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Pattern Synthesis of a Resonant Slot on a Broad Wall of the Rectangular Waveguide Using Amplitude and Phase Control

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Abstract—A design technique to develop the desired pattern with uniform spacing between elements for a resonant linear slot array on the broad wall of a rectangular waveguide is discussed in this study. First, linear array pattern synthesis is used to achieve the amplitude and phase of the array element. Then both radiation pattern synthesis and array input impedance matching are achieved using the least-squares method. In addition, the error function is created by combining the three terms of impedance matching, array pattern synthesis, and slot design equations. Genetic algorithm (GA) and the conjugate gradient (CG) technique are used to minimize the acquired error function. The utilized approach results in precise pattern synthesis, good impedance matching, development of appropriate design equations, and power loss minimization. The computing needs were also reduced using the suggested antenna design. The approach is particularly beneficial since it integrates slot parameter dimensions and impedance matching with array pattern synthesis, resulting in a faster and more accurate design. Full-wave simulation Software HFSS was utilized to validate the suggested design method. Moreover, the measurements were conducted on a prototype designed to validate the simulation's accuracy and the designed antenna practicality, and excellent agreements between theoretical predictions and simulation results were achieved.

1. INTRODUCTION

Because of their widespread application in telecommunications and radar systems, rectangular waveguide slot arrays are in great demand [1]. Since these antennas are used in high-power transceivers, the phase and amplitude of the slots' voltages on a rectangular waveguide must be precisely managed in order to produce a desired pattern with the desired shape [2, 3]. Stevenson was the one who conducted the initial experiments on resonant slots [4]. Oliner provided several equations and also developed a non-resonant slots circuit [5]. Eventually, Elliott has provided design equations for slot arrays [6]. A method is proposed in [7] for a non-resonant longitudinal slot on a broad-wall rectangular waveguide with equal spacing between slots to achieve maximal input matching; nevertheless, it should be noted that it is expanded for a center inclined slot. The procedure was also improved in [8] with the assistance of the design error function and pattern error function to attain maximum input matching and the required sum pattern as well. Genetic algorithm is used in [9] to obtain the optimum values of loads and their optimum position along the dipole, and the optimization of the antenna's size for low frequency is done in [10, 11].

In this study, we use the least-squares method for both syntheses of the radiation pattern with the specified side lobe level and matching of the input impedance to design and create a linear slot array with the required pattern. The error function is built using the three terms of impedance matching,

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array pattern synthesis, and slot design equations for optimization. The genetic algorithm is coupled with the conjugate gradient approach to minimize the error function. Finally, the offsets and slots angle values are determined to accomplish the desired matching on the input terminal and also lead to the lowest absorbed power in the load and results in good output matching. Prediction models and trend analysis are well suited by the least-squares analysis technique. The closest relationship between the variables is provided, and the computing technique is straightforward and simple to use. Particularly for array synthesis, the least squares approach allows us to get the required pattern more quickly and readily than other techniques, including Orchard method synthesis, according to our expertise in pattern synthesis. This approach is quite useful for creating a lengthy linear array [12].

In order to change the slot's amplitude and phase when the slot's length is in resonant mode, this study varies the slot's offset from the waveguide's longitudinal direction and its angle from the slot's central axis. In order to focus on these factors and solely utilize these parameters to control the amplitude and phase of the slot, we approximated the distances between the slots to be half the wavelength of the waveguide. In order to control the amplitude and phase of the slot as well as pattern synthesis, which was not the focus of this article, the parameter controlling the distance between the slots should also be taken into consideration if the spacing between the slots was irregular. So in this paper we are going to control the settings for amplitude and phase adjustment.

2. DESIGN PROCEDURE

When the equation is computed using the offset and slots' angle, the following facts must be taken into account as significant phases in the design process:

- Along the waveguide, TE_{10} is the only dominant mode that is propagated.
- The width of the slot must be substantially less than its length. (It is around one-tenth or less).
- The waveguide material is thought to be a perfect conductor with a very thin thickness.

The slot's configuration on a broad wall of rectangular waveguide is shown in Figure 1.

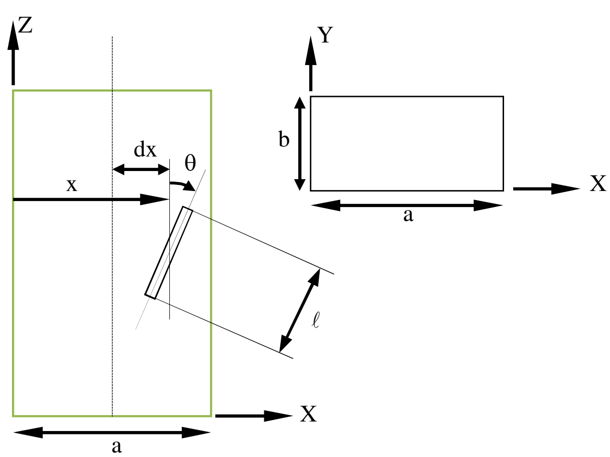

Figure 1. Slot configuration on waveguide.

2.1. Formulation

To create the required pattern, we need to consider Equation (1):

$$\frac{V}{A} = \left|\frac{V}{A}\right| e^{j\alpha} = \frac{8.1}{a}\left[\frac{\cos\left(\frac{\pi}{2}\left(\cos(\zeta-\theta)\right)\right)}{\sin(\zeta-\theta)}e^{j\pi x/a} + \frac{\cos\left(\frac{\pi}{2}\left(\cos(\zeta+\theta)\right)\right)}{\sin(\zeta+\theta)}e^{-j\pi x/a}\right] \tag{1}$$

in which A is the amplitude of propagated wave in the dominant mode, a the waveguide's broad wall surface area, x the slot's displacement, θ the slot's deviation, and angle ζ is a parameter given by Equation (2):

$$\zeta = \sin^{-1}\frac{\lambda}{2a} = \cos^{-1}\frac{\lambda}{\lambda_g} \tag{2}$$

In Equation (2), λ is the wavelength (in the operation frequency), and λ_g is the waveguide wavelength. To achieve the desired pattern, the value of slot displacement must be established using Equation (1) in terms of slot specifications and its location.

3. SLOT ARRAY DESIGN PROCEDURE

Array synthesis is used to create the required pattern, which is provided by Equation (3):

$$\Theta_n = \chi_n e^{i\alpha_n} \tag{3}$$

where χ_n and α_n are the amplitude and phase of n'th lot, respectively.

The coupling coefficients equation reported in [13] is used here to attain the required voltage standing wave ratio (VSWR). The coupling coefficient is proportional to the n'th excitation voltage amplitude, and result can be determined using Equation (4), which is based on the ratio of radiated power from the slot to absorbed power in the matching load.

$$\gamma_n = \frac{\chi_n^2(1-\rho_L)}{\Re_{N-1} - \Re_n(1-\rho_L)} \tag{4}$$

where N is the number of elements, ρ_L the value of absorbed power on the matching load, and $\Re_s = \sum_{m=0}^{s} \chi_m^2$, which is calculated using the equation presented in [14].

A slot array antenna with 50 or more components has an absorption around 2% of the input power, which looks fairly reasonable. The slot's excitation phase is the same as the array element's phase. The voltage amplitude is calculated by (5) and the equation reported in [15]:

$$\left|\frac{V}{A}\right|_n \cong \sqrt{12.74\frac{kk_g b}{a}\gamma_n} \tag{5}$$

where $k = 2\pi/\lambda$, $k_g = 2\pi/\lambda_g$, and a and b are the dimensions of the waveguide's aperture.

The error function must be constructed to compute the offset and deviation angle of any slot. So we need to know the amplitude and phase of voltage distributions in the slot to create the error function. A complex slot characteristics graph, as well as genetic algorithms and conjugate gradient methods, may be used to calculate the offset values and their deviation angles.

So the error function is as follows:

$$\xi_n = w_{1,n}\xi_{1,n} + w_{2,n}\xi_{2,n} \tag{6}$$

where

$$\xi_{1,n} = \left|\left|\frac{V}{A}\right|_n - |\Lambda_n|\right|^2 \text{ and } \xi_{2,n} = \||\alpha_n| - |\angle\Lambda_n|\|^2 \tag{7}$$

$w_{1,n}w_{2,n}$ are the weight functions of the error function:

$$\Lambda_n = \frac{8.1}{a}\left[\frac{\cos\left(\frac{\pi}{2}\left(\cos\left(\zeta-\theta_n\right)\right)\right)}{\sin\left(\zeta-\theta_n\right)}e^{j\pi x_n/a} + \frac{\cos\left(\frac{\pi}{2}\left(\cos\left(\zeta+\theta_n\right)\right)\right)}{\sin\left(\zeta+\theta_n\right)}e^{-j\pi x_n/a}\right] \tag{8}$$

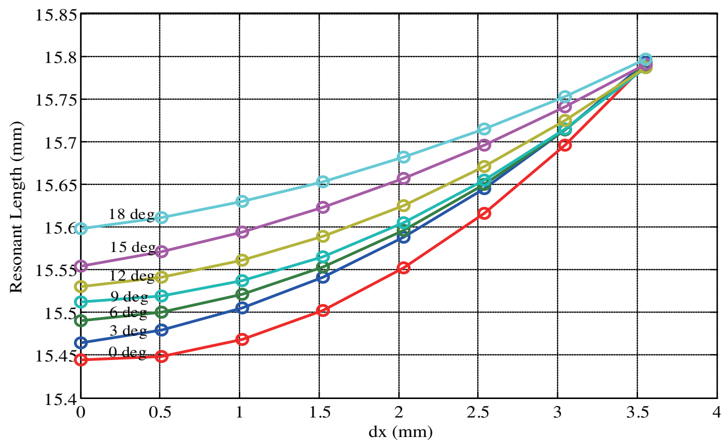

Figure 2. Resonant slot length against offset (dx).

4. RESONANT SLOT LENGTH CALCULATION

Figure 2 shows a graph of resonant slot length versus the offsets that may be used to derive resonant slot equations in terms of the polynomial term (which is dependent on the slot's offset and slope angle). The curve's fitting characteristic can be used to compute the length of resonant slots.

The fitting curve is obtained by Equation (9):

$$L_n(t_n, dx_n) = q_2(t_n)dx_n^2 + q_1(t_n)dx_n + q_0(t_n) \tag{9}$$

where $q_i(t)$ is

$$q_i(t_n) = p_4 t_n^4 + p_3 t_n^3 + p_2 t_n^2 + p_1 t_n + p_0 \tag{10}$$

And the coefficients are tabulated in Table 1.

Table 1. Resonant slot length equation coefficients.

$L_N(T_N, DX_N) \times 10^3$	$q_2(t_n)$	$q_1(t_n)$	$q_0(t_n)$
P_4	0.070178	−0.011342	1.63150×10^{-4}
P_3	2.66400	0.423430	-4.91223×10^{-3}
P_2	31.7090	−4.95007	0.044612
P_1	−144.577	19.6202	0.172869
P_0	742.311	−6.55757	608.048

5. ARRAY SYNTHESIS

Polynomial term array or Schelkunoff polynomial term is an N-element slot array with d spacing between the adjacent slots:

$$AF(\omega) = \frac{I_N}{I_0} \prod_{n=1}^{N-1} (\omega - \omega_n) \tag{11}$$

where ω_n are the roots of the Schelkunoff polynomial term. Root displacement on the Schelkunoff unit circle is used to create sum and difference patterns. The shaped pattern is generated as a consequence of null filling by roots displacement on the complex plane. In general, we suppose the Schelkunoff polynomial term roots as:

$$\omega_n = e^{a_n + jb_n}$$

If M_1 is considered as a root on the Schelkunoff unit circle, a universal term for any type of array may be defined. If $a_n = 0$, M_1 is a deep null in beam, and M_2 is filled null in that ($a_n \neq 0$); therefore, the term of array factor can be defined as Equation (11):

$$AF(\omega) = \frac{I_N}{I_0}(\omega + 1) \prod_{n=1}^{M1} (\omega - e^{jb_n}) \prod_{n=M1+1}^{M2} (\omega - e^{a_n + jb_n}) \tag{12}$$

given $\omega = e^{j\psi}$, the power in terms of dB is:

$$\begin{aligned} G(\psi) &= 10 \log \left(\frac{I_N}{I_0} \right) + 10 \log(2(1 + \cos(\psi))) + \sum_{n=1}^{M1} 10 \log(2(1 - \cos(\psi - b_n))) + \kappa \\ \kappa &= \sum_{n=M1+1}^{M1+M2} 10 \log(1 + e^{2a_n} - 2e^{a_n} \cos(\psi - b_n)) \end{aligned} \tag{13}$$

Equation (11) is applicable to the even number of elements by having the root in $\omega = -1$, so $M1 + M2 = N - 2$.

However, if we have the odd number of elements the term $\omega+1$ will be removed, so $M1+M2 = N-1$. As a result, various types of patterns may be created, such as symmetrical, asymmetrical, filled, or deep null patterns. For even values of N, we must assume $\omega = 1$ to have a different pattern with the arbitrary null; therefore, Equation (13) should be changed as follows:

$$\begin{aligned} G(\psi) &= 10 \log \left(\frac{I_N}{I_0} \right) + 10 \log(2(1 - \cos(\psi))) + \sum_{n=1}^{M1} 10 \log(2(1 - \cos(\psi - b_n))) + \kappa \\ \kappa &= \sum_{n=M1+1}^{M1+M2} 10 \log(1 + e^{2a_n} - 2e^{a_n} \cos(\psi - b_n)) \end{aligned} \tag{14}$$

For array synthesis, pattern nulling is obtained by controlling only the amplitude, position, and phase of the antenna array elements. To clarify the definition of deep null and filled null we can say that, by applying the deep null displacement in the array synthesis, the value of the integer M_1 can be used to indicate the quantity of side lobe level (SLL level). In order to create various patterns, like the top flat pattern or CSC^2 pattern, we must fill the deep nulls and determine the power level. In this article, the desired pattern shape is denoted by the integer M_2.

6. ERROR FUNCTION OF PATTERN SYNTHESIS

The acquired equations are utilized to synthesize the desired pattern as well as create and minimize the error function.

Given $\psi_n = (b_n + b_{n+1})/2$, the n'th lobe level may be determined by computing the value of $G(\psi)$.

As a result, Equation (14) is used to represent the error function:

$$error_{n,sll} = |G(\psi_n) - SLL_n|^2 \tag{15}$$

Ripple (RP) is a term used to describe a section of a pattern that is loaded by filled nulls and was originally described as a shaped region. Variations between side lobe levels in loaded nulls can be controlled by positive number RP, (i.e., in $a_n + jb_n$) also considering $\psi_n = (b_n + b_{n+1})/2$ peaks between loaded nulls can be controlled.

So, the shaped area's error function is as follows:

$$\text{error}_{n,RP} = \begin{cases} |G(\psi_n) - DL(\psi_n)|^2 + |G(\psi_n) - G(a_n + jb_n)|^2 & \text{if } |G(\psi_n) - G(a_n + jb_n)| > RP \\ 0 & \text{else} \end{cases} \tag{16}$$

where $DL(\psi_n)$ is a surface that must have a pattern surrounding it.

However, the following equation is the total error function.

$$\xi_{total\ error} = w_3 \sum_{n=1}^{M1} error_{n,sll} + w_4 \sum_{n=M1+1}^{M1+M2} error_{n,RP}$$

where w_3 and w_4 are weighting variables that are decided by our preferences. As an instance, if it is not required to have a full null, $w_4 = 0$ can be used.

7. RESONANT SLOT VOLTAGE AND PHASE GRAPH IN TERMS OF ANGLE AND OFFSET

After defining an acceptable VSWR and desirable array pattern, each array's desired phase values must be determined considering Figure 3 and also based on the array's placement on a waveguide. The geometrical specifications of slots, such as offset, angle, and their length, as well as the position of slots on a waveguide, are then determined using Equation (1). In practice, different offset values for the slots and adjustments to the slots' angles with respect to the central axis are required in order to compute and report the characteristics of slot excitation as shown in Figure 3.

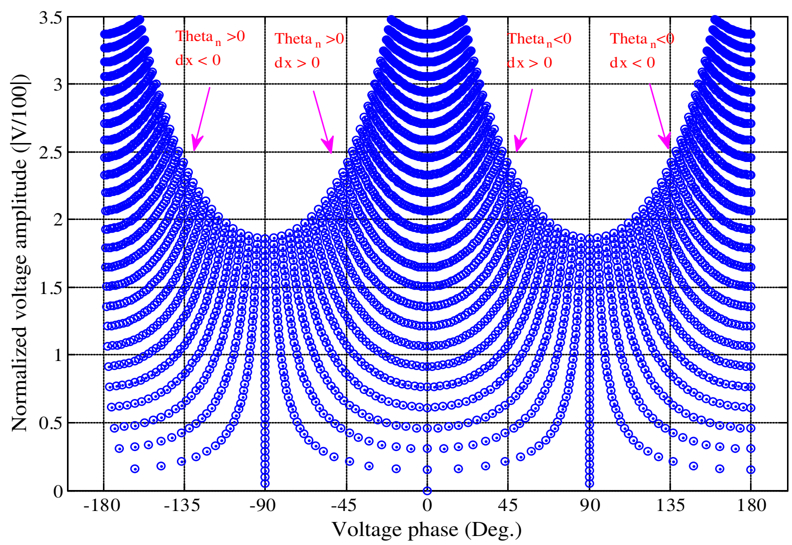

Figure 3. Characteristics of slot excitation.

8. SUM, DIFFERENCE, AND SHAPED BEAM PATTERN

8.1. Sum Pattern

For the sum pattern synthesis, a 10-element array of slots with $d = 0.5\lambda_g$ spacing between them is constructed. The desired levels of the first two side lobes for the design are -20 dB, whereas the other lobe levels are -30 dB. The weight components of the error function that contain filled nulls are assumed zero because we only have deep nulls. Table 2 lists the geometrical specifications, and Figure 4 shows the radiation pattern of the proposed antenna at 9.375 GHz.

8.2. Difference Pattern

The same technique is utilized for difference pattern synthesis as used for sum pattern synthesis to obtain values of offset, slots' slope angle, and slots resonant length. For difference pattern synthesis, same as the sum pattern, the first two side lobes levels are intended to be -20 dB, and the other lobes

Table 2. Geometrical specifications of slots to achieve sum pattern.

Slot number	Slot length (mm)	Slot displacement (mm)
1	15.4548	0.7186
2	15.4789	−1.2031
3	15.4900	1.3664
4	15.5059	−1.5661
5	15.5466	1.9848
6	15.5576	−2.0831
7	15.5253	1.7788
8	15.5127	−1.6447
9	15.5018	1.5179
10	15.4636	−0.9294

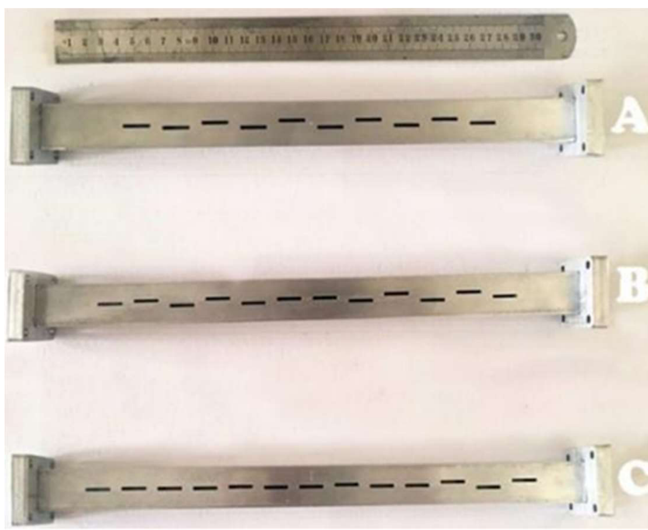

Figure 4. Fabrication of (a) sum pattern by 10 slots, (b) differential pattern by 12 slots and top flat pattern by 13 slots.

levels are supposed to be −30 dB. Table 3 shows the geometrical parameters of the proposed antenna, which includes a twelve-slot array with $d = 0.5\lambda_g$ spacing between the slots.

8.3. Shaped Beam Pattern

To create the top flat pattern some nulls must be filled. For flat pattern synthesis, the element numbers are considered to be 13, and both the left and right sides of the main lobe nulls are filled. Intended ripple level is 1 dB, and the side lobe level is 20 dB. The geometrical specifications of slots to achieve the shaped beam pattern are tabulated in Table 4.

9. MANUFACTURING, MEASUREMENT, AND COMPARISON

A bunch of typical resonant slot arrays on broad wall of the rectangular waveguide antenna is manufactured for the test, as shown in Figure 4. The prototypes include 10 slots to create sum pattern,

Table 3. Geometrical specifications of slots to achieve a different pattern.

Slot number	Slot length (mm)	Slot displacement (mm)
1	15.4731	−0.7369
2	15.4886	1.4468
3	15.5230	−1.8173
4	15.5250	1.7913
5	15.4897	−1.4671
6	15.4483	0.5937
7	15.4483	0.5961
8	15.4926	−1.5187
9	15.5426	1.9873
10	15.5590	−2.2071
11	15.5194	1.8952
12	15.4987	−0.9963

Table 4. Geometrical specifications of slots to achieve the shaped beam pattern.

Slot number	Slot length (mm)	Slot displacement (mm)
1	15.4441	0.1389
2	15.4445	0.2374
3	15.4441	−0.0693
4	15.4442	−0.1895
5	15.4464	0.3964
6	15.4446	−0.2465
7	15.4445	−0.2331
8	15.4518	0.6275
9	15.4469	−0.4236
10	15.4473	−0.4450
11	15.4997	1.4919
12	15.5584	−2.0899
13	15.5257	1.7833

12 slots to make differential pattern and top flat pattern by 13 slots. The measurement data are provided for the comparison and validation in Figures 5, 6, and 7. A coaxially fed monopole, a slotted waveguide, and a coax to waveguide feed and coax to waveguide SMA termination are all part of the fabricated antenna (we may also use an absorber for waveguide termination, such as Eccosorb LS14). The antenna has two ports, with one serving as the excitatory terminal and the other as a 50-Ohm male pin SMA terminator coax connector plug (TCCP) for absorbing traveling waves. A comparison between the proposed antenna and other slot arrays is conducted and listed in Table 5.

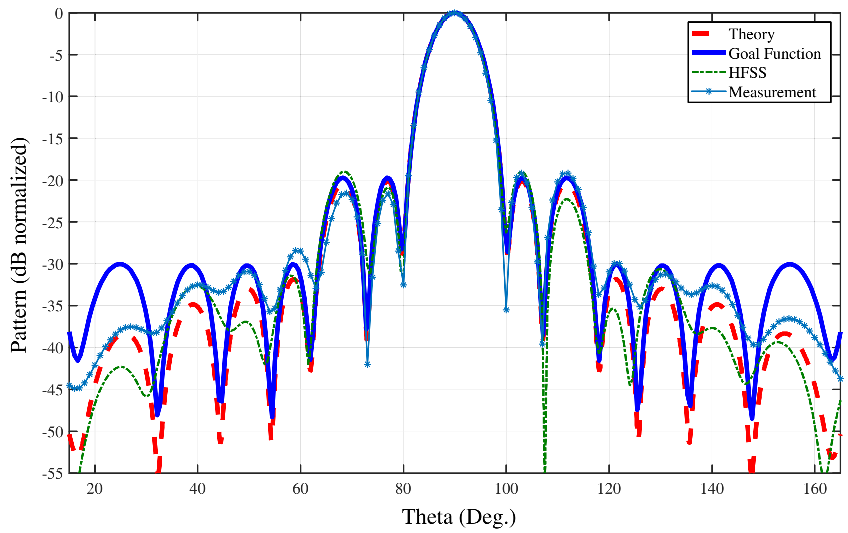

Figure 5. Sum pattern for 10 slots.

Table 5. Performance comparison of slot array antennas.

Ref.	**Element number**	f_0 **(GHz)**	**Feeding Mechanism**	**Imp. BW (%)**	**Peak Gain (dBic)**	**Peak SLL**	**size**
[16]	16×16	10	End feed	2.70	24.5	-31.3 dB (E-plane) -30.3 dB (H-plane)	9×8.67
[17]	10×10	10	End feed	2.40	22	-32.3 dB (E-plane) -33.8 dB (H-plane)	7×5.67
[18]	4×16	27.5	Center feed	2.18	21.4	-17.9 dB (E-plane) -19.1 dB (H-plane)	10.3×5.7
[19]	4×32	24	Center feed	1.71	22.8	-13.5 dB (E-plane) 2 -21.0 dB (H-plane)	15.6×3.2
[20]	6×16	10	differential feed	7.15	21.92	-29.1 dB (E-plane) Design -29.4 dB (H-plane)	14×4.83
proposed design	1×14	9.375	Standing	26	18	20 dB	35.683 mm

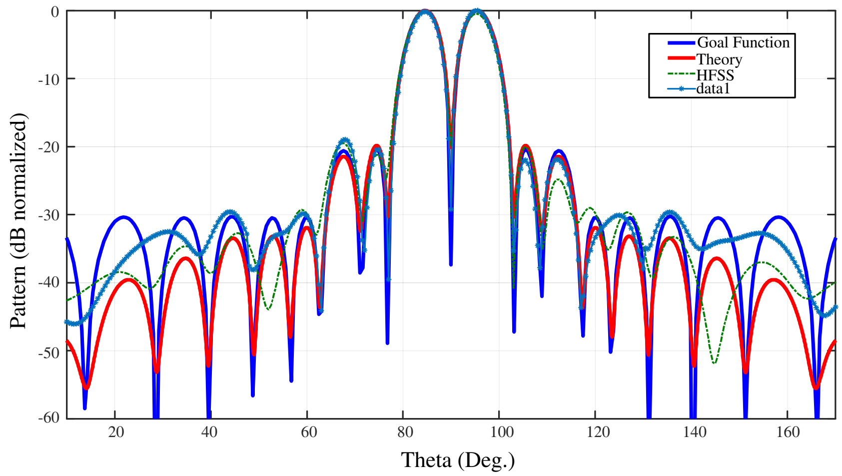

Figure 6. Difference pattern for 12 slots.

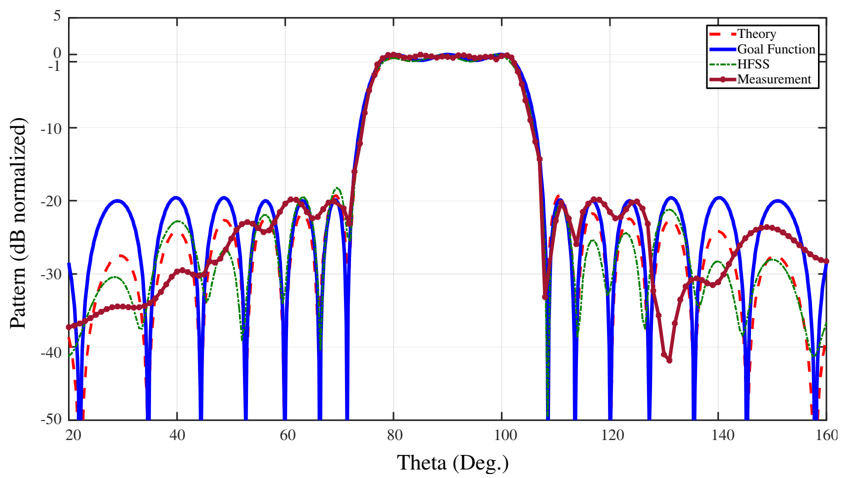

Figure 7. Shaped beam pattern (Top flat pattern) for 13 slots.

10. CONCLUSION

Through this work, we propose a method to have the most optimal design of rectangular waveguide slot arrays. The amplitude and phase of resonant slot voltage on the rectangular waveguide wall are adjusted using the suggested method. The offset and angle of resonant slots were calculated using the least-squares method and by the construction of the error function. The optimum values of phase and amplitude of each resonant slot are also determined using the pattern synthesis technique mentioned in this paper. According to the suggested analytical work, the length of the antenna can be calculated easily, and also by computing the offsets, the slot antenna array with the highest efficiency and lowest absorbed power on load (load loss) can be designed, and the suggested procedure is ideal for designing multiple elements long arrays utilizing the provided analytical work. The length of the antenna can be simply determined, so a slot antenna array with the maximum efficiency and lowest absorbed power on load (load loss) may be created. The suggested approach in this paper is appropriate for creating multiple elements long slot arrays.

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