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Adaptive Beamforming Algorithm Based on MVDR for Smart Linear Dipole Array with Known Mutual Coupling

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Abstract—In this paper, minimum variance distortionless response (MVDR) algorithm for adaptive Beamforming is applied to a linear array under known mutual coupling among half wavelength dipole (HWD) antennas. This algorithm will minimize the signals from all interference directions while keeping the desired signal undistorted. The problem of calculating mutual coupling coefficient of the array HWD antennas formed into a matrix has been considered. The obtained results show the effectiveness of the proposed method, in which the optimum weighting of adaptive antenna arrays is accomplished by computing the weight vector that achieves maximum towards the desired signal and nulls towards interferers. Also, performance evaluation of this algorithm in terms of complexity, convergence speed, and amplitude response will be present. It is shown from the simulation results that the performance of the beamforming algorithm considering the mutual coupling effect can be improved by the proposed compensation method. We also simulate the signal-to-interference-plus-noise ratio (SINR) with different input signal-to-interference ratio (SIR). The different results obtained are in good agreement with those of the literature.

1. INTRODUCTION

Recently wireless communication engineers are faced with the challenge of designing highly efficient systems that rely on adaptive intelligent antennas to improve coverage and capacity [1, 2]. Smart antenna plays a major role to improve the performance of wireless communication systems [3]. This antenna uses an advanced signal processor which improves the performance of a wireless communication system with a good choice of formation adaptive beamforming algorithm [1]. Adaptive beamforming is a technique of combining the weighted received signals on an array of HWD antennas to achieve maximum reception in the direction of desired user while signals from other directions are rejected [4]. These antennas are located near each other, and thus the effects of mutual coupling become more significant [5, 6]. This is because when an antenna is radiating, some of the energy in one antenna is coupled into an adjacent antenna [7, 8]. The adaptive beamformer is extremely sensitive to the influence of mutual coupling in arrays [9–13]. An obvious way to overcome the impact of mutual coupling in arrays is good HWD antenna array design. To improve the performance of the beamformer, many adaptive beamforming methods have been proposed for enhancing signals at desired direction while attenuating interference from other directions. In the current paper, we calculate the weights of the MVDR beamforming algorithm that requires the computation of the covariance matrix $\bar{R}$, and the signal received (x) of an adaptive dipole array system. The $\bar{x}$ vector is a function of the mutual coupling matrix $\bar{C}$. The theory proposed to calculate the mutual coupling between HWD antennas is based on basic electromagnetic concepts [14]. The weights obtained by the MVDR beamforming algorithm are used to plot the variation of radiation patterns of linear array of HWD antennas. It is well known that the data

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dependent MVDR beamformer has better resolution and much better interference rejection capability than the conventional data independent beamformers, provided that the steering vector corresponding to the signal of interest is accurately known. The simulation results show that the MVDR beamformer is very sensitive to mutual antenna coupling in antenna arrays. Obtained results include a large number of examples to illustrate important concepts and show how the major algorithms work.

2. THEORETICAL MODEL

2.1. The Covariance Matrix

The fundamental proposed smart antenna is a dipole array with smart signal processing algorithms used to identify the direction of arrival (DOA) of the signal and use them to calculate beamforming vectors. The general methodology of beamforming is shown in Figure 1, which consists of an array of HWD antennas, complex weights, sum, and a signal processing unit. A beamformer is a collection of antennas which are linearly arranged so that their output can be steered electronically. The received signal at these antennas is used to compute the complex weights which are adaptively updated based on signal samples [6]. The first part of the presented design is begun by calculating the covariance matrix. Supposing that the L uncorrelated narrow band sources emitting plane wave signals $S_L(t)$ come at a certain angle θ, the wave will be received by all elements but at different times. In ideal situations, the steering vector is assumed to be exactly known which depends on the array geometry and signal location. In practice, interactions between the HWD antennas will result in mutual coupling, which distorts the ideal steering vector significantly. In this situation, the true steering vector should be modified, then the received signal $x_M(t)$ is give by [6]:

$$x_M(t) = \sum_{i=1}^{L} c(\theta_i) \cdot e^{-j2\pi \frac{d\cdot(M-1)\cdot\sin(\theta_i)}{\lambda}} \cdot s_i(t) + b_M(t) \tag{1}$$

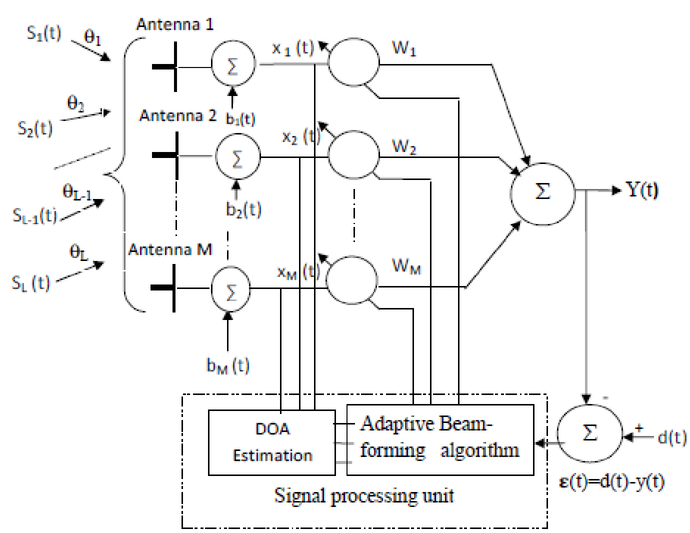

Figure 1. Structure of an adaptive dipole array system.

We can write formula (1) in the following form:

$$\bar{x}(t) = \bar{C}(\theta) \cdot \bar{A}(\theta) \cdot \bar{S}(t) + \bar{b}(t) \tag{2}$$

In Equation (2), $\bar{b}(t)$ is the additive noise vector with component of variance σ^2, and $\bar{S}(t)$ is the original signal from the L sources. $\bar{C}(\theta)$ denotes the mutual coupling matrix (MCM) composed of the mutual coupling coefficients. $\bar{A}(\theta)$ denotes the steering vector for the Lth signal with the M antennas given by:

$$\bar{x}(t) = [x_1(t), \ldots, x_M(t)]^T \tag{3}$$

$$\bar{b}(t) = [b_1(t), \ldots, b_M(t)]^T \tag{4}$$

$$\bar{S}(t) = [s_1(t), \ldots, s_L(t)]^T \tag{5}$$

$$\bar{A}(\theta) = [\bar{a}(\theta_1), \bar{a}(\theta_2), \ldots, \bar{a}(\theta_L)]^T \tag{6}$$

The steering vector $\bar{a}(\theta_i)$ is given by:

$$\bar{a}(\theta_i) = \left[1, e^{-j2\pi\frac{d\sin(\theta_i)}{\lambda}}, \ldots, e^{-j2\pi\frac{d(M-1)\sin(\theta_i)}{\lambda}}\right] \tag{7}$$

When an incoming wave, carrying a baseband signal $s(t)$, impinges at an incident angle θ_i on the antenna array, the steering matrix $\bar{A}(\theta)$ of the linear array to the source arriving from direction θ_i, $(i = 1 \ldots L)$ is obtained as:

$$\bar{A}(\theta) = \begin{bmatrix} 1 & 1 & & 1 \\ e^{-j2\pi\frac{d\sin(\theta_1)}{\lambda}} & e^{-j2\pi\frac{d\sin(\theta_2)}{\lambda}} & & e^{-j2\pi\frac{d\sin(\theta_L)}{\lambda}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j2\pi\frac{d(M-1)\sin(\theta_1)}{\lambda}} & e^{-j2\pi\frac{d(M-1)\sin(\theta_2)}{\lambda}} & \cdots & e^{-j2\pi\frac{d(M-1)\sin(\theta_L)}{\lambda}} \end{bmatrix}_{M\times L} \tag{8}$$

where the dimension of $\bar{A}(\theta)$ is L by M matrix, with columns corresponding to M directional antennas.

The array of HWD antennas is viewed as a multiport network, and it is shown that the coupling matrix can be simply related to the generalized impedance matrix of the multiport network. In order to calculate the mutual coupling matrix $\bar{C}$, we consider that the interactions between the HWD antennas of the array will result in mutual coupling, and the array is composed of M coupled HWD antennas, which we classically model as a M port network. $\bar{C}$ can be written as [6, 15]:

$$\bar{C} = (\bar{Z}_A + \bar{Z}_T) \cdot \left(\bar{Z}_{ij} + \bar{Z}_T \bar{I}\right)^{-1} \tag{9}$$

where

$\bar{Z}_A$: are defined as the antenna impedance of isolated antennas.

$\bar{Z}_T$: antenna termination impedance.

$\bar{Z}_{ij}$: is called the mutual impedance between the ith and jth HWD antennas.

We can rewrite $\bar{C}$ as follows

$$\bar{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1M} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{2M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{M1} & c_{M2} & c_{M3} & \cdots & c_{MM} \end{bmatrix} \tag{10}$$

According to Equation (2), the coupling matrix only affects the signal and not the noise. If the coupling is known, it is relatively easy to compensate for mutual coupling. Different methods of compensation algorithms can be applied, such as the open-circuit voltage method [8], S-parameter method [9], full-wave electromagnetic method of moments [16], calibration method [17], and finally receiving mutual impedance methods. Additionally, the value of the coupling is approximately the same along the diagonals, and thus it can be modeled by using only one parameter for each subdiagonal resulting in a coupling matrix of Toeplitz structure matrix [17]. Also, note that the coupling from element q to m is the same as the coupling from m to q which results in a symmetric Toeplitz matrix

($c_{i,j} = c_{1+|i-j|}$, where $c_1 = 1$). Based on these observations, a compensated coupling model can be formulated as:

$$\bar{C} = \begin{bmatrix} 1 & c_2 & c_3 & \dots & c_M \\ c_2 & 1 & c_2 & \dots & c_{M-1} \\ c_3 & c_2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & c_2 \\ c_M & c_{M-1} & \vdots & c_2 & 1 \end{bmatrix} \tag{11}$$

As shown in Figure 1, the array weights $\bar{w}(n)$ are iteratively computed based on array output $\bar{y}(n)$, and a reference signal $\bar{d}(n)$ approximates the desired signal and previous weights. The error $\bar{e}(n)$ is used to calculate new weights and is given by

$$\bar{e}(n) = \left[\bar{d}(n) - \bar{y}(n)\right] \tag{12}$$

The minimum mean square error (MMSE) criterion is used to find the required properties of the filter weight values which minimize the squared error:

$$E[\bar{e}^2(n)] = E\left[\bar{d}^2(n)\right] - 2 \cdot \bar{w}^H \cdot E\left[\bar{d}(n) \cdot \bar{x}(n)\right] + \bar{w}^H \cdot E\left[\bar{x}(n) \cdot \bar{x}^H(n)\right] \cdot \bar{w} \tag{13}$$

Similarly

$$E[\bar{e}^2(n)] = E\left[\bar{d}^2(n)\right] - 2 \cdot \bar{w}^H \cdot E\left[\bar{d}(n) \cdot \bar{x}(n)\right] + \bar{w}^H \cdot \bar{R}_{xx} \cdot \bar{w} \tag{14}$$

Therefore,

$$E[\bar{e}^2(n)] = E\left[\bar{d}^2(n)\right] - 2 \cdot \bar{w}^H \cdot \bar{R}_{d\cdot x} + \bar{w}^T \cdot \bar{R}_{xx} \cdot \bar{w}] \tag{15}$$

In this relation, $\bar{R}_{d\cdot x}$ and $\bar{R}_{x\cdot x}$ are defined by:

$$\bar{R}_{xx} = E\left[\bar{x}(n) \cdot \bar{x}^T(n)\right] \tag{16}$$

$$\bar{R}_{d\cdot x} = E\left[\bar{d}(n) \cdot \bar{x}^T(n)\right] \tag{17}$$

where

$\bar{R}_{x\cdot x}$: is the input autocorrelation matrix.

$\bar{R}_{d\cdot x}$: is the input to output cross correlation vector.

The gradient of the mean squared error is

$$\bar{\nabla} \cdot E\left[\bar{e}^2(n)\right] = \frac{\partial E\left[\bar{e}^2(n)\right]}{\partial \bar{w}} = -2 \cdot \bar{R}_{d\cdot x} + 2 \cdot \bar{w}^T \cdot \bar{R}_{xx} \tag{18}$$

In order to reach the optimal weight value, the algorithm will have to update the gradient every cycle and step in the gradient's direction. If the gradient of the weight vector $\bar{w}$ is zero, the MMSE is at its minimum. This leads to:

$$-2 \cdot \bar{R}_{d\cdot x} + 2\bar{R}_{xx} \cdot \bar{w}_{opt} = 0 \tag{19}$$

The optimal weight vector $\bar{w}_{opt}$ can be found by determining where the gradient is zero. In this case we have,

$$\bar{w}_{opt} = \bar{R}_{xx}^{-1} \cdot \bar{R}_{d\cdot x} \tag{20}$$

The gradient of the cost function can be approximated by the steepest descent method. Although the gradient descent method makes it possible to iteratively obtain solutions which can reduce the computation time, the problem of the ignorance of the statistics remains an obstacle to the application of this method. When the necessary information is not available or inaccurate, it is not possible to design an optimal system. When the auto and inter-correlation functions are not known, we will then approach the optimal system by using a feedback loop and a minimization algorithm: this is called the adaptive system. Adaptive systems have many advantages that overcome these difficulties and achieve good performance in real applications [18]. We will present, in what follows, the MVDR adaptive algorithm which is the most representative of beamforming algorithms [19–21].

2.2. MVDR Beamforming

Capon method is an adaptive beamforming technique which is developed in order to reduce the negative aspects of conventional beamforming. This method overcomes the low-resolution problem associated with the Bartlett method. In order to overcome the limitations of Bartlett's method such as its inability of resolution of two closely placed sources, the method of Capon which is also called the minimum variance distortionless response method (MVDR) was proposed by Capon in 1967 [22]. It makes the outputting power with minimum interference and noise in the desired direction through adjusting a weight factor, while ensuring the output desired signal without minimum distortion. The beamformer output is given by [23].

$$\bar{y}(t) = \bar{w}^H \cdot \left[\bar{x}_d(t) + \bar{x}_I(t) + \bar{b}(t)\right] \tag{21}$$

$$\bar{n}(t) = \bar{x}_I(t) + \bar{b}(t) \tag{22}$$

$$\bar{y}(t) = \bar{w}^H \cdot \bar{x}_d(t) + \bar{w}^H \cdot \bar{n}(t) \tag{23}$$

The output of the beamformer is given by

$$\bar{y}(t) = \bar{w}^H \cdot \bar{s}(t) \cdot \bar{a}_d + \bar{w}^H \cdot \bar{n}(t) \tag{24}$$

The principle of MVDR formatter is to find the weighting vector $\bar{w}(k)$ which minimizes the overall output power of the formatter while maintaining unity gain in the direction of desired (θ_d). This minimization can be solved using the method of Lagrange multipliers. Using the method of Lagrange multipliers, we can transform the constrained optimization above into an unconstrained one. The Lagrangian function is given by

$$h\left(\bar{w}, \bar{\lambda}\right) = \bar{w}^H \cdot \bar{R}_a \cdot \bar{w} + \bar{\lambda} \cdot \left(1 - \bar{w}^H \cdot \bar{a}_d\right) \tag{25}$$

The power at the output signal of the beamformer can be calculated using:

$$\bar{R}_a = E\left(\bar{y}^2(t)\right) = \bar{w}^H \cdot \bar{R} \cdot \bar{w} \tag{26}$$

where $\bar{R} = E\left(\bar{x}(i) \cdot \bar{x}^H(i)\right)$ is the covariance matrix of $\bar{x}(i)$.

Equating the terms to zero and solving for $\bar{w}$, we obtain

$$\frac{\partial h\left(\bar{w}, \bar{\lambda}\right)}{\partial \bar{w}^*} = 0 \tag{27}$$

We find the following expression:

$$\bar{w} = \bar{\lambda} \cdot \bar{R}_a^{-1} \cdot \bar{a}_d \tag{28}$$

For a distortion-free response, one must add the constraint that the derivative of the Lagrangian function with respect to λ equals zero.

$$\frac{\partial h\left(\bar{w}, \bar{\lambda}\right)}{\partial \bar{\lambda}} = 0 \tag{29}$$

We find the following expression:

$$1 - \bar{w}^H \cdot \bar{a}_d = 0 \tag{30}$$

Using Equation (28), we find:

$$1 - (\bar{\lambda} \cdot R_a^{-1} \cdot \bar{a}_d)^H \cdot \bar{a}_d = 0 \tag{31}$$

we have $(\bar{R}_a^{-1})^H = (\bar{R}_a^{-1})$ which has Hermitian symmetry, and the expression for $\bar{\lambda}^*$ is given by:

$$\bar{\lambda}^* = \frac{1}{\bar{a}_d^H \cdot \bar{R}_a^{-1} \cdot \bar{a}_d} \tag{32}$$

Using Equations (28) and (32) leads to the following MVDR weighting [24, 25].

$$\bar{w} = \frac{\bar{R}_a^{-1} \cdot \bar{a}_d}{\bar{a}_d^H \cdot R_a^{-1} \cdot \bar{a}_d} \tag{33}$$

The total far field radiation patterns of linear array of HWD antennas is the product of the element factor $E(\theta)$ and array factor (AF) [6, 26, 27]. AF depends on the geometric arrangement of the array elements, the spacing of the HWD antennas (d), and the weights $\bar{w}$ obtained by the MVDR algorithm.

In order to determine the efficiency of the adaptive beamforming technique, it is necessary to calculate the SINR (signal-to-interference-plus-noise ratio). The objective of the signal enhancement of adaptive beamforming technique is to make the output SINR greater than the input SNR (signal-to-noise ratio). Consequently, the quality of the adaptive beamforming technique output signal may be enhanced compared to the noisy signal. The SINR is defined as the ratio of desired signal power divided by the undesired signal power [28]:

$$SINR = \frac{E\left(\left|\bar{y}_d(t)\right|^2\right)}{E\left(\left|\bar{y}_n(t)\right|^2\right)} = \frac{E\left(\left|\bar{w}^H \cdot \bar{x}_d\left(t\right)\right|^2\right)}{E\left(\left|\bar{w}^H \cdot \left(\bar{x}_I\left(t\right) + \bar{b}(t)\right)\right|^2\right)} = \frac{\bar{w}^H \cdot \bar{R}_d \cdot \bar{w}}{\bar{w}^H \cdot \bar{R}_n \cdot \bar{w}} \tag{34}$$

3. NUMERICAL RESULTS

In order to observe the effect of HWD antennas separation d on mutual coupling of a linear array, we have simulated, in Figure 2, the real and imaginary parts of the mutual coupling between two HWD antennas as a function of the elements separation d. As the distance between two elements increases, the magnitude of mutual coupling impedance diminishes and approaches zero.

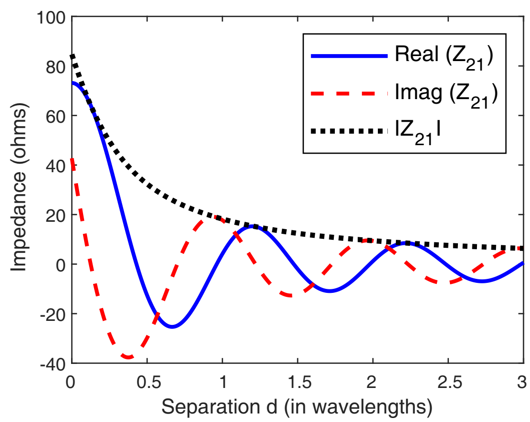

Figure 2. Real, imaginary and magnitude parts of mutual coupling impedance versus elements separation d the two HWD antennas.

In this section, the performance of the proposed algorithm is evaluated by the numerical simulations in terms of array pattern. In these simulations, a linear array of HWD antennas is considered. The performance of MVDR algorithm has been developed with the MATLAB language. The noise is complex Gaussian with mean zero. The optimum parameters of excitation obtained by formula (33) are used to plot the variation of radiation patterns (RP) of linear array of HWD antennas, shown in Figures 3–6. The magnitude of the initial pattern is determined by the initial choice of weights, whereas the magnitude of the final pattern is determined by the strength of the desired signal, direction, strength of the interfering signal, and the noise in the system. To estimate the correlation matrices, we use 200 snapshots.

In Figure 3, the effects of distance among 16 HWD antennas on the RP of linear array are determined as a function of angle θ. When $d = 0.5\lambda$, the results clearly show that the zeros are oriented towards the interferer directions at 60° and −60° and the main beam towards the desired user direction at 30°. In case the distance between HWD antenna array elements is less than ($\lambda/2$), the MVDR algorithm is not able to detect the wanted signal at $\theta = 30°$. This condition means that the HWD antennas should not be spaced less than half a wavelength apart. However, the closer they are,

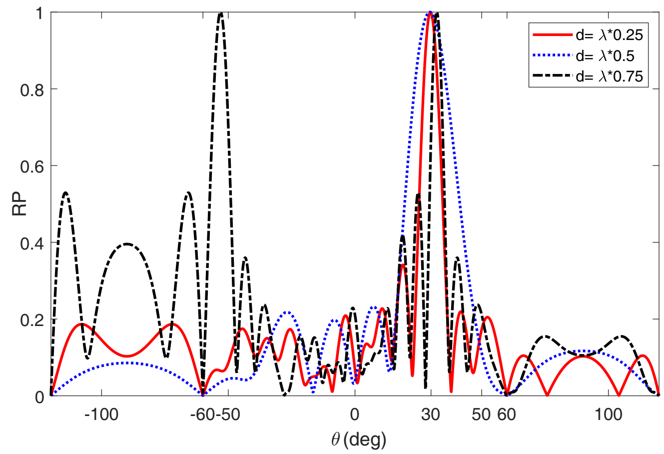

Figure 3. Radiation pattern versus θ for different values of d.

the greater the mutual couplings are in the near field between HWD antennas. When the distance ($d = 0.75\lambda$) and the sidelobes level (SLL) reach the level of the main lobe, significant parasitic lobes appear.

As displayed in Figure 4, the variation of RP of linear array of ($M = 4$, $M = 8$, and $M = 16$) HWD antennas separated by a distance is equal to a half wavelength. The proper choice of this value avoids the mutual coupling effects between half-wavelength dipoles. The results clearly show that the zeros are oriented towards the interferer directions at 60° and −15° and the main beam towards the desired user direction at 30°. We also observe that the increase in the number of antennas leads to an improvement in the beamwidth, reduction of sidelobes level (SLL) and number of secondary lobes of factor array. Also, when increasing the number of antennas, there is an improvement in the direction of arrival estimation. We also observe that when $M = 4$, the resulting signal is not able to detect the useful signal at $\theta = 30°$. The program converges after 30 iterations, and this number is found to be sufficient to obtain satisfactory patterns with a desired performance. The results obtained by this approach are satisfactory and show the interest of the application of the MVDR algorithm in optimizing the performance of HWD antenna arrays.

Figure 5 shows the simulation result for the case of four sources $\theta_{I1} = -60°$, $\theta_{I2} = -15°$, $\theta_{I3} = -75°$, $\theta_d = 30°$. In this simulation, we will fix the inter-element distance ($d = 0.5\lambda$) and the number of HWD antennas ($M = 16$). To plot the variation of radiation pattern, we use formula (33). We notice that the MVDR algorithm is able to detect the useful signal at 30 degrees and good results in the direction of the interfering sources.

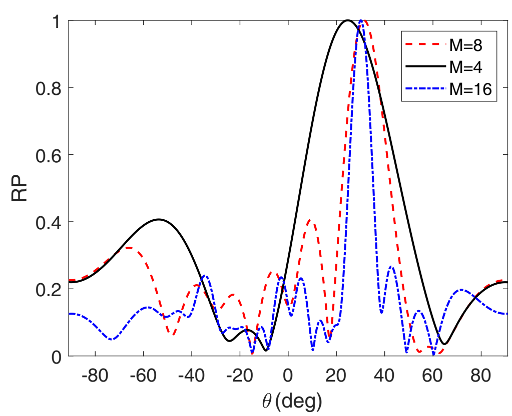

Figure 4. Radiation pattern versus θ for different values of M.

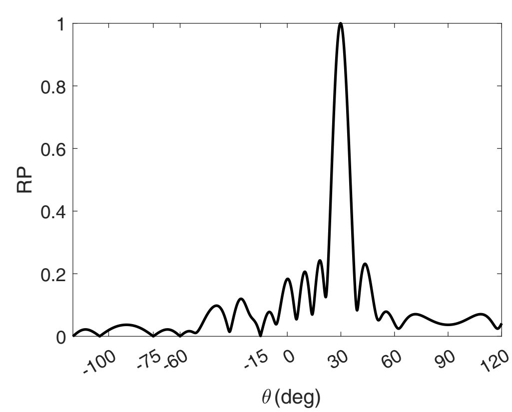

Figure 5. Radiation pattern versus θ for four sources.

In order to examine the performance of MVDR method, some simulations are performed for three cases; array with coupling (conventional algorithm), array without mutual coupling, and array with coupling (compensated algorithm). A linear array with 16 HWD antennas is considered here. According to the results found in Figure 6, we can conclude that the known mutual coupling between antenna elements may seriously distort the performance of MVDR beamforming algorithm. One way of attenuating these effects is using the compensated algorithm. The MVDR method gives one clearly higher peaks close to the true DOAs for array with coupling using the compensated algorithm.

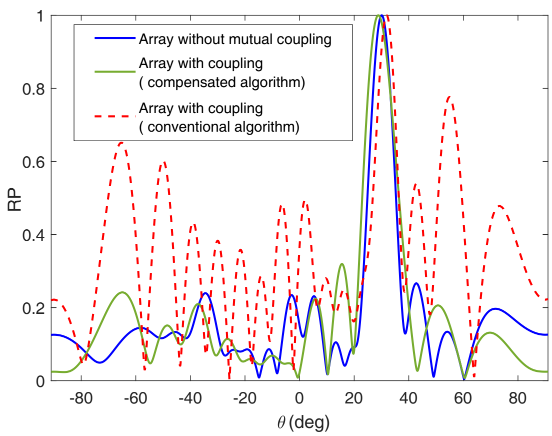

Figure 6. Radiation pattern versus θ for three cases; array with coupling, array without mutual coupling and array with compensated algorithm.

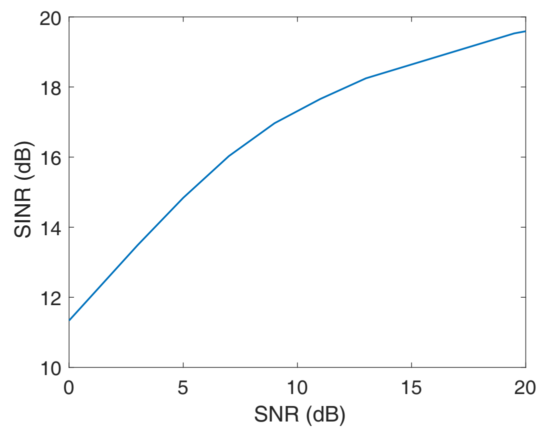

Figure 7. SINR versus SNR with MVDR algorithm.

In Figure 7, we present the results obtained using formula (34). To simplify analytical process, assuming that interference to noise ratio (INR) is equal to 20 dB, the input SNR is equal to 0 dB to 20 dB, respectively. It is clear that the SINR is higher at higher SNR.

Figure 8 shows the variation of error between reference signal and array output (MSE), we can see that the MVDR algorithm has very fast convergence rate compared to least mean squares (LMS) algorithm. According to Figures 3–6, we notice that the side lobe level (SLL) is high for MVDR beamforming which is effective as long as interfering signals arrive at angles away from the angle of arrival of the useful signal.

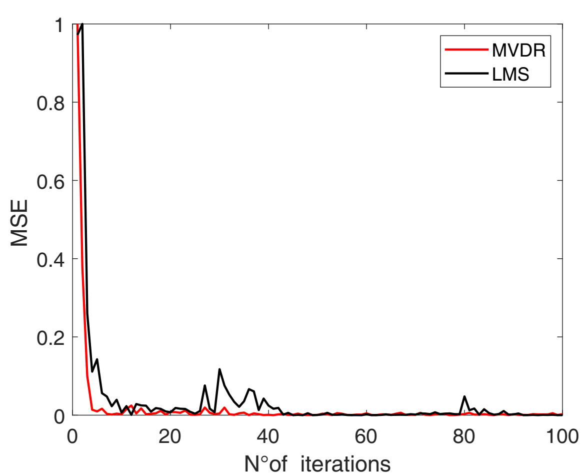

Figure 8. Mean square error (MSE) versus the number of iterations.

4. CONCLUSIONS

In this work, we have considered the MVDR adaptive beamforming algorithm for an adaptive HWD antenna array. This algorithm serves to direct the main beam in the direction of the desired signal and place nulls in the direction of interfering signals. The optimum parameters of excitation obtained by MVDR algorithm are used to plot the variation of radiation patterns of linear array of HWD antennas. It is seen that the MVDR algorithm has good performance except that it has a higher SLL and lower null depths than least mean squares algorithm. It is shown that $(\lambda/2)$ is the optimum spacing between the antenna array elements. The known mutual coupling between HWD antennas may seriously distorts the array output signal and degrades the performance of beamforming using MVDR algorithm. One way of attenuating these effects is to using the compensated algorithm. The outcomes given by SINR as a function of SNR are fulfilling and clear which makes us infer that the MVDR algorithm introduced in this study suggests an efficient algorithm, precise and dependable in cellular network. In addition, this algorithm is more appropriate for massive MIMO systems due to its good precision and lack of a requirement for complicated mathematical functions.

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