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Polarization Reversal of Oblique Electromagnetic Wave in Collisional Beam-Hydrogen Plasma

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Abstract—Energetic ion or electron beams cause plasma instabilities. Depending on plasma and the beam parameters, an ion beam leads to change in the dispersion relation of Alfven waves on interacting with magnetoplasmas as it can efficiently transfer its energy to the plasma. We have derived dispersion relation and the growth rates for oblique shear Alfven wave in hydrogen plasma. The particles of the beam interact with the Shear Alfven waves only when they counter-propagate each other and destabilize left-hand polarized mode for parallel waves and left-hand as well as right-hand polarized modes for oblique waves, via fast cyclotron interaction. The collisions between beam ions and plasma components affect the growth rate and the frequency of generated Alfven waves, differently for right-hand (RH) and left-hand (LH) polarized oblique Alfven modes. For $(\omega + k_z v_{bo} > \omega_{bc})$, the most unstable mode is the LH polarized oblique Alfven mode, and it is the RH polarized oblique Alfven mode for $(\omega + k_z v_{bo} < \omega_{bc})$, which shows a polarization reversal after resonance condition. Numerical results indicate that the growth rates increase with increase in angle of propagation. The maximum growth rate values in the presence or absence of beam increase due to obliquity of wave.

1. INTRODUCTION

Alfven waves are a type of magneto-hydro-dynamic (MHD) waves which are low frequency waves below the ion cyclotron frequency travelling in a magnetized conducting fluid like plasma existing in space. These waves transport electromagnetic energy and communicate information concerning changes in plasma currents and magnetic field topology. At low frequencies, there are two different modes of electromagnetic propagation i) a compressional wave in which magnetic field strength and density change and ii) a shear wave in which only the direction of magnetic field varies. In the present paper, we are concerned with shear Alfven wave only. The shear Alfven wave propagates with the wave magnetic field vector perpendicular to background field. Gekelman et al. [1] have studied various properties related to shear Alfven waves.

The shear Alfven wave is nearly incompressible and hence, more readily excitable by either external perturbations, e.g., solar wind, antenna or intrinsic collective instabilities (Chen and Zonca [2]). In a cylindrical column, the dispersion of shear wave was determined by Jephcott and Stocker [3]. In last three decades, many experiments on Alfven waves have been done where some fundamental properties of shear Alfven wave have been explored. Oblique shear Alfven wave, which is transverse with strong magnetic field variation perpendicular to the wave motion, can trade energy between the different frequencies which might propagate through plasma. This also means that energy can be exchanged with the particles in the plasma, in some cases, trapping particles in the troughs of the wave and carrying them along.

Plasma instabilities are caused by the energetic ion or electron beams. These beams are ever-present in the variety of space and astrophysical plasmas. Ion beam and interaction of radiation with plasma

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allocate a free energy source which can excite different wave modes in a plasma. Plasma waves can be stabilized or destabilized when they interact with electron or ion beams [4–11], and collisions generally have a stabilizing effect on the plasma waves [12, 13]. An ion beam can destabilize Alfven waves and whistler waves if the beam speed is sufficiently large. The shear Alfven wave instability depends on the free energy stored in the particle distribution function in a velocity space, and in a magnetized plasma, ion neutral collisions result in the change of its dispersion relation [14]. For the study of Alfven waves employing parallel ion beams, several laboratory experiments have been performed. In a laboratory magnetoplasma, Tripathi et al. [15] demonstrated excitation of propagating shear Alfven wave and established that the ambient plasma density and electron temperature were increased significantly by the ions beam. Zhang et al. [16] have experimentally explained the Doppler shifted cyclotron resonance between fast Lithium ions and shear Alfven waves in the helium plasma of the Large Plasma Device. In the partially ionized solar chromospheres, the collisions between various particles are an efficient dissipative mechanism for Alfven waves [16, 17].

Many physicists have shown that both Alfven waves and magnetosonic waves can be excited nonlinearly [18–21] and have explained about interactions of Alfven waves with ions [20]. Hollweg and Markovskii [21] have analytically studied the instabilities generated by cyclotron resonances of ions with obliquely propagating waves in coronal holes and solar wind. Li and Lu [22] have investigated the interaction between oblique propagating Alfven waves and minor ions in the fast solar wind stream. Hellinger and Mangeney [23] studied structure of an oblique shock wave by means of numerical simulations, and they found that proton beam generates whistler waves at slightly oblique propagation. They also showed that for dense proton beams, the oblique modes have an important role in the nonlinear stage of electromagnetic instability [24]. Verscharen and Chandran [25] studied the polarization properties of both oblique and parallel propagating waves in the presence of ion beam and found that minimum beam speed required to excite such instabilities was significantly smaller for the former mode than for latter mode with $k \times B_0 = 0$. Maneva et al. [26] examined the comparative behavior of parallel vs oblique Alfven cyclotron waves in the observed heating and acceleration alpha particles in the fast solar wind. They aimed to find which propagation angles were the most efficient in preferentially heating the alpha particles within the considered low frequency turbulent wave spectra. Gao et al. [27] found that the obliquely propagating Alfven waves can be excited by alpha/proton instability, and background proton component & alpha component can be heated resonantly by Alfven waves. The study of oblique modes is very important in the thermodynamics of minor ions in the solar wind and may also find application within the Earth's bow shock as well as to basic plasma processes.

In the present paper, we study the reaction of oblique shear Alfven waves with an ion beam forced into magnetized plasma parallel to the magnetic field. In Section 2, instability analysis is given. The dispersion relation and growth rate of Alfven waves for fast and slow cyclotron interactions are derived in the absence as well as in the presence of plasma and beam collisions, and the results are discussed. Finally, the conclusion is given in Section 3.

2. INSTABILITY ANALYSIS

Assume a plasma in a dc background magnetic field $B_s \parallel \hat{z}$, with electron density n_{eo}, ion density n_{io}, electron mass m_e, ion mass m_i, electron charge $-e$, and ion charge e. A positively charged particle-beam having mass m_b, charge q_b, and density n_{bo} passes parallel to the magnetic field, through the plasma along the z-direction with velocity v_{bo}. Consider that an electromagnetic Alfven wave is propagating through the plasma with electric field

$$\vec{E} = Ae^{-i\left(\omega t - \vec{k}\cdot\vec{r}\right)}, \quad \text{where} \quad \vec{k} = k_x\hat{x} + k_z\hat{z}.$$

The magnetic field of the electromagnetic wave is given as $\vec{B} = c\vec{k} \times \vec{E}/\omega$.

The equation of motion, governing the drift velocity of plasma ions, plasma electrons, and beam particles, is

$$m\left[\frac{\partial\vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right] = -e\vec{E} - \frac{e}{c}\vec{v} \times \left(\vec{B}_s + \vec{B}\right) - \nu m\vec{v}, \tag{1}$$

where ν is the collision frequency of beam particles and plasma components. In equilibrium (i.e., in the absence of the wave), $v_{\mathbf{i}} = 0$ and $v_e = 0$. When wave is present, $\vec{E}$ and $\vec{B}$ of the wave are treated as small or perturbed quantities and result in perturbed velocities υ_{i1}, υ_{e1}, and υ_{b1} for plasma ions, plasma electrons, and beam particles, respectively.

On linearizing Eq. (1), we obtain

$$\frac{\partial \vec{\upsilon}_1}{\partial t} = -\frac{e\vec{E}}{m} - \vec{\upsilon}_1 \times \hat{z}\omega_{ce} - v\vec{\upsilon}_1, \tag{2}$$

where $\omega_{ce} = \frac{eB_s}{mc}$ is the electron cyclotron frequency.

By using equation of continuity, we have calculated the number densities of electrons and ions as

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{\upsilon}) = 0, \tag{3}$$

Current densities of different components have been determined using the relation

$$\vec{J} = ne\vec{\upsilon}. \tag{4}$$

To obtain dispersion relation, the wave equation is given as

$$\nabla^2\vec{E} - \nabla\left(\vec{\nabla} \cdot \vec{E}\right) + \frac{\omega^2}{c^2}\vec{E} = -\frac{4\pi i\omega}{c^2}\vec{J}. \tag{5}$$

2.1. Oblique Shear Alfven Wave

We consider an elliptically polarized Alfven wave propagating in X-Z plane with electric vectors also in X-Z plane, i.e., $\vec{E} = E_x\hat{x} + E_z\hat{z}$ and $\vec{k} = k_x\hat{x} + k_z\hat{z}$, $k_z = k\cos\theta$ and $k_x = k\sin\theta$.

Obtaining x, y, z-components of Eq. (2), we calculate the perturbed plasma electron velocities as

$$\upsilon_{e1x} = \frac{\mathrm{ie}\omega E_x}{m_e\omega_{ce}^2}, \tag{6}$$

$$\upsilon_{e1y} = -\frac{eE_x\omega_{ce}}{m_e\omega_{ce}^2}, \tag{7}$$

$$\text{and} \quad \upsilon_{e1z} = \frac{eE_z}{m_e i\omega}. \tag{8}$$

Substituting Eqs. (6), (7), and (8) in Eq. (4), we obtain the perturbed plasma electron current densities as

$$J_{e1x} = -n_{eo}\frac{e^2}{m_e}\frac{i\omega E_x}{\omega_{ce}^2}, \tag{9}$$

$$J_{e1y} = n_{eo}\frac{e^2}{m_e}\frac{\omega_{ce}Ex}{\omega_{ce}^2}, \tag{10}$$

$$\text{and} \quad J_{e1z} = -\frac{n_{eo}e^2E_z}{m_e i\omega}. \tag{11}$$

The perturbed plasma ion current densities are obtained from Eqs. (9), (10), and (11), by replacing e by $-e$, m_e by m_i, and ω_{ce} by ω_{ci}.

Similarly, the perturbed beam electron velocities are obtained using Eq. (2) as

$$\upsilon_{b1x} = \frac{q_b}{m_b}\frac{(\nu - i\bar{\omega})\,\bar{\omega}E_x}{\omega\left[(\nu - i\bar{\omega})^2 + \omega_{bc}^2\right]} + \frac{q_b}{m_b}\frac{\upsilon_{bo}k_x(\nu - i\bar{\omega})\,E_z}{\omega\left[(\nu - i\bar{\omega})^2 + \omega_{bc}^2\right]}, \tag{12}$$

$$\upsilon_{b1y} = -\frac{q_b}{m_b}\frac{\bar{\omega}\omega_{bc}E_x}{\omega\left[(\nu - i\bar{\omega})^2 + \omega_{bc}^2\right]} - \frac{q_b}{m_b}\frac{\upsilon_{bo}k_x\omega_{bc}E_z}{\omega\left[(\nu - i\bar{\omega})^2 + \omega_{bc}^2\right]}, \tag{13}$$

$$\text{and} \quad \upsilon_{b1z} = \frac{q_bE_z}{m_b(\nu - i\bar{\omega})}, \tag{14}$$

where ω_{bc} is the cyclotron frequency of beam particles and $\bar{\omega} = \omega - k_z\upsilon_{bo}$.

The perturbed beam particle current densities calculated using Eq. (4) are

$$J_{b1x} = \frac{q_b^2}{m_b}\frac{n_{bo}\left(\nu - i\bar{\omega}\right)\bar{\omega}E_x}{\omega\left[\left(\nu - i\bar{\omega}\right)^2 + \omega_{bc}^2\right]} + \frac{q_b^2}{m_b}\frac{n_{bo}v_{bo}k_x\left(\nu - i\bar{\omega}\right)E_z}{\omega\left[\left(\nu - i\bar{\omega}\right)^2 + \omega_{bc}^2\right]}, \tag{15}$$

$$J_{b1y} = -\frac{q_b^2}{m_b}\frac{n_{bo}\bar{\omega}\omega_{bc}E_x}{\omega\left[\left(\nu - i\bar{\omega}\right)^2 + \omega_{bc}^2\right]} - \frac{q_b^2}{m_b}\frac{n_{bo}v_{b0}\omega_{bc}E_z}{\omega\left[\left(\nu - i\bar{\omega}\right)^2 + \omega_{bc}^2\right]}, \quad \text{and} \tag{16}$$

$$J_{b1z} = \frac{q_b^2}{m_b}\frac{n_{bo}v_{bo}k_x\left(\nu - i\bar{\omega}\right)\bar{\omega}E_x}{\omega\left[\left(\nu - i\bar{\omega}\right)^2 + \omega_{bc}^2\right]} + \frac{q_b^2 n_{bo}}{m_b}\left[\frac{\omega}{\bar{\omega}\left(\nu - i\bar{\omega}\right)} + \frac{v_{bo}^2k_x^2\left(\nu - i\bar{\omega}\right)}{\omega\bar{\omega}\left[\left(\nu - i\bar{\omega}\right)^2 + \omega_{bc}^2\right]}\right]E_z \tag{17}$$

Substituting Eqs. (9), (10), (11), (15), (16), (17) and corresponding current densities of plasma ions in Eq. (5), we obtain

$$\begin{aligned} \eta E_x + \mu E_z &= 0 \\ \mu E_x + \lambda E_z &= 0 \\ \text{or} \quad \begin{pmatrix} \eta & \mu \\ \mu & \lambda \end{pmatrix}\begin{pmatrix} E_x \\ E_z \end{pmatrix} &= 0, \end{aligned} \tag{18}$$

where

$$\begin{aligned} \eta &= -k_z^2 + \frac{\omega^2}{c^2} + \frac{\omega_{pe}^2\omega^2}{c^2\omega_{ce}^2} + \frac{\omega_{pi}^2\omega^2}{c^2\omega_{ci}^2} + \frac{\omega_{pb}^2 i(\nu - i\bar{\omega})\bar{\omega}}{c^2[(\nu - i\bar{\omega})^2 + \omega_{bc}^2]}, \\ \mu &= k_xk_z + \frac{\omega_{pb}^2 i v_{bo}k_x\left(\nu - i\bar{\omega}\right)}{c^2\left[\left(\nu - i\bar{\omega}\right)^2 + \omega_{bc}^2\right]}, \\ \lambda &= -k_x^2 + \frac{\omega^2}{c^2} - \frac{\omega_{pe}^2}{c^2} - \frac{\omega_{pi}^2}{c^2} + \frac{\omega_{pb}^2 i\omega^2}{c^2\bar{\omega}\left(\nu - i\bar{\omega}\right)} + \frac{\omega_{pb}^2 v_{bo}^2k_x^2 i\left(\nu - i\bar{\omega}\right)}{c^2\bar{\omega}\left[\left(\nu - i\bar{\omega}\right)^2 + \omega_{bc}^2\right]}, \end{aligned} \tag{19}$$

$$\omega_{pe}^2 = \frac{4\pi n_{eo}e^2}{m_e}, \quad \omega_{pi}^2 = \frac{4\pi n_{io}e^2}{m_i} \quad \text{and} \quad \omega_{pb}^2 = \frac{4\pi n_{bo}q_d^2}{m_d}.$$

The dispersion relation can be obtained by taking

$$\eta\lambda - \mu^2 = 0. \tag{20}$$

For elliptically polarized mode of propagation, $E_x = \pm iE_z$, which gives the dispersion relation as

$$\begin{aligned} \left(\omega^2 - \omega_s^2\right) = & \frac{-\omega_{pb}^2\left[\left(\bar{\omega}^2 - k_xv_{bo}\nu\right)\left(\omega_{bc}^2 - \bar{\omega}^2\right) - 2\bar{\omega}^2\nu\left(k_xv_{bo} + \nu\right)\right]}{\left(\omega_{bc}^2 - \bar{\omega}^2\right)^2(1 + A)} \\ & + i\frac{\bar{\omega}\left[\left(k_xv_{bo} + \nu\right)\left(\omega_{bc}^2 - \bar{\omega}^2\right) + 2\nu\left(\bar{\omega}^2 - k_xv_{bo}\nu\right)\right]}{\left(\omega_{bc}^2 - \bar{\omega}^2\right)^2(1 + A)}, \end{aligned} \tag{21}$$

$$\text{where} \quad \omega_{s\mp}^2 = \frac{k_z^2c^2 \mp ik_xk_zc^2}{(1 + A)}, \quad \text{and} \quad A = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2}. \tag{22}$$

Equation (22) is the modified dispersion relation of RH (subscript-) and LH (subscript+) elliptically polarized oblique shear Alfven wave in the absence of beam.

In terms of θ, the angle of propagation of shear Alfven wave with respect to the ambient magnetic field, we can write Eq. (22) as

$$\omega_{s\mp} = \omega_A\frac{1}{\sqrt{1 - \tan^2\theta/2}} \mp i\omega_A\frac{1}{\sqrt{\cot^2\theta/2 - 1}}. \tag{23}$$

where $\omega_A = \frac{k_zc}{\sqrt{1+A}}$ is the Alfven frequency. Eq. (23) indicates that the RH polarized oblique wave damps, and LH polarized oblique wave grows due to obliquity. The real frequency for both the modes increases with angle θ, and the growth (or attenuation) of LH (or RH) waves also increases with angle θ.

From Eq. (21), we can say that both fast and slow cyclotron interactions are possible for both LH and RH polarized oblique Alfven waves, in contrast to only slow (or fast) cyclotron interaction for RH (or LH) polarized parallel propagating Alfven waves. In Fig. 1, the dispersion curves of Alfven waves are plotted for different wave propagation angles $\theta(= 0°, 30°, 45°, 60°, \text{ and } 80°)$, plotted using Eq. (23) along with the beam mode for beam velocity $v_{bo} = 1.69 \times 10^8\,\text{cm/s}$. The frequencies and the corresponding parallel wave numbers of the unstable modes obtained from the points of intersection between the beam mode and plasma modes are given in Table 1. The unstable frequency of Alfven waves 'ω' gradually increases while the unstable wave number 'k_z' decreases with increase in the angle of wave propagation θ.

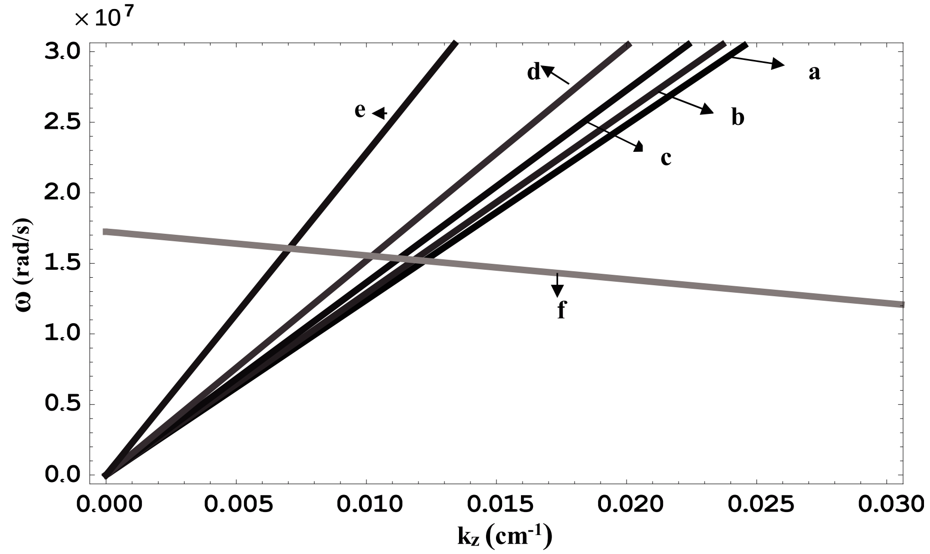

Figure 1. Dispersion curves of oblique shear Alfven wave for different angles of propagation θ, (a) 0°, (b) 30°, (c) 45°, (d) 60° and (e) 80° and the beam mode (f) with velocity $1.69 \times 10^8\,\text{cms}^{-1}$.

Table 1. Angle of propagation, unstable wave numbers and frequencies of oblique shear Alfven waves and maximum growth rates of wave in the absence and presence of beam-plasma collisions.

θ	**0°**	**30°**	**45°**	**60°**	**80°**
$\mathbf{k}_z$ **(cm^{-1})**	0.01223	0.01184	0.01126	0.01021	0.00704
$\boldsymbol{\omega}$ **($\times 10^7$ rads^{-1})**	1.5178	1.5245	1.5342	1.5514	1.6054
$\boldsymbol{\omega/\omega_{bc}}$	0.8801	0.8840	0.8896	0.8996	0.9309
γ **($\times 10^5$ s^{-1})**	0	1.94053	3.08662	4.52084	8.08631
$\boldsymbol{\gamma_{wc}}$ **($\times 10^5$ s^{-1}) $\boldsymbol{\nu = 10^4}$ s^{-1}**	1.71571	1.75076	1.80275	1.89856	2.24340

Assuming fast cyclotron interaction and considering perturbed quantities

$$\omega = \omega_{s\mp} + \Delta \quad \text{and} \quad \bar{\omega} = \omega_{bc} + \Delta, \text{ we get from Eq. (21)}$$

$$\Delta^3 = \frac{-\omega_{pb}^2 \bar{\omega}^2 \nu}{4\omega_s \omega_{bc}^2 (1 + A)} \left(\mp k_x v_{bo} + i\omega_{bc}\right), \tag{24}$$

$$\text{or} \quad \Delta = \left[\frac{\omega_{pb}^2 \nu k v_{bo}}{4\omega_s (1 + A)}\right]^{1/3} \left(\pm \sin\theta/3 + i\cos\theta/3\right). \tag{25}$$

In Eq. (25), a positive imaginary part or the growth rate establishes instability, while a negative imaginary part indicates damping. A positive (or negative) real part means that the frequency of wave increases (or decreases) on interaction between beam mode and wave mode. The frequency of RH polarized mode increases while the frequency of LH polarized mode decreases with the increase in angle of propagation of wave (θ) with respect to ambient magnetic field, and both the modes become unstable due to beam-wave interaction. The growth rate increases monotonically as the angle of propagation increases, but the increase in maximum growth rate in the presence of beam collisions is nominal. The frequency and growth rate of waves depend mainly on the value of $(\frac{\omega+k_z v_{bo}}{k_x v_{bo}})$ [cf. Eq. (24)]. $\omega + k_z v_{bo} = \omega_{bc} = k_x v_{bo}$ corresponds to the case of parallel propagation. The frequency increases/decreases for RH/LH polarized waves by a factor of 0.5 as θ varies from 0° to 90° due to beam-wave interaction, and the maximum growth rate at $\theta = 80°$ becomes 1.3 times of maximum growth rate at $\theta = 30°$.

If the collisions are ignored, Eq. (21) becomes

$$\left(\omega^2 - \omega_{s\mp}^2\right) = \frac{\omega_{pb}^2 \left(\omega + k_z v_{bo}\right) k v_{bo}}{\left[\left(\omega + k_z v_{bo}\right)^2 - \omega_{bc}^2\right]\left(1 + A\right)} \exp\left(\mp i\theta\right) \tag{26}$$

Assuming perturbed quantities for fast cyclotron interaction, we get

$$\Delta = \left[\frac{\omega_{pb}^2 \left(\omega + k_z v_{bo}\right) k v_{bo}}{4\omega_{bc}\omega_s \left(1 + A\right)}\right]^{1/2} \left(\cos\theta/2 \mp i \sin\theta/2\right) \tag{27}$$

Using Eq. (26) we plot, in Fig. 2, the growth rate of oblique shear Alfven wave with beam collision frequency γ_{wc} (in sec^{-1}), and using Eq. (27) we plot, in Fig. 3, the growth rate of oblique shear Alfven wave without collision frequency γ, as a function of k_z, for the same parameters used for plotting dispersion curves. Fig. 2 shows that the maximum growth rate as well as the spectrum of the unstable mode increases with the increase in angle of propagation θ. Collisions have induced a growth in elliptically X-Z polarized wave for parallel propagation.

In the absence of collisions, for angle of propagation $\theta = 0°$, the condition is analogous to the interaction of pure Alfven wave, as E_z does not affect the interaction for parallel propagating waves. The frequency then increases, and growth rate is zero. The beam generates LH polarized waves and stabilizes the RH polarized wave.

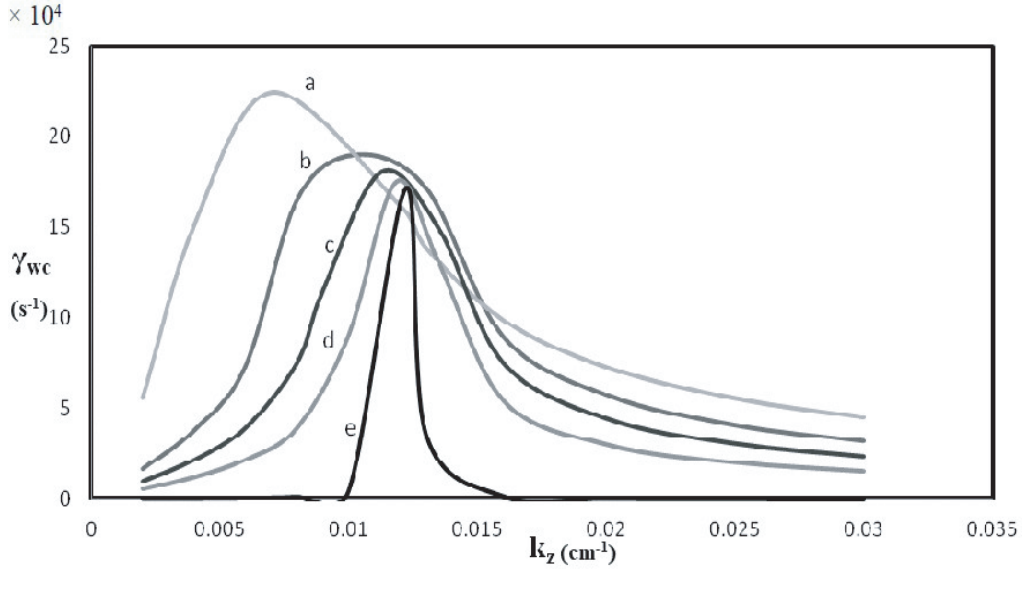

Figure 2. Growth rate γ_{wc} (in sec^{-1}) of the unstable mode as a function of k_z (in cm^{-1}) for different angles, (a) 80°, (b) 60°, (c) 45°, (d) 30° and (e) 0° of propagation of shear Alfven wave in the presence of beam-plasma collisions.

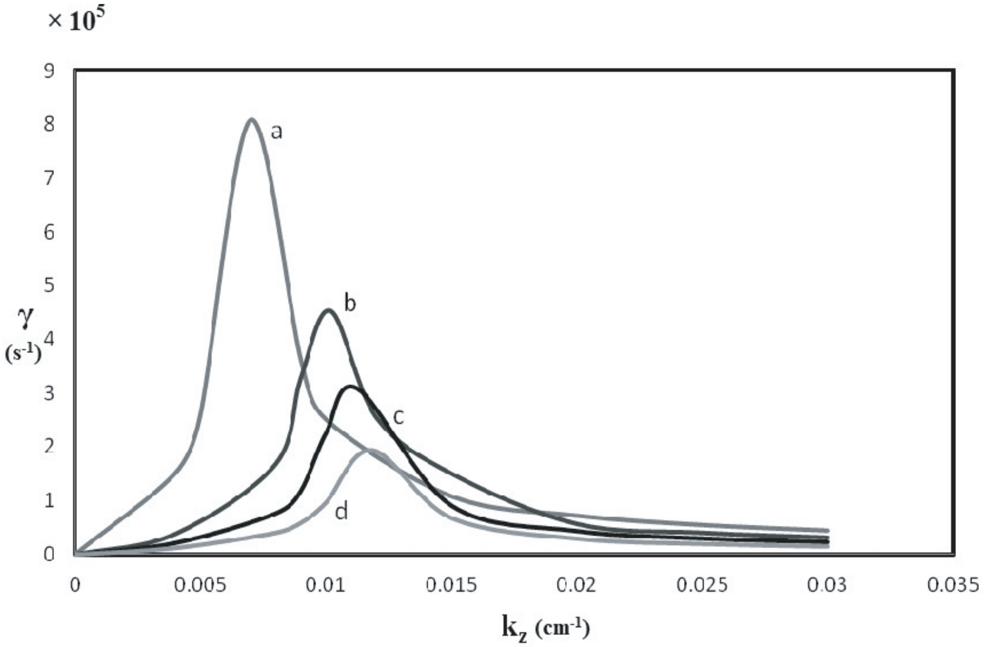

Figure 3. Growth rate γ (in $\sec^{-1}$) of the unstable mode as a function of k_z (in cm^{-1}) for different angles, (a) 80°, (b) 60°, (c) 45° and (d) 30° of propagation of shear Alfven wave.

Equation (27) shows that the maximum growth rate of the unstable LH polarized oblique Alfven waves occurs at more oblique angles. The produced waves heat/accelerate the plasma ions. On the other hand, the RH polarized oblique Alfven wave loses their energy and heat the beam ions. Li and Lu [22] have also observed similar results using hybrid simulations. The maximum growth rate becomes four-fold as θ changes from 30° to 80°, and the unstable wave spectrum decreases with θ. The maximum growth rate of the LH polarized mode excited at nearly perpendicular propagation is

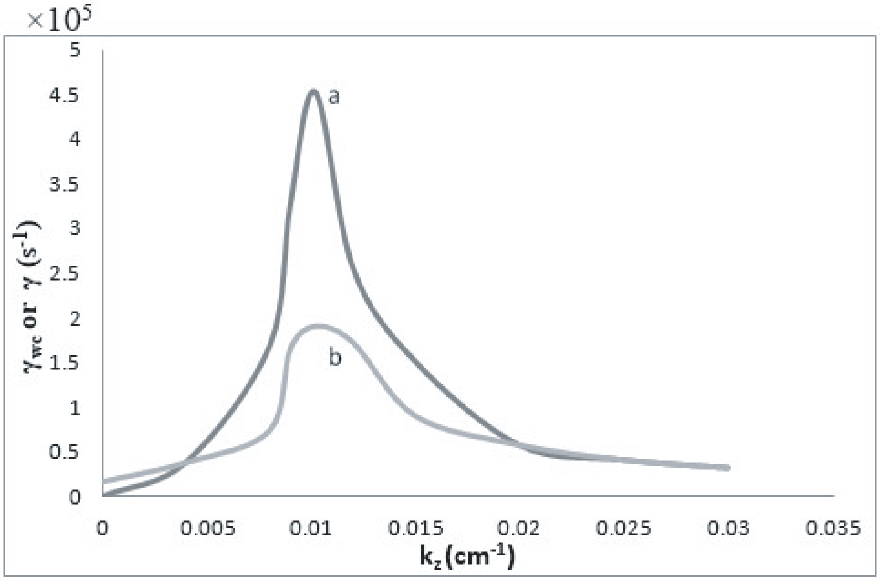

Figure 4. Growth rates, (a) γ and (b) γ_{wc} (in $\sec^{-1}$) of the unstable mode as a function of k_z (in cm^{-1}) for an angle of propagation of shear Alfven wave 60° in the absence and presence of beam-plasma collisions.

$0.05\omega_{bc}$ occurring at 80°. The maximum frequency of the unstable wave mode decreases slightly with an increase in propagation angle. From Table 1, we may conclude that the maximum growth rate values in the presence or absence of beam collisions increase due to obliquity of wave.

Assuming slow cyclotron interaction and considering perturbed quantities $\omega = \omega_{s\mp} + \Delta$ and $\bar{\omega} = -\omega_{bc} + \Delta$, we get from Eq. (26)

$$\Delta = \left[\frac{\omega_{pb}^2 \bar{\omega}^2 \nu k v_{bo}}{4\omega_s \omega_{bc}^2 (1+A)}\right]^{1/3} \left\{\cos(\pi/6 \mp \theta/3) + i \sin(\pi/6 \mp \theta/3)\right\}. \tag{28}$$

The behaviours of RH and LH polarized waves are reciprocal to each other. As the angle of propagation θ increases, the frequency of RH mode (or LH mode) increases (or decreases), while the growth rate of RH mode (or LH mode) decreases (or increases) by small factors.

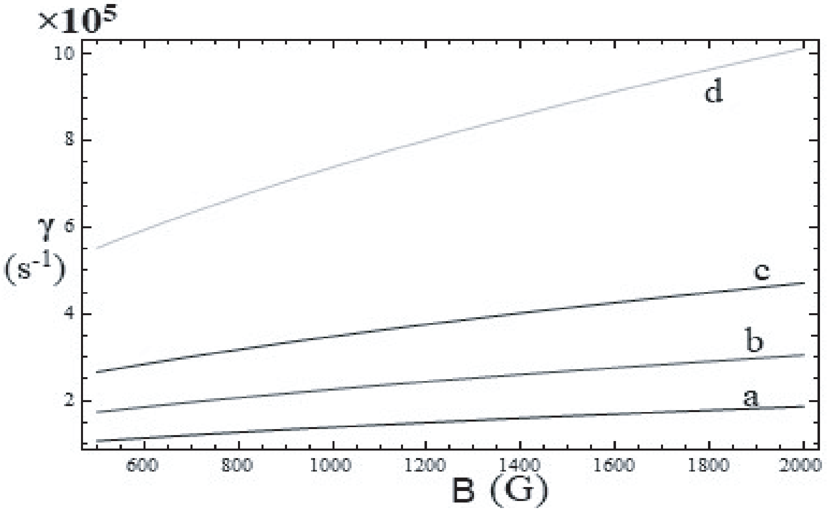

Figure 5. Growth rate γ (in sec^{-1}) of the unstable mode as a function of B (in gauss) for different angles, (a) 30°, (b) 45°, (c) 60° and (d) 80° of propagation of shear Alfven wave.

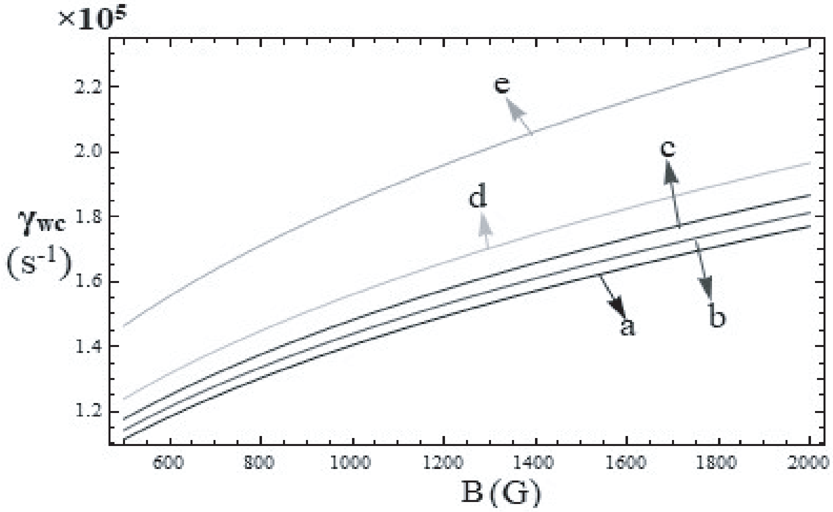

Figure 6. Growth rate γ_{wc} (in sec^{-1}) of the unstable mode as a function of B (in gauss) for different angles, (a) 0°, (b) 30°, (c) 45°, (d) 60° and (e) 80° of propagation of shear Alfven wave in the presence of beam-plasma collisions.

In the absence of collisions, for slow cyclotron interaction, we get

$$\Delta = \left[\frac{\omega_{pb}^2 k v_{bo}}{4\omega_s\,(1+A)}\right]^{1/2} (\pm\sin\theta/2 + i\cos\theta/2)\,. \tag{29}$$

The frequency of RH mode increases, while that of LH mode decreases in this interaction, while both the modes grow with time.

Figure 4 shows the comparison of growth rates of unstable shear Alfven modes in the presence and absence of beam plasma collisions as a function of k_z at wave propagation angle of 60°, which justifies that the growth rate decreases due to collisions. A rise in the value of ambient magnetic field raises the maximum growth rate in both the cases as shown in Fig. 5 and Fig. 6. However, the rate of increase in growth rate is slightly more in the presence of collisions. The variation of growth rate of the unstable mode with the number densities of electrons in the plasma are shown in Fig. 7 and Fig. 8 for different angles of propagation without and with beam plasma collisions. Increase in the number

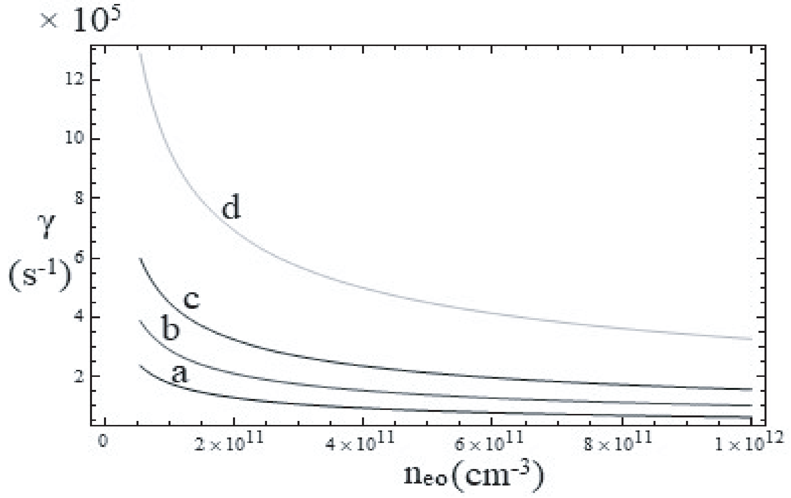

Figure 7. Growth rate γ (in $\sec^{-1}$) of the unstable mode as a function of n_{eo} (in cm^{-3}) for different angles, (a) 30°, (b) 45°, (c) 60° and (d) 80° of propagation of shear Alfven wave.

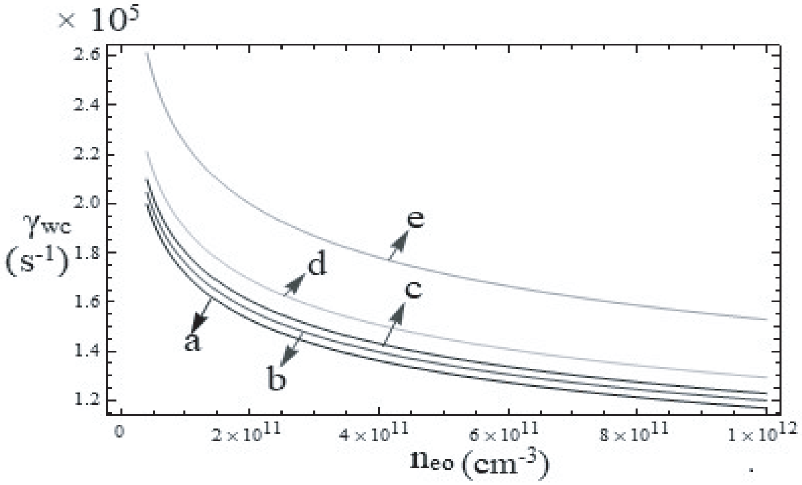

Figure 8. Growth rate γ_{wc} (in $\sec^{-1}$) of the unstable mode as a function of n_{eo} (in cm^{-3}) for different angles, (a) 0°, (b) 30°, (c) 45°, (d) 60° and (e) 80° of propagation of shear Alfven wave in the presence of beam-plasma collisions.

density of electrons decreases the relative density of beam ions in plasma, thus decreasing the growth rate of modes. It can also be seen from the growth rate expressions that an increase in the ion beam velocity increases the growth rate. Similar results for growth rates have also been observed by Xiang et al. [28], and they have also observed a higher growth rate for RH waves than the LH waves.

3. CONCLUSION

In this paper, we show that an ion beam can efficiently transfer its energy to plasma through wave generation. The beam ions are resonant with both the parallel and oblique shear Alfven modes. The frequency of waves may increase or decrease depending upon the polarization of waves. For oblique shear Alfven waves, the waves grow irrespective of their polarization via beam-wave cyclotron interaction, and the beam ions do not show Landau resonance. The numerical values of the growth rate are however much smaller in the presence of collisions, indicating a collisional damping of waves. The growth rates and frequencies of generated waves are also sensitive to the beam properties. Our results indicate that the growth rates of LH and RH Alfven modes vary with the velocity of ion beam, number density of plasma electrons, ambient magnetic field, and collisions in plasma. A stronger magnetic field supports the Alfven wave generation. For $(\omega + k_z v_{bo} > \omega_{bc})$, the most unstable mode is the LH polarized oblique Alfven mode, and for $(\omega + k_z v_{bo} < \omega_{bc})$, it is the RH polarized oblique Alfven mode, indicating a polarization reversal after resonance condition. In this case, the beam-wave interaction could take place only for propagation of Alfven waves and beam ions in opposite direction, with respect to the ambient magnetic field, and only via fast cyclotron interaction. The growth rate is more for parallel shear Alfven waves than oblique shear Alfven waves, making them the dominant modes. The study of oblique modes may find application within the Earth's bow shock as well as to basic plasma processes and are important in the thermodynamics of minor ions in the solar wind.

The collision of beam ions with plasma components affects the growth rate as well as the frequency of generated waves, while the collision between plasma components damps the waves. The effect of beam collisions can be ignored for beam velocities more than $\sim 10^8$ cm/s, but as the velocity of beam decreases, the role of collisions becomes more and more significant, and they decrease the growth rate of Alfven waves. The effect of beam and plasma collisions becomes more significant for heavy ion beams or in a complex multi-component plasma, which will be a subject of our future work.

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