis pursuit de-noising (BPDN) problem is solved using the alternating direction method of multipliers (ADMM) [7] to avoid the computational complexity for real-time implementations. Quasi-stationary DOA signals are approximated using less number of sensors than sources criteria [8]. An integrated artificial neural network (ANN) based estimation proves its effectiveness on coherent signals by reconstructing the weighted noise subspace method [9]. Monopulse ratio (MR) based estimation is introduced for cyclic prefix (CP) orthogonal frequency division multiplexing (OFDM) millimeter wave systems [10]. Minimum redundancy array (MRA) concepts are briefed which are significant for the antenna reconfiguration process [11].

Furthermore, the integration of orthogonal matching pursuit (OMP) and majorization minimization (MM) approach [12] shows low computational expenses and grid mismatch for DOA estimates. Single snapshot and associated sources are not appropriate for the MUSIC and ESPRIT techniques; therefore, a fast iterative interpolated beam former (FI IB) technique [13] is replaced for the intrinsic spatial smoothing in MIMO radar. For many sources of approximation, complex non-negative matrix factorization (CNMF) on the spatial covariance matrix is inspired for beam formation. It is noticed that CNMF-based DOA has a greater number of reconfigurations, including varied channel numbers, microphone spacing, reverberation times, source types, and angular positions [14]. MUSIC and Newton approaches [15] are introduced for 6 GHz continuous wave radar systems in which a 54% rise in Newton's computational speed is observed comparatively. Electronically steerable parasitic array radiator (ESPAR) antenna is aided by the received signal strength (RSS) resulting in lower DOA estimation errors [16]. The newly developed Doppler coherent angle of interest (AOI) detection and DOA estimation are introduced for the radar technology for navigation using environment mapping [17]. A novel NNSBL algorithm is formulated to virtually address the source localization problems for sparse arrays [18, 19]. An investigation on the bi-static passive radar [20] DOA estimation using the SBL framework is conducted. Effective and robust reconstruction of interference plus noise covariance matrix is demonstrated for DOA findings via SBL [21]. Cumulative and hybrid form basis vector merged with optimized antenna reconfiguration method is adopted for underdetermined direction calculation for a NULA [22] and an arbitrary linear array [23]. On-grid variational Bayesian inference (VBI) method for a sparse array DOA estimation is proposed in [24]. Then, with unknown prior knowledge, good performance in DOA, array gain, and phase error estimation is observed [25].

The important aim of this paper is to overcome the shortcomings in the literature such as high computational complexity, low accuracy, and huge errors for high SNR values and to scrutinize a better sparse Bayesian learning approach that enhances the DOA estimation performance parameters. In this regard, compressive sensing-based DOA estimation is drawing attention because of its low computational complexity, robustness, and accurate source direction estimation in a sparse MIMO channel. The main contributions in this paper can be classified into two aspects. Firstly, an advancement in the stochastic NNSBL algorithm to solve the DOA problems by a novel multiplicative multi-kernel-based manifold matrix is developed. Then, the unique expectation maximization (EM) method is used for sparse signal reconstruction during the beam forming process. The second is for a NULA of antennas, and degrees of freedom (DOF) can be enhanced by using the CSA. The advantage of the antenna reconfiguration procedure is its limited simple steps to optimize the considered objective function as RMSE for various SNR values. The experimental results show that the proposed algorithm achieves optimized results compared with conventional NNSBL and accurate convergence plots for different SNR levels. The rest of the paper is organized as follows. Underdetermined DOA estimation using a novel multiplicative basis vector for an MK NNSBL algorithm is introduced in Section 2. The optimized antenna reconfiguration model is discussed in Section 3 with the overall algorithm's flowchart. Section 4 presents the optimized results with a discussion of the proposed method and comparison plots and finally followed by a conclusion in Section 5.

2. UNDERDETERMINED DOA ESTIMATION USING MULTIPLICATIVE BASIS VECTOR

The NNSBL algorithm is used for underdetermined DOA estimation with the multiplicative basis vector for MIMO wireless communications. The underdetermined condition is the number of sources which is greater than or equal to the number of sensors that are exploited. Using the prior values NNSBL maximizes the posterior values. This implementation shows improved beam forming accuracy by considering multiplicative multi-kernel basis vector and optimized stochastic antenna reconfiguration model. The two ways considered to optimize the DOA estimation are as follows:

- The inclusion of multiplicative basis vectors generation inside the manifold matrix for beam steering purpose.
- The adjustment of the antenna positions until the fine reception of source signals.

2.1. Multi-Kernel Basis Vector-Based NNSBL Algorithm for Sparse Channel

The NNSBL algorithm is implemented using a multi-kernel basis vector in a multiplicative approach. BPDN-based sparse representation DOA estimate methods are offered for approximating the DOA. The application of the conventional approach is described, and the advancement in the current DOA estimation algorithm in the sparse reconstruction paradigm is briefed. The new development is the multiplicative multi-kernel basis vector implementations of the BPDN-based DOA estimation algorithm.

2.2. Signal Model

Consider 'N' narrowband far-field source signals impinging on an omnidirectional antenna from all directions. An analysis of a NULA geometry separated by a distance in terms of integer multiples of half-wavelength ($\lambda/2$) is performed. Multiple snapshots are preferred to reach optimized values and accuracy. The MRA enhances the DOF. The distance considered for antenna reconfiguration in MIMO applications is an independent variable. So, let $[0, d_1, ..., d_{M-1}]$ be the distance between the reference point and the individual antenna. If the total elements are 'M', then the differential co-array is taken into consideration as 'Ω' given by

$$\Omega = \{d_{m1} - d_{m2}\}_{m1=0,1,...,M-1;\ m2=0,1,...,M-1}$$

Given that M antennas are acquiring naturally uncorrelated signals from N distant sources, M antennas are provided with an additional DOF option. Narrowband sources $S_n(t)$, $n = 1, 2, ..., N$. The proposed implementation computes the DOA estimate using spatially white Gaussian noises as the channel. The received signals over T snapshots with noise are represented as,

$$x(t) = As(t) + n(t) \tag{1}$$

Equation (1) represents the t^{th} snapshot's array received vector $x(t)$, the signal from the sending source $s(t)$, and channel noise $n(t)$. The aggregate steering vectors from all N sources are contained in the manifold matrix A, where $A = [a(\theta_1), a(\theta_2), ..., a(\theta_N)]$. Each steering vector is composed of $a(\theta_n) = [1, v(d_{m1}, \theta_n), ..., v(d_{M-1}, \theta_N)]$ where $n = 1, 2, ..., N$, which is defined as $v(d_m, \theta_n) = \exp[-j2\pi(d_m/\lambda)\sin\theta_n]$, and d_m is the element spacing. The further covariance matrix is defined in Equation (2).

$$R_x = E\left\{x(t)x^H(t)\right\} = A\,\mathrm{diag}\left(\sigma_1^2, \sigma_2^2, ..., \sigma_N^2\right)A^H + \sigma_n^2 I_M \tag{2}$$

Vectorizing the covariance matrix of Equation (2) is calculated using Equation (3).

$$y = \mathrm{vec}(R_x) = \mathrm{vec}(AR_sA^H) + \sigma_n^2\,\mathrm{vec}(I) = (A^* \circledast A)g + \sigma_n^2 I_M \tag{3}$$

where $\circledast$ denotes the Kramar Rao product; $g = [\sigma_1^2, \sigma_2^2, ..., \sigma_N^2]^T$ is the variance vector; $\{\cdot\}^H$ is the conjugate transpose, $1_m = [e_1^1, e_2^T, ..., e_M^T]^T$ is the unit diagonal vector. From the manifold matrix, the virtual manifold matrix is found as $\overline{A} = (A^* \circledast A)$. The sample covariance matrix is defined as $\widehat{R}_x = \sum_{t=1}^{T} x(t)x^H(t)/T$. The residual error of the covariance matrix is an asymptotic complex Gaussian distribution since the incident signals are characterized as circularly symmetric Gaussian distributions. Equation (4) gives the residual error between the estimated DOA '$\widehat{y}$' and the true DOA 'y' as,

$$\widehat{y} - y = \mathrm{vec}(\widehat{R}_x) - \mathrm{vec}(R_x) \sim C\mathbb{N}(0, R_x^T \otimes R_x/T) \tag{4}$$

Let $\widetilde{R}_x = R_x^T \otimes R_x/T$, with $\otimes$ as Kronecker product. By using (3), Equation (4) is transformed to

$$\widehat{y} \sim C\mathbb{N}\left(\overline{A}g + \sigma_n^2 1_M, \widetilde{R}_x\right) \tag{5}$$

The BPDN formulation in Equation (5) is solved using the NNSBL solution as given in Equation (6). The predefined grid space $\Theta = [\theta_1, \theta_2, ..., \theta_N]$ is used for the solution.

$$\widehat{y} \sim C\mathbb{N}\left(\Phi w + \sigma_n^2 1_M, \widetilde{R}_x\right) \tag{6}$$

The DOA estimation problem is solved using over complete dictionary matrix 'Φ'. The basis vector for matrix 'A' is formed by using every direction in the grid. The non-negative sparse matrix 'w' has zeros at all other locations and ones where genuine DOA is present which reflects the compressive sensing approach. '$\mathbb{N}$' defines the non-negative Gaussian distribution. The key parameter RMSE is defined as the square root of the mean of the square of all of the error values and is expressed as,

$$\text{RMSE} = \sqrt{1/N \sum_{i=1}^{N} \left(\widehat{y}_i - y_i\right)^2} \tag{7}$$

3. OPTIMIZED ANTENNA RECONFIGURATION MODEL

The optimized distances between the antennas are populated in this paper as the reconfiguration method, and it is performed through the CSA to acquire the best position of the antenna until a minimal RMSE occurs for the particular DOA estimation.

3.1. Cuckoo Search Algorithm (CSA)

Meta-heuristic algorithms are used to solve most stochastic problems. CSA is one such algorithm. The ability of the cuckoo to manipulate the host bird by laying its egg in the host bird's nest is the bio-inspired concept of optimization. The main aim is to use new or possible best solutions to restore the less good ones to achieve an optimal solution. The concept is designed as a mathematical meta-heuristic solution for any stochastic problems. In this attempt of antenna reconfiguration, CSA runs on the outer loop, and NNSBL runs on the inner loop to find a DOA solution during the implementation. With the objective function as RMSE, the antenna reconfiguration is developed by populating the inter-antenna distance which is considered constant in many traditional executions. The inter-antenna distance is considered the stochastic independent variable that is populated and iterated for the best RMSE. The search of inter-antenna distance using the CSA provides a better DOA estimation with minimized RMSE. The pseudo-code for developing the algorithm is as follows.

There are two parts to the cuckoo search methodology. One is the local refinements, and the other is the global search step.

Step 1: Populate the number of nests so that each cuckoo randomly selects the nest for laying one egg at a time that shows the local refinements.

Mathematically every egg acts as a solution, and it is preserved in the nest along with a cuckoo's egg as an artificial one. To better understand eggs in the nest are treated as a set of solutions, and a cuckoo egg is considered a new solution.

Step 2: High-quality eggs in the best nest will be carried over to the next generation that shows the global best for the first iteration.

The high-quality eggs are treated as the best solution close to optimal value mathematically. In this sense, cuckoo egg which is most similar to the host bird gets the opportunity to become big in the next generation. Now, the new solution will replace the old optimal value.

Step 3: The number of the host bird's nests is fixed. The probability $Pa\{0, 1\}$ is the probability of the host bird detecting the cuckoo egg. Specifically, the two probabilities that a host bird can have is to throw away the cuckoo egg or to leave the nest and build a new one.

Step 4: Mathematical model based on the above three rules is described as Equation (8).

Cuckoo search finds the new solution using the Levy flight concept as given in Equation (8),

$$x^{(t+1)} = x^t + \alpha \oplus \text{Levy}(\lambda) \tag{8}$$

where $x^{(t+1)}$ is the new solution, x^t the current solution, α the step size considered as 1 in many cases, and λ the Levy exponent equal to 1.5. The operator $\oplus$ indicates the entry-wise multiplication.

The Levy flight is defined as the random walk (small step) made by the cuckoo in a stepwise manner to reach the destination. Levy distribution (Levy (λ)) is the series of random walks defined as random step length, and it is calculated using Equation (9)

$$\text{Levy} \sim u = t^{-\lambda}, \quad (1 < \lambda \leq 3) \tag{9}$$

where 'u' is a normal stochastic variable, and 't' shows the current iteration.

Step 5: Evaluate the new positions of the nest that optimizes the considered objective function RMSE as per the proposed method. Again iteratively find the global best.

3.2. Proposed Method Flowchart

Based on the above discussed theoretical analysis, the overall flowchart is shown in Figure 1.

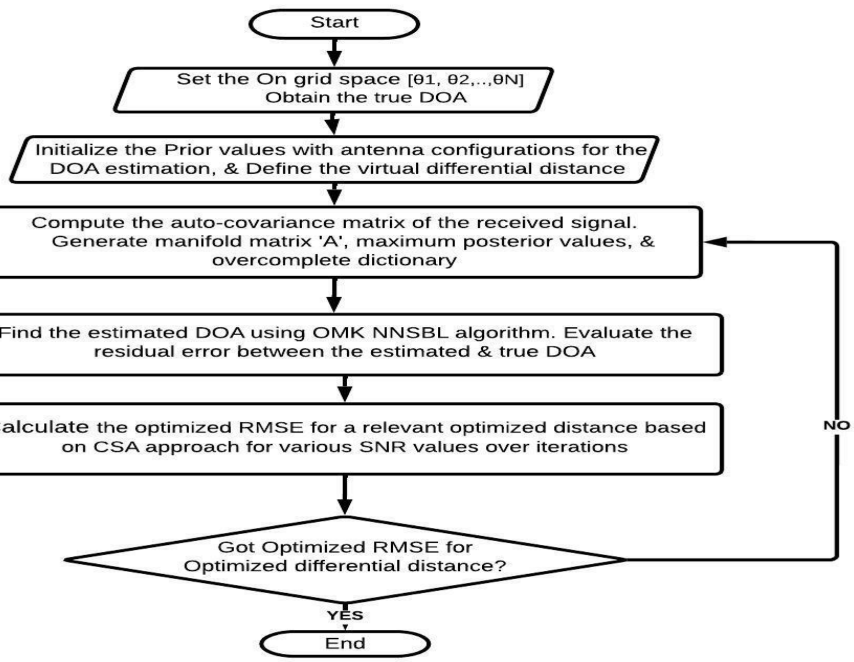

Figure 1. Flowchart of the proposed method.

4. RESULTS AND DISCUSSIONS

Based on the proposed algorithm, a novel antenna reconfigurable DOA estimation problem is resolved using a unique CSA approach that runs on an outer loop. A multiplicative multi-kernel basis vector framework is created for the optimization of better DOA estimation executes in an inner loop. The noise considered is AWGN, and the grid interval is 1 degree in all the cases for the number of snapshots 'T'. MATLAB-based simulation experiments were executed on the computer. The proposed algorithm's antenna signal arrangement is depicted in Table 1. The number of antennas, carrier frequency, and angle of source signals are also indicated in Table 1.

4.1. Random Generation of Different Signals and Its Variations

This section presents the randomly generated input signals from fifteen multiple-source antennas that are shown in Figure 2. These signals are uncorrelated in nature which avoids most of the interference. Time-varying source signals are affected by the random Gaussian noise in the MIMO sparse channel during transmission. Thus, all the noise-added source signals simultaneously impinge on the receiving

Table 1. Antenna signal configuration for the proposed DOA estimation algorithm.

Parameters	Configuration
Number of Antennas	13
Antenna Array type	Non-Uniform Linear Array
Angle range (Degrees)	−60 to 60
Min to Maxrange coverage (Degrees)	−70 to 70
Carrier frequency	200 MHz
Propagation velocity	380 m/s
Interval of angle Searching	1 Degree
Angles of source signals (Degrees)	−68.4 −55.1 −42.2 −30.2 −22.6 −10.4 −6.2 2.4 7.3 15.6 23.2 31.4 42.2 55.2 68.2

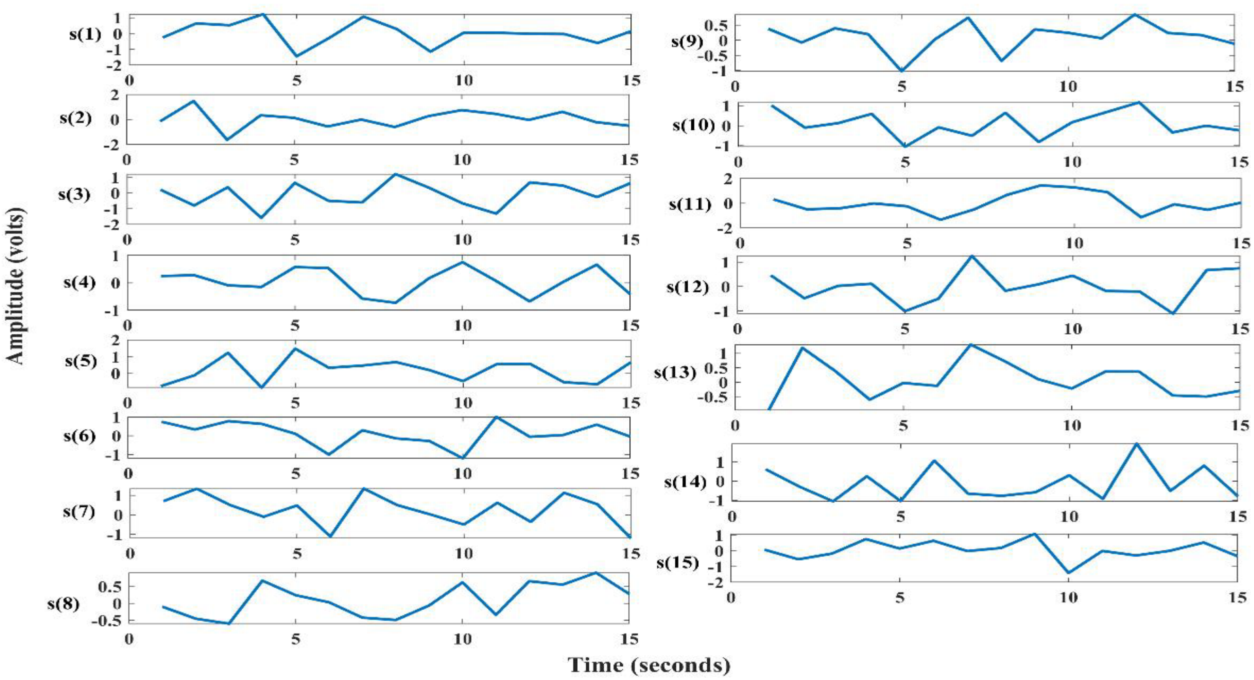

Figure 2. Randomly generated input signals from multiple sources.

antenna sensors that are further considered for the beam steering procedure using thirteen 'A' manifold matrices.

The respective noise signal at each antenna sensor is given in Figure 3. The noise added to the signal is varied between −20 dB and 40 dB. Transmitted source signals which are impinging on the thirteen sensors are represented in a matrix form. The optimized differential distance is varied in this implementation by the CSA technique with the beam forming at the sensor.

The most attractive thing about a sparse array is its capability to resolve more input sources than the number of receiving sensors. Initially, the proposed algorithm sets the entire priors as unknown values during MK NNSBL implementation. Further, these values are maximized by the EM method to obtain the posterior values. Antenna deployment positions usage is denoted as [1, 9, 6, 8, 7, 5, 4, 10, 12, 3, 14, 2, 13]. All the noises in Figure 3 are added to the source signals in Figure 2 respectively to obtain the combined signal as represented in Figure 4.

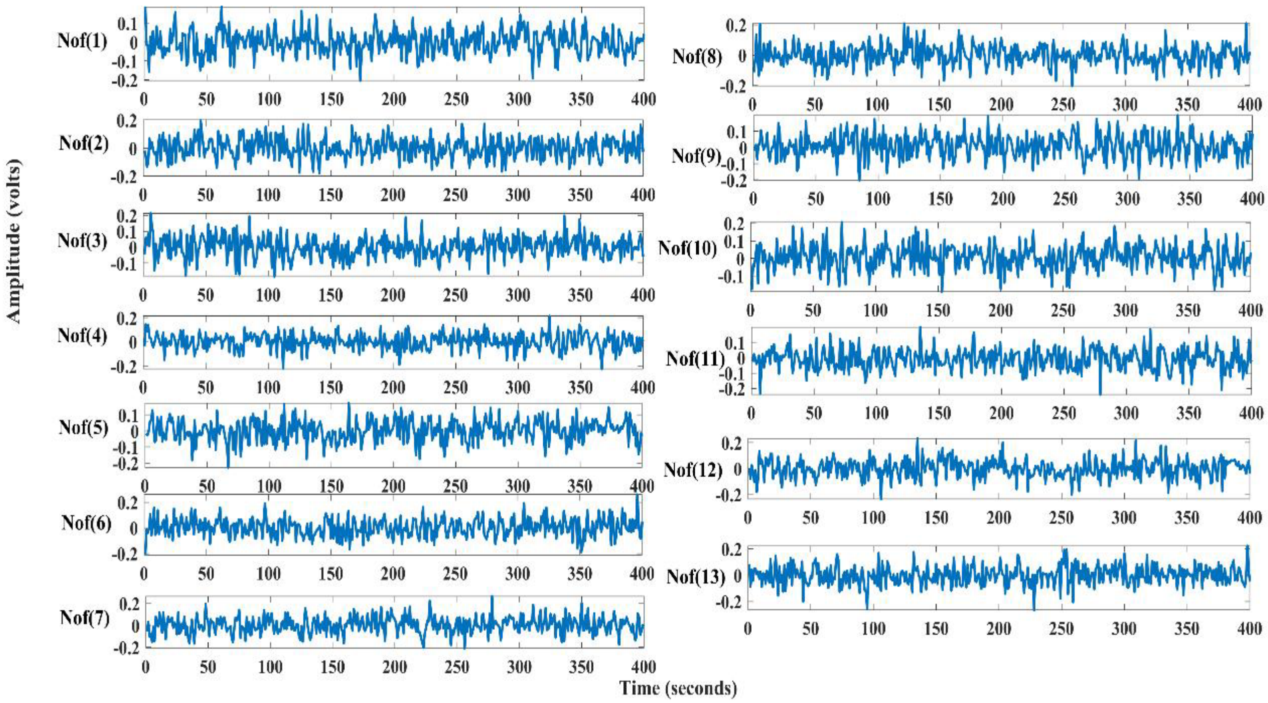

Figure 3. Random noise at respective receiving antenna elements.

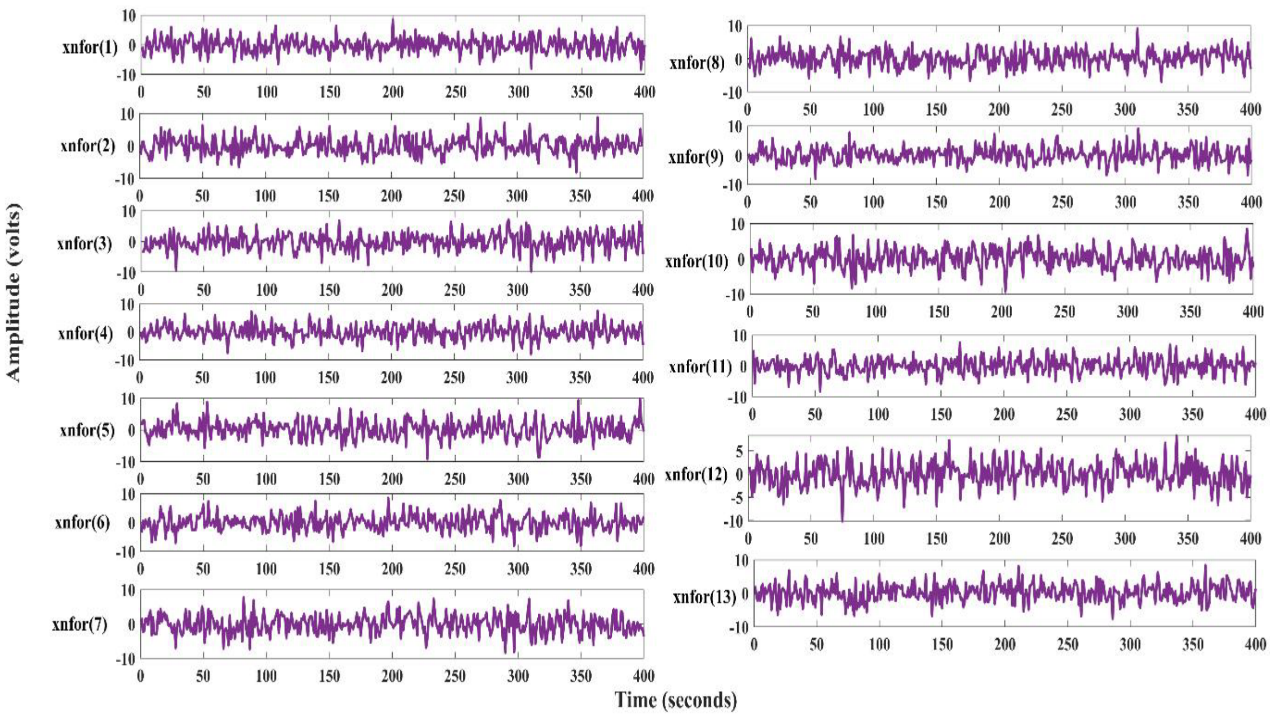

Figure 4. Input signal added with the noise in AWGN channel received at the antenna element.

Unique manifold matrix includes the steering vectors which are formed by combining all the multiplicative multi-kernel basis vectors. These steering vectors are responsible for steering the beam that is formed at the sensor for the fine DOA determination and to reduce the error. Since these vectors are stochastic in nature, it needs a basis pursuit method for proper DOA peaks approximation. The complete random generation of different signals and their variations is performed using 400 snapshots and 100 Monte Carlo numerical simulations. The multiplicative basis vector-based manifold matrix thus developed for the DOA estimation is shown in Figure 5.

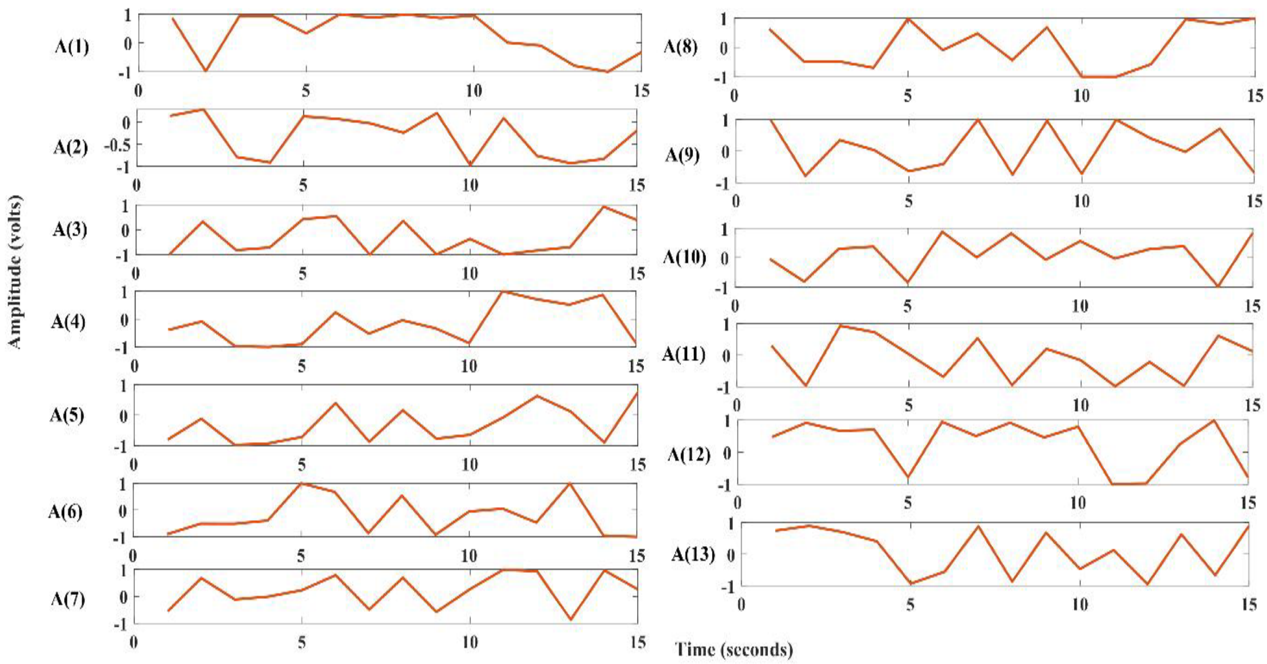

Figure 5. Representation of manifold matrix "A" for all the antenna elements in an array.

4.2. Performance Comparison of Optimized MK NNSBL Algorithm (OMK NNSBL) over Conventional Method

This experiment examines the performance of the proposed method based on the assumption of uncorrelated incoming sources. An advancement in NNSBL is made to obtain multi-kernel NNSBL (MK NNSBL). This MK NNSBL is applied with an optimization technique for better RMSE over various SNR levels. The specifications of the optimization method CSA are defined as shown in Table 2.

Table 2. Cuckoo search algorithm specifications.

Parameter	Value
Number of Population	20
Total number of iterations	15
Objective Function (Degrees)	RMSE of DOA estimation
Distance range (Centimeter)	0.3 to 1 time of the wavelength

CSA is used to finalize the distance between antennas which provides the best DOA estimation for the NULA. The convergence of the optimization is important for the best DOA estimation since the objective function is RMSE defined as the difference in actual and estimated DOAs for all the antennas. The distance is populated and checked for the optimized RMSE (minimization) with the variation in antenna distance. The CSA progressively populates the antenna distance towards RMSE minimization for every iteration. Reduced RMSE is obtained for the optimized multi-kernel NNSBL (OMK NNSBL)

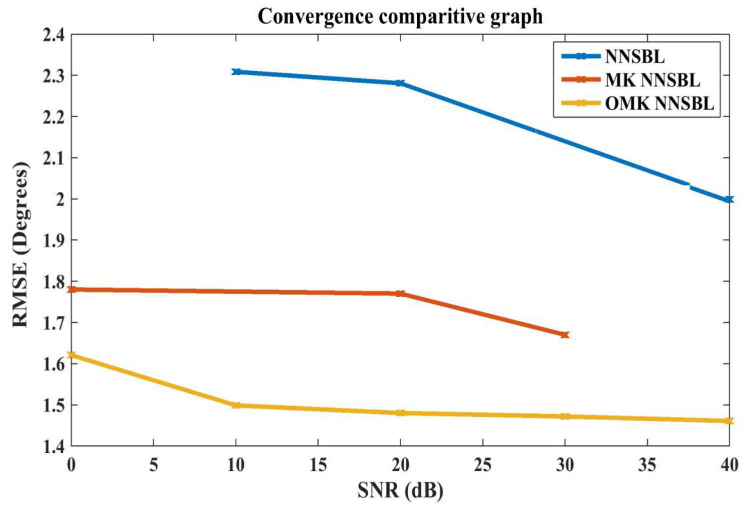

Figure 6. RMSE versus SNR performance comparison of different methods in underdetermined condition.

algorithm that reflects the sharp DOA estimation, and it is compared with the MK NNSBL and NNSBL methods as shown in Figure 6.

The minimum redundancy array considers the distance between the sensors as noncontinuous. This distance is virtually varied using CSA until the least RMSE is obtained called optimized distance. The DOA estimation is perfect when the true signal inclines with the estimated one. Sharp spectrum peaks resulting in the lower error are defined as optimized DOA estimation. The best obtained DOA with corresponding distance related to OMK NNSBL implementation is given in Table 3.

The results of RMSE versus SNR performance comparison of different methods in underdetermined condition obtained in Figure 6 are measured and tabulated in Table 4.

Table 3. OMK NNSBL DOA output with the corresponding distance.

Optimized Distance (Centimeter)	Optimized DOA (Degrees)	Actual Angles (Degrees)
0.3	−53.28125	−68.40
0.545560512937412	−47.65625	−55.10
0.456400380356047	−42.03125	−42.20
0.301609444714322	−36.09375	−30.20
0.486400231067017	−26.90625	−22.60
0.528558088426088	−13.46875	−10.40
0.606154901720926	−10.59375	−6.20
0.856372470004193	−2.59375	−2.40
0.3	5.46875	7.30
0.639161694155916	11.09375	15.60
0.789178192327613	14.34375	23.20
0.729572611755399	36.15625	31.40
0.579451515125934	49.28125	42.20
0.731277782570519	54.03125	55.20
0.665700858622444	59.15625	68.20

Table 4. Convergence comparative results of NNSBL, MK NNSBL, and OMK NNSBL algorithms.

SNR (dB)	NNSBL RMSE (Degrees)	MK NNSBL RMSE (Degrees)	OMK NNSBL RMSE (Degrees)
−10	2.3642	1.59	1.5370
0	2.0390	1.78	1.6208
10	2.3086	1.641	1.4986
20	2.2807	1.77	1.4801
30	2.3148	1.65	1.4718
40	1.9989	1.67	1.4608

4.3. Effectiveness of the Proposed Algorithm

RMSE convergence graph is simulated for 400 snapshots. By varying SNR in terms of −10, 0, 10, 20, 30, and 40 dB, DOA estimation is performed for 20 iterations. Every iteration includes all the SNR value simulations. Specific SNR level includes 100 Monte Carlo trails for the OMK NNSBL method. It is observed that the RMSE is reduced from 2.3 to 1.44 for the increase in SNR values as shown in Figure 7. Therefore, the proposed method is proven to be performing effectively better than the NNSBL and MK NNSBL methods with the increase in SNR values. To ultimately handle the RMSE performance degradation for high SNR values, the on-grid scenario is preferred for the execution.

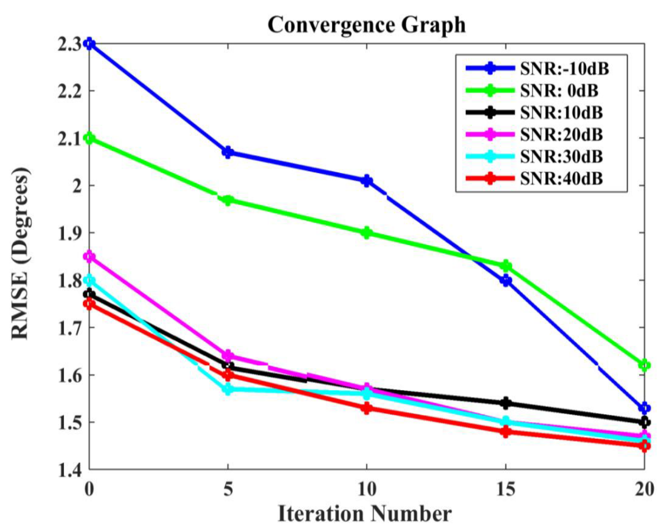

Figure 7. RMSE convergence graph of OMK NNSBL method at different SNR levels based on multiple iterations.

5. CONCLUSION

Most of the DOA estimation algorithms are computationally complex and consume huge processing cost for data transmissions over sparse arrays. It is not easy to achieve optimized results. To overcome this, sparse reconstruction methods are adopted for achieving robustness, least errors, accuracy, and low computation complexity. Two approaches are considered in this paper simultaneously for optimization purposes. First is the inclusion of multiplicative multi-kernel basis vectors inside the manifold matrix for beam steering purposes based on a non-negative sparse Bayesian learning algorithm. Second is the reconfiguration of the antenna iteratively using the cuckoo search algorithm model. Through MATLAB-based implementation the reconfiguration of antennas has improved the DOA estimation accurately. Results are compared with different wavelengths of the randomly generated signals. Root mean square error is reduced to an optimal value with the increase in signal-to-noise ratio values. Overall, the performance of the novel method is found to be satisfactory and is competitive with the other conventional methods that can be used for 5G massive multiple input multiple output applications.

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