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Optical Neural Networks for Holographic Image Recognition

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Abstract—Inspired by neural networks based on traditional electronic circuits, optical neural networks (ONNs) show great potential in terms of computing speed and power consumption. Though some progress has been made in devices and schemes, ONNs are still a long way from replacing electronic neural networks in terms of generalizability. Here, we present a complex optical neural network (cONN) for holographic image recognition, within which a high-speed parallel operating unit for complex matrices is proposed, targeting the real-imaginary-splitting and column splitting. Based on the proposed cONN, we have numerically demonstrated the training-recognition process on our cONN for holographic images converted from handwritten digit datasets, achieving an accuracy of 90% based on the back-propagation algorithm. Our training-verification integrated architecture will enrich the further development and applications of on-chip photonic matrix computing.

1. INTRODUCTION

As an important development within the computer discipline, artificial neural networks bring revolutionary solutions for image processing [1, 2], speech recognition [3, 4], data prediction [5], etc., but have gradually encountered bottlenecks in hardware implementation in terms of latency and power consumption [6, 7]. As one of the paths to breaking through Moore's Law, on-chip optical computing has been attracting attention for its high speed, low theoretical consumption, and natural suitability for parallel computing [8]. The field of optical computing was stagnant for a period of time due to severe limitations in device performance including low integration density and weak nonlinearity. In recent years, however, with the improvement of micro-nano optoelectronics technology and the huge demand for arithmetic power for AI applications, optical computing has gained more possibilities and attention, and thus become the frontier and hot spot of micro and nano optoelectronics research [9, 10].

In order to utilize the optical mechanism to implement neural networks, finding the corresponding implementation of matrix-vector multiplication (MVM) is necessary, which is the core operation form in the neural network [11–13]. The exploration of ONNs can be traced back to the 1990s when Reck et al. proposed a revolutionary idea to implement matrix operations on arrays of light intensity signals using beam splitters [14]. Their proposed method for splitting higher-order matrices into 2-order unitary (or orthogonal) matrices is called Reck model, and an alternative called Clement model which is based on a similar principle later proposed in 2016 [15]. The construction ideas of these two models are similar (actually both of them could be considered as two special solutions to the same mathematical problem), and the former is easier to obtain parameters, while the latter is more conducive to the

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integration of devices. Based on these two models, several on-chip optical neural network schemes have been proposed to make ONNs more feasible, most of which adopt MZI (Mach-Zehnder interferometer) arrays rather than the original beam splitters to perform MVM, considering the integration capability and compatibility [16]. Besides, spatial light computing, which is another alternative method for ONN implementation and always operates by well-designed or programmable metasurfaces, sacrificing space occupation for ultra-large computing power, is also a trend of development [17–19]. In addition, the practicability of utilizing fast Fourier transform [20] or frequency multiplexing [21, 22] to achieve a similar effect has been demonstrated as unconventional routes to achieve ONNs on chips.

3D imaging, especially holographic imaging, offers broad application prospects in areas ranging from medical detection [2, 23] to topographic mapping [24, 25] due to its high information density and interference resistance. After holograms are obtained, neural networks with self-learning and self-adaptive features are often regarded as powerful tools for follow-up work, such as identification and classification. Concurrently, the processing of 3D imaging has been extensively studied in conventional electronic networks, but its ONN solution has scarcely progressed since holographic image recognition is based on complex information, while previous ONNs always focus on calculations in the real domain. Therefore, an ONN extended to enable complex operations would be a direct and effective way to fill this gap.

Here in our work, an optical neural network, which can realize high-speed complex-matrix optical operations for holographic image recognition is proposed based on existing theory. To achieve the intended performance, the operation unit in our network is designed to be a high-speed parallel operating unit for complex matrices, which targets the real and imaginary parts splitting, as well as the column splitting. A physical feedback loop based on optical nonlinear devices and photodetectors was further designed to achieve the full functionality. The proposed cONN was validated by numerical simulation. We first verified the convergence of our cONN through random initial inputs and predictions of 2-by-2 scale data. Then a recognition task of 20-by-20 scale holographic handwritten digital images, which were generated by the method of the computational hologram, was carried out on our cONN, and acceptable results were obtained.

2. BASIC DESIGN AND THEORICAL MODELING

MZI is an optical device consisting of an optical waveguide and phase-modulation components. By electro-optical [26, 27] or thermo-optical effects [28], MZI typically multiplies the input two-dimensional vector (optical signal) with a two-dimensional orthogonal matrix defined by MZI configuration. When only one of the ports is utilized, the regular MZI (Fig. 1(a)) will become an MZI attenuator (Fig. 1(b)), which realizes the multiplication operation of scalars in a cosine manner. As mentioned before, a high-dimensional orthogonal matrix can be split and represented by a combination of regular MZI and MZI attenuators [29]. Thus, assuming that an operation matrix M is required by the specific task, the orthogonal matrices U, V and the diagonal matrix Σ can be obtained by performing the singular value decomposition (SVD) on the matrix ($M = U\Sigma V^T$). For U and V, the decomposition based on the Clement model would be done to obtain the modulation parameters for each regular MZI, while Σ corresponds to a row of MZI attenuators. Thus, an adjustable representation of any desired matrix would be achieved.

To derive our final design from the analysis above, we first implement the RI-splitting (real-imaginary-splitting) and column splitting of a matrix separately. Though the optical signal itself can be considered as spontaneously carrying complex information in its amplitude and phase, such property can hardly be directly applied to complex operations because photodetector is not capable to record the phase signals that change with space during the propagation. Therefore, performing complex operations on the real domain becomes an alternative method. According to the principle of linear algebra, we can obtain the expression (Eq. (1)) for converting complex matrix-vector multiplication into real form. In the expression, A and B respectively represent the operator matrix and the input vector, both containing complex information, while the subscripts R and I represent the real and imaginary parts of the original matrix (or vector). Here we present the operation as a sum of two terms, which corresponds to the idea that the real and imaginary components of weights are adjusted independently at the hardware level to

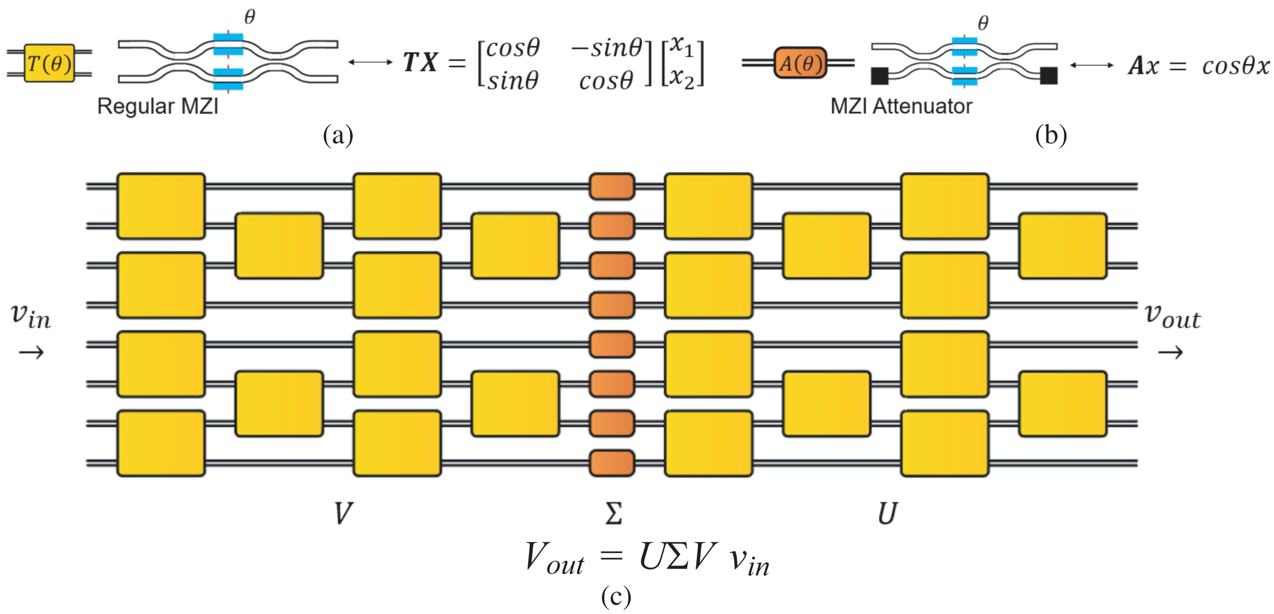


Figure 1. The scheme of the linear arithmetic unit for complex matrix operations. Regular (a) MZI and (b) MZI attenuator can undertake matrix operations and scalar operations respectively. (c) Two types of MZIs are organized in a specific form according to the clement model principle and our proposal of dealing with complex matrices, assuming multiplication between 2×2 complex matrices.

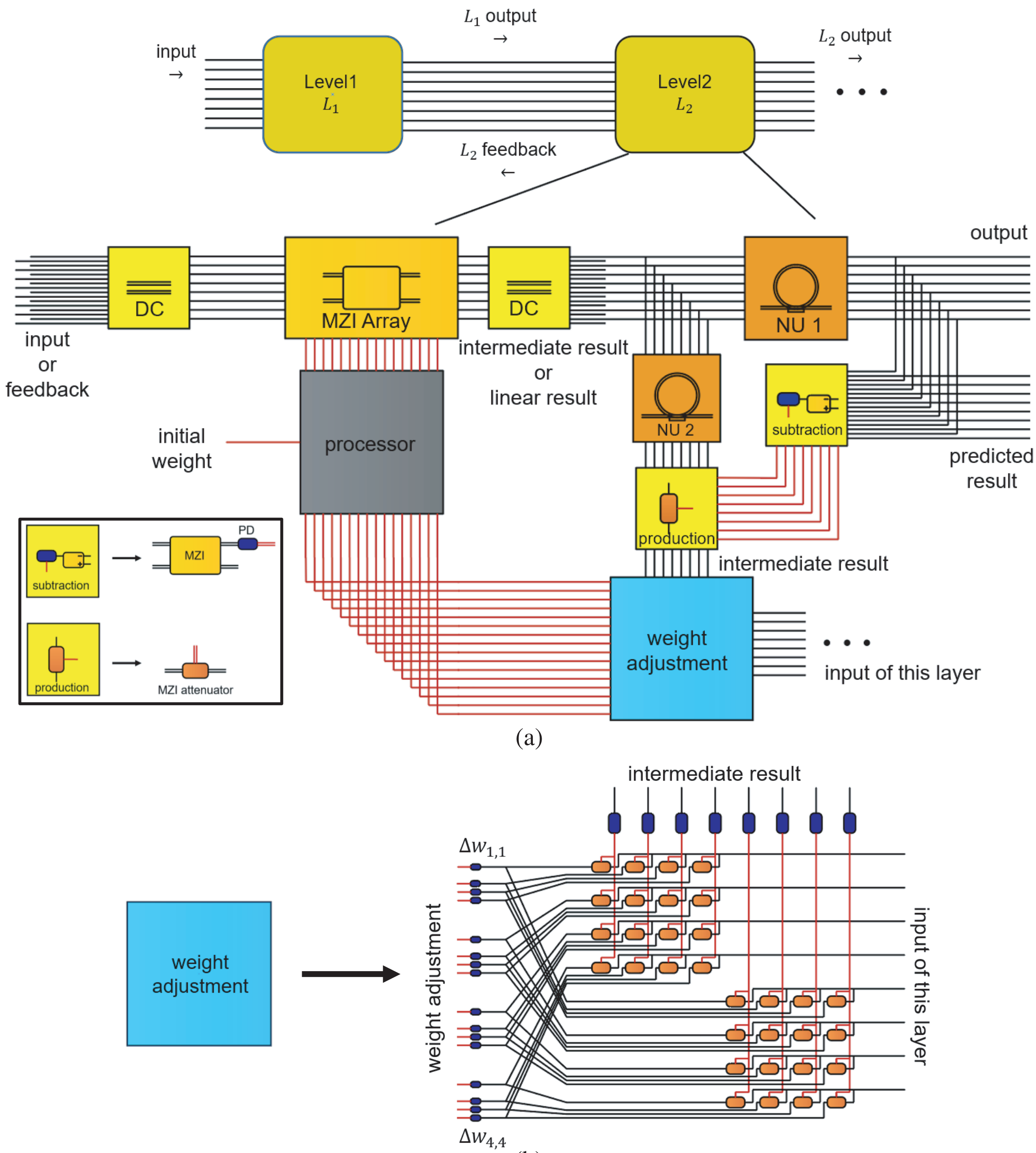
prevent parameter redundancy due to matrix expansion.

$$\begin{bmatrix} A_R & -A_I \\ A_I & A_R \end{bmatrix} \cdot \begin{bmatrix} B_R \\ B_I \end{bmatrix} = \begin{bmatrix} A_R & 0 \\ 0 & A_R \end{bmatrix} \cdot \begin{bmatrix} B_R \\ B_I \end{bmatrix} + \begin{bmatrix} 0 & -A_I \\ A_I & 0 \end{bmatrix} \cdot \begin{bmatrix} B_R \\ B_I \end{bmatrix} \tag{1}$$

In practical tasks, the processed information is often expressed in the form of a matrix (images for example), and if it is sent column by column. The overall operation speed would be mostly limited by the clocking frequency of the system, and the advantages of high speed and parallelization of optical operations cannot be fully utilized. Therefore, we consider splitting and splicing the information of matrix form into one-dimensional vectors by column, and directly participating in matrix operations. This process can be described by Eq. (2) in terms of matrix operations. Combining the two components above, a linear arithmetic unit for complex matrix operations is proposed (Fig. 1(c)).

$$[A]_{n\times n} \cdot [B]_{n\times n} \rightarrow \begin{bmatrix} A & & \\ & \ddots & \\ & & A \end{bmatrix}_{n^2\times n^2} \cdot \begin{bmatrix} B \\ \vdots \\ B \end{bmatrix}_{n^2} \tag{2}$$

The BP (back-propagation) algorithm [30] is an elementary and effective method for training weights in neural networks, which is relatively simple to implement in hardware. According to the principle of the BP algorithm, we implement the algorithm with optoelectronic devices. For example, we utilize the photoelectric conversion (100% efficiency) and optical nonlinearity to implement the linear operations and activation functions in the algorithm (See SI Section 1 for details). As a result, an on-chip neural network circuit including waveguides, optical nonlinearities, photodetectors, etc. is conceived and shown in Fig. 2. In specific network levels, nonlinearities with two different types of functions (NU1 and NU2 in Fig. 2(a)) need to satisfy a specific relationship where one is a derivative function of the other. Meanwhile, in contrast to electronic networks, the information in the ONN is carried on the light intensity. In digital circuits where information is carried on logical signals based on voltage, branches from the same node share the same information, while in optical circuits, the signal intensity would be split into parts with constant total energy. Therefore, the energy changes in photodiodes, as well as branching of the optical path, need to be considered in the feedback intensity. In order to reduce the device size, some structural optimizations have been made (Fig. 2(b)), effectively enhancing the efficiency of the structure.



(a)

(b)

Figure 2. Hierarchies of the network. (a) The overview of a multi-layer network and a typical single layer in our network, which consists of MZIs, waveguides, optical nonlinearities and photodetectors to perform computation and training tasks. In single layer, the red lines represent the circuit and the black lines represent the optical path, and the two types of nonlinear units (NU1 & NU2) are assumed to be micro-ring structures here. In order to achieve a feedback loop, MZIs and photodetectors (PDs) are utilized to perform vector subtraction and multiplication of the corresponding elements of the vectors. (b) The weight adjustment block, which is designed to fit our objective of highly parallel complex number operations.

3. RESULTS

Due to technical limitations and the idealization of the scheme, our results are firstly based on numerical demonstration. To verify the feed forward operation and backward propagation adjustment, we build a simulation network without hidden layers, in which all the optoelectronic devices are modeled (Fig. 3(a)). The latency of each device is considered in 1 nanosecond. For the simulation, we randomly generate two complex matrices of 2×2, one of which is the input, and the other is decomposed in advance and written into the MZI array as the operation matrix (or initial weight). Here, for simplicity, we ignore the effect of the branch on the signal and assume the waveguide is lossless. Then we do the simulation, letting the photoelectric signal flow in the circuits to perform the feed forward operation and backward propagation adjustment. The results are shown in Figs. 3(b), (c), from which it is shown that when the signal is stable, the result is the same as the pre-calculated value.

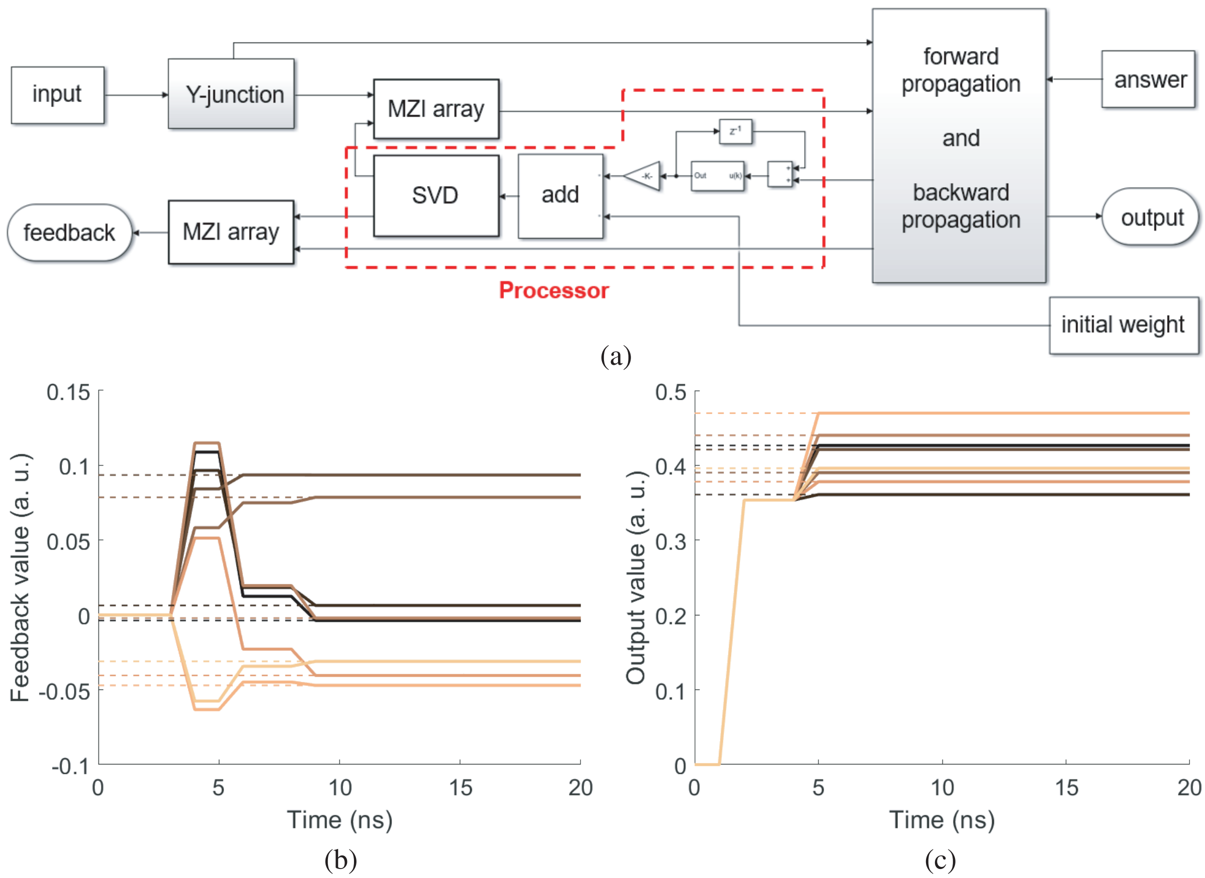


Figure 3. The scheme and results of preliminary verification for a single-level network (See SI Section 2 for details). (a) Scheme for simulation of 2 by 2 matrix with each value within $[-1, 1]$. Y-junctions (and some more in the back propagation block) are added to reflect the role of branches in the actual optical path, which were omitted in further simulations, and a processor is used to control the timing, update the weight and perform SVD operations. The results obtained from (b) feedback port and (c) output port, in which solid lines correspond to sequential results and correct results for dashed lines, verifying the forward calculation and backward feedback (between front and rear levels).

A vector containing 0's and 1's is further generated as the predefined results, and a multi-cycle simulation was done. Within 50 iterations the desired output signals rapidly approached the maximum, and minimum for other signals. The Cross-Entropy Error (CEE) loss between the output and predefined results is converged to 0. Furthermore, to verify the feasibility of the network for image recognition, we build a two-level network with a hidden layer for the recognition of 20×20 grayscale and holographic images. Considering the feasibility of the actual footprint of the chip with the expectation of about $20\,\text{mm}^2$ for a single level, the network is designed to process 16 images in one shot with $16 \times 400 = 6400$ input neurons, $16 \times 25 = 400$ neurons in the hidden layer, and $16 \times 10 = 160$ output neurons (Fig. 4(a)). A specific time sequence for simulation is designed to enable the network to perform the training and

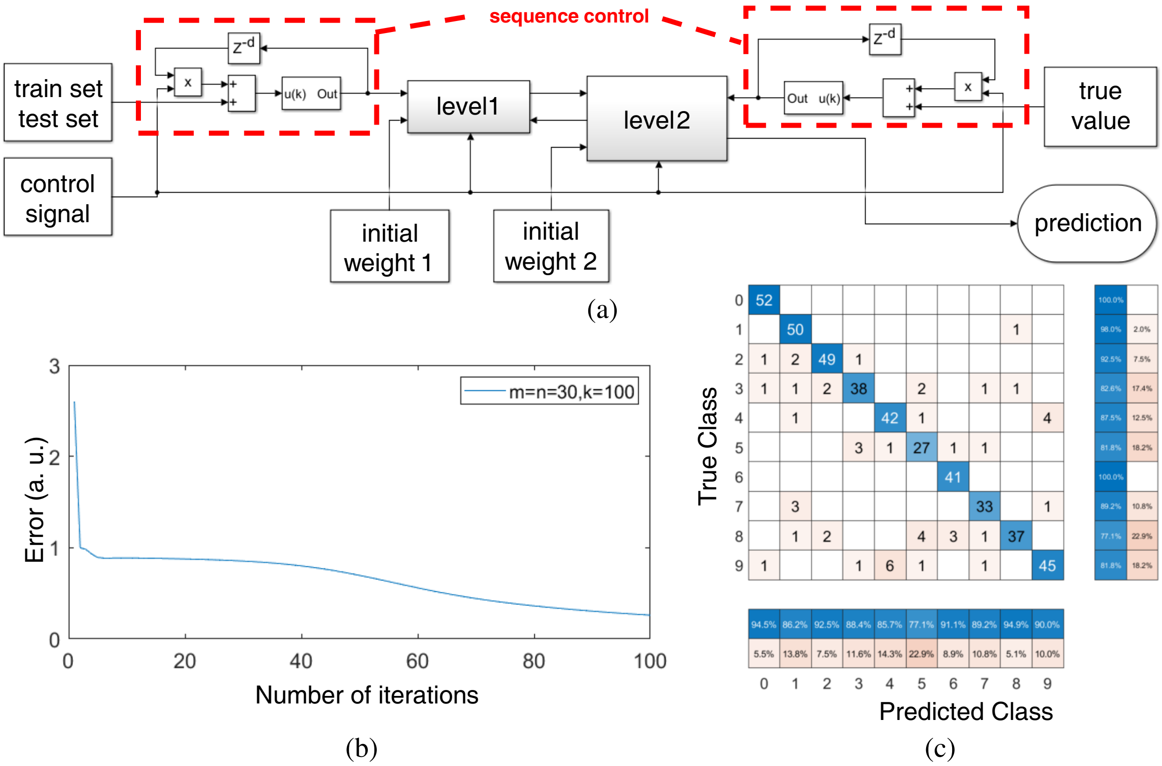


Figure 4. The results for the task of handwritten number recognition. (a) A network with two levels (3 layers). An external sequence control module with a control signal to achieve a one-time implementation of training-testing is added. The (b) MSE and (c) confusion chart obtained from the simulation demonstrate the validation of the network.

validation sequentially: 1) Preprocessing stage: 16$\mathbf{n}$ images are extracted as the training set; 16$\mathbf{m}$ images are extracted as the validation set; and a random initial weight matrix is generated. 2) Training stage: The training set images are sent to the network in groups of 16 for a total of $\mathbf{n}$ groups and repeated $\mathbf{k}$ times in $\mathbf{n} \times \mathbf{k}$ training cycles. 3) Switching stage: Cut off the data input and feedback path for a cycle to ensure the completion of the previous stage. 4) Test stage: The test set images are sent to the network in groups of 16 for a total of $\mathbf{m}$ groups, and the results are collected in $\mathbf{m}$ validation cycles. Regarding the computational holography, we convert those greyscale images to holographic images (See SI Section 3 for details) and do the simulation for both cases (greyscale and holographic) with $\mathbf{n} = \mathbf{m} = 30$, $\mathbf{k} = 100$. The results of holograms are shown in Figs. 4(b), (c). In the case of randomly selected training and validation groups, the final recognition accuracy reaches 90%, which is well acceptable considering that there is no elaborate advanced algorithm in our scheme.

4. DISCUSSION

In our work, we focus on the complex operations in ONNs driven by the application of holographic image recognition and propose our scheme as cONNs. Compared with other ONNs designed and applied for handwritten digital recognition [31, 32] (See SI Section 4 for details), our scheme gives comparable results and manages to process holographic (complex) image information. Using MZI arrays, we perform matrix operations based on image pixel data, and hence the demand for MZI array size is directly related to the size of the input image. Thus, when it comes to ultra-large scale, the access rate of matrix weights becomes a limiting factor that confines the speed of network operations. Therefore, efficient data addressing in optical signal storage needs to be achieved. Other assumptions are made in the design as well, including the feasibility of optical nonlinearity. To achieve a wide output range and specialized features determined by BP algorithms, optical nonlinearity neuron devices would be desired. Materials based on metallic quantum wells [33–35], which own the giant nonlinearities, and the non-Hermitian optical resonators [36–38] might provide a solution for this.

5. CONCLUSION

We have proposed a complex optical neural network that can realize high-speed complex-matrix optical operations for holographic image recognition. To realize the target, we design a linear arithmetic unit for high-speed complex matrix operations, and inspired by the BP algorithm, a photoelectric circuit was proposed to perform the training process. In the simulation of the recognition task, the effectiveness and accuracy of our network have been verified. To make the network more efficient and practical, further research on design and devices is still needed.

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