# Period Doubling Analysis on the SBD in Microwave Transmission Line

# Yong Xia<sup>1, \*</sup> and Xiaowei Shi<sup>2</sup>

Abstract—A simple electromagnetic system composed by a Schottky barrier diode (SBD) and linear lossless microstrip line is introduced in this paper. The period doubling phenomenon in this circuit system is studied following the train of thought, which is adopted in the field of the driven Resistor-Inductor-Diode (RLD) series circuit. The related phenomenon and its origins have been studied extensively in RLD circuit. However, it will not be able to exhibit period doubling and chaos if SBD is applied in RLD circuit. We show period doubling phenomena in a microwave circuit containing SBD for the first time. This paper reveals that the given circuit can produce cycle period doubling at microwave frequency by means of simulations and experiments. Physical verification and theoretical explanation have been given in this paper.

## 1. INTRODUCTION

With the development of nonlinear theory people have a new understanding of many complex phenomena in nature. Whether a simple circuit can produce period doubling and chaos is one of the most challenging problems in the field of chaos [1–3]. The driven RLD circuit is studied widely because of its typicality [4– 6]. With further research in nonlinear systems, many types of diodes, such as ordinary diode, SiC, and PIN diode, have been put to use, but SBD has never been adopted [7,8]. It is well known that the working mechanism of SBD is majority carrier translation motion, which is different from other types of diodes. So, it has been pointed out that SBD will not exhibit period doubling phenomenon [9].

Because the diode is chosen as the core component of this circuit, the origins of nonlinear phenomena are studied by changing various diodes [10–12]. Previous studies have shown that nonlinear capacitance and reverse recovery time are the main necessary conditions for producing chaos and period doubling. There have been different schools of thought debating which is the main reason, but some viewpoints consider that the two views are essentially the same [4].

At present, the research on RLD circuit mainly focuses on low frequency. Controlled GHz-scale chaos and period doubling are hot topics.

Transmission lines can effectively transfer information or energy from one place to another. The research on the time-delayed system based on transmission lines has attracted many scholars' attention. Some simple electromagnetic systems connecting diode and negative resistance through transmission line are also studied [13, 14]. This paper presents a transmission line connected to SBD at one end and power source at the other end. It is found that the period doubling phenomenon can emerge in microwave circuit due to the combination of microstrip line and SBD at GHz frequencies. The measured results agree with the simulated ones essentially.

Received 27 June 2022, Accepted 31 August 2022, Scheduled 14 September 2022

<sup>\*</sup> Corresponding author: Yong Xia (yxia@stu.xidian.edu.cn).

<sup>&</sup>lt;sup>1</sup> School of Advanced Materials and Nanotechnology, Xidian University, China. <sup>2</sup> School of Electronic Engineering, Xidian University, China.

#### 2. PREVIOUS RESEARCH STATUS

As everyone knows, the p-n junction contained in a Si crystal is the basic nonlinear element. Because the p-n junction is a nonlinear charge-storage element, it should be noted that nonlinear capacitance and reverse recovery time are essential ingredients for the behavior of diodes.

What causes the period doubling and chaotic behavior is the key question. The driven RLD circuit containing a p-n junction is a very simple chaotic circuit, which has been widely investigated. A lot of research work has been done for observing the chaotic phenomena and explaining the reason.

The RLD circuit diagram is shown in Fig. 1. In the process of the research on RLD circuit, there are two contrasting views on the reason for the phenomenon of period doubling and chaos. The two schools show different models. One view is that the nonlinearity of the reverse-recovery time can explain the phenomenon completely, and the diode model is shown in Fig. 2(a). Another view is that a piece-wise linear capacitance model is the main cause, and the diode model is shown in Fig. 2(b) [11].

It has been noted that the frequency and amplitude of signal source determine the process of causing chaos. Resistance element in the circuit affects the performance of the circuit.

Many researchers believe that relevant phenomena are attributed to the reverse-recovery time. When the diode state is switched, the minority of carriers do not have enough time to synchronize with the input signal. When the mode of diode is reversed, the minority of carriers cannot have enough time to recombine. As a result, the history effects will lead to period doubling and chaos.

Not only do the researchers have a one-sided understanding of the problem, but also the forms of circuit are the same in essence. There is no point in making such a distinction.

It should be pointed out that the period doubling and chaotic phenomenon can be observed from many microwave circuits including nonlinear terminating elements, such as tunnel diode and Chua's diode [15, 16].

The electronic system composed of diode and negative resistance is shown in Fig. 3. It is important to understand that the negative resistance is a key effect on producing chaos, while the function of the transmission line is more considered from the view of delay effect.

The next section will propose this kind of mechanism combining with the microstrip line and SBD.



Figure 1. The driven RLD series circuit.



Figure 2. (a) The model basing on history effects. (b) The nonlinear capacitance and nonlinear resistance model.





Figure 3. Schematic of a transmission line with nonlinear oscillator.

Figure 4. The interaction with transmission line and SBD.

## 3. A NEW PERIOD DOUBLING CIRCUIT

The ideal transmission lines are the simplest transmission lines, whose the characteristics are interconnections without losses [17, 18]. In this paper, we shall analyze the interaction with transmission line and SBD represented in Fig. 4.

We simulate the circuit in Fig. 4 with Advanced Design System (ADS) software. Here f0 = 1 GHzand pw = 10 dbm. The selected SBD is HSMS286B. The period doubling phenomenon will appear by adjusting the TLIN of TL1 and TL2, and the test results are shown in the Fig. 5–Fig. 8. If we change the input parameter, a similar result can be obtained. The fact is that period doubling can be realized on SBD by changing the length of microstrip line. Here, adjusting the length of transmission line is the same as adjusting the reactance value in an RLD circuit [7].



Figure 5. Single period TL2 = 38 mm.



Figure 7. Period tripling TL2 = 72 mm.



Figure 6. Period doubling TL2 = 60 mm.



Figure 8. Period quadrupling TL2 = 88 mm.

There is no doubt that the joint action of transmission line and SBD exhibits the phenomenon. With the change of the length of the transmission line, the waveform will change periodically. The periodic change of simulation results is shown in Fig. 9. The rule of change is 1-2-3-4-3-2-1-2-3 ....

The physical test of material object is shown in Fig. 10. There is a set of coupled strip lines for every channel, and there is not a ground plane underneath the substrate.

The test channels in the material object are designed according to the length of the transmission line. The length of channel 1 is the shortest, 3 mm. The length of channel 12 is the longest, 155.6 mm. According to the original idea, the length of transmission line is based on the simulation results.



Figure 9. The change rule of period.



Figure 10. The material object.



Figure 11. The spectrum of test point in channel 2.

The test is done with a spectrum analyzer. The test spectrum of voltage on resistance (VR) point is shown in Fig. 11. The input signal is a standard sine wave. Here, pw = 10 dbm and f0 = 1 GHz. We will select a few typical channels, and the test results are listed as follows:

The channel 2: 1G = -13.6 dbm, 2G = -22 dbm, 3G = -29 dbm, 4G = -36 dbm; The channel 4: 1G = 1.5 dbm, 2G = -31 dbm, 3G = -38 dbm; The channel 8: 1G = -20 dbm, 2G = -23 dbm, 3G = -27 dbm; The channel 10: 1G = -11 dbm, 2G = -25 dbm, 3G = -30 dbm, 4G = -35 dbm; The channel 11: 1G = -0.33 dbm, 2G = -35 dbm.

#### Progress In Electromagnetics Research Letters, Vol. 106, 2022

According to the test results, we will draw the corresponding waveform in time domain in Figs. 12–16.

Because the selection of transmission line length is based on the simulation results, there is a big gap between the test and simulation, but the waveform has the ability to explain the basic problem. A period change is easy to observe from the result of physical testing, which is similar to simulation result. Period doubling and period tripling occur. In the next section, we will analyze and explain the phenomenon in theory.



Figure 12. The waveform of channel 2.



Figure 14. The waveform of channel 8.



Figure 13. The waveform of channel 4.



Figure 15. The waveform of channel 10.

### 4. PHYSICAL CHARACTERISTICS ANALYSIS

Based on the simulated and measured results of the previous section, this section analyzes the circuit given in this paper further.

Firstly, we will analyze the characteristic of transmission line. The model of the ideal transmission line is shown in Fig. 17.

The equations describing the voltage and current along the transmission line are

$$-\frac{\partial v}{\partial x} = L\frac{\partial i}{\partial t}, \quad -\frac{\partial i}{\partial x} = C\frac{\partial v}{\partial t}, \quad 0 < x < d, \quad t \ge 0$$
(1)

where L and C are, respectively, the inductance and capacitance per-unit-length of the transmission line.

Xia and Shi



Figure 16. The waveform of channel 11.



**Figure 17.** Sketch of a lossless translation line.

In the analysis of transmission lines, two-port networks are used to model the transmission lines with lumped circuits, as shown in Fig. 18.

Figure 18. Two ports model.

 $\overset{I}{\overset{}_{+}}$  $v_1$ 

Twp-port TL

Figure 19. The simplified model of the transmission line.

The relationship between voltage and current at both ends of the line may be expressed as follows:

$$v_1(t) = v(x = 0, t), \quad i_1(t) = i(x = 0, t)$$
(2)

$$v_2(t) = v(x = d, t), \quad i_2(t) = -i(x = d, t)$$
(3)

We use the general solution of the line Equation (1):

$$v(x,t) = v^{+}(t - x/c + T) + v^{-}(t + x/c)$$
(4)

$$i(x,t) = \frac{1}{Rc} \left[ v^+ \left( t - x/c + T \right) - v^- \left( t + x/c \right) \right]$$

$$T = d/c$$
(5)

In the case of removing the SBD in Fig. 4, we will measure the spectrum of endpoint when the endpoint is open. The test results are listed as follows:

The channel 8: 
$$1G = -20 \text{ dbm}$$
, The channel 11:  $1G = 0 \text{ dbm}$ 

So, we will get different values with different transmission line lengths. This shows that when the terminal is open, the amplitude of the signal varies greatly due to the different lengths of the transmission line. We get the difference in length between channel 8 and channel 11 close to 60 mm, which is close to  $\lambda/4$ . According to the characteristics of the transmission line, the amplitude of the main frequency will change from the maximum value to the minimum value after  $\lambda/4$  wavelength. It should be noted that the scene in this paper is very similar to this theorem.

#### Progress In Electromagnetics Research Letters, Vol. 106, 2022

Next, we will analyze the phenomenon from the perspective of SBD. When the SBD is replaced with ordinary diodes, capacitors and other nonlinear components, respectively, there is no period doubling phenomenon. So, different boundary conditions produce different nonlinear reflections and results. We study diode and SBD from the view of physical structure and equivalent circuit. For different carrier translation motions, in the case of forward bias the model of SBD is shown in Fig. 20.

SBD differs from ordinary diode not only in the magnitude of I, but also in the switching frequency. The switching frequency of SBD is much higher than that of ordinary diodes due to the multi-carrier transmission. Due to the difference of parameter, the response of SBD to the excitation signal is also different from that of ordinary diode. The nonlinearity of SBD can be considered from the nonlinear two port network. The diagram of the nonlinear two port network is shown in Fig. 21.



Figure 20. The sketch map of physical structure for SBD.

Figure 21. The diagram of the nonlinear two port network.

When the input signal is a sinusoidal signal, the response of sinusoidal excitation will contain fundamental frequency and new sinusoidal components. The following formulas are given:

$$x = V_1 \cos(\omega_1 t + \phi_1) \tag{6}$$

$$y(t) \approx \sum_{k=0}^{n} h_k \cos\left[(\omega_1 t + \phi_1)k\right]$$
(7)

So, high order harmonics will be generated when the microwave goes through the nonlinear equipment. Due to the nonlinearity of SBD, the times of harmonic frequency will be caused. In the circuit of Fig. 10, we will observe the 2G, 3G, and 4G harmonics. Based on the previous physical test and simulation results, the test waveform will change periodically after changing the length of the transmission line. Meanwhile, it is found that every interval changes periodically too. The observed doubling of the period results from the doubling of the frequency.

The difference in length between channel 4 and channel 8 is 60 mm, which is the same as the difference in length between channel 8 and channel 11. The distance of 60 mm is close to  $\lambda/4$ . The waveform of channel 4 is single period, and the waveform of channel 8 is tripling period. The waveform of channel 11 is single period again. The period doubling phenomenon across different circuits with varying length of the transmission line may be observed. The amplitude of each frequency point plays an important role in determining the waveform of time domain.

A single cycle will appear if the amplitude of 1G is very large, and the amplitude of 2G is very small. It can be proved by channel 4 and channel 11.

The period doubling waveform will appear under the condition of the similar amplitudes of 1G and 2G. Because the amplitude of the main frequency decreases greatly, we will observe the period tripling in channel 8 for the amplitude of high order harmonics is similar to the amplitude of 1G.

To summarize, we will observe single period at the maximum magnitude position of excitation wave. After  $\lambda/4$  wavelength, the excitation wave with minimum magnitude appears. The amplitude of the main frequency shows periodic variation every  $\lambda/4$ . Here, the period tripling will appear because the magnitude of 1G signal is very low. It is noted that the different period waves at certain position origin from the different amplitudes of the main frequency. The length of the microstrip line determines the amplitude of the dominant frequency. High order harmonics are generated as the result of the nonlinearity of SBD. The occurrence of the period doubling is mainly derived from the joint action of transmission line and SBD.

So far, we explain these phenomena theoretically. The reason of period doubling is different from RLD circuit completely.

## 5. CONCLUSION

Previous researchers point out that SBD does not exhibit bifurcations for its mechanism of majority carrier. In this paper, the period doubling is observed in the given circuit through the interaction with transmission line and SBD at GHz frequencies. The result has a guiding role for the design of the circuit containing SBD. It provides a new way for the design of the detection circuit, frequency multiplier, and microwave communication.

## REFERENCES

- Linsay, P. S., "Period doubling and chaotic behavior in a driven an harmonic oscillator," *Phys. Rev. Lett.*, Vol. 47, No. 19, 1349–1352, 1981.
- 2. Singh, J. P. and B. K. Roy, "Simplest hyperchaotic system with only one piecewise linear term," *Electronics Letters*, Vol. 55, No. 7, 378–380, Apr. 2019.
- Lai, Q., P. D. K. Kuate, F. Liu, and H. H. Iu, "An extremely simple chaotic system with infifnitely many coexisting attractors," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, Vol. 67, No. 6, 1129–1133, Jun. 2020.
- Carroll, T. L. and L. M. Pecora, "Parameter ranges for onset of period doubling in the diode resonator," *Phys. Rev. E*, Vol. 66, 046219-1–046219-8, 2002.
- 5. Tanaka, S., S. Higuchi, and T. Matsumoto, "Sheet structure in global bifurcations of a driven R-L-diode circuit," *Phys. Rev. E*, Vol. 54, 6014–6028, 1996.
- De Moraes, R. M. and S. M. Anlage, "Effects of UHF stimulus and negative feedback on nonlinear circuits," *IEEE Trans. Circuits Syst. I, Fundamental Theory and Applications*, Vol. 51, No. 4, 748–754(7), Apr. 2004.
- 7. Lu, L. and Z. W. Du, "Chaos in a simple microwave circuit with a PIN diode," Antennas Propagation and EM Theory (ISAPE), 963–965, 2010.
- Xu, F. and P. Tan, "Analysis of Period-doubling Bifurcation and Chaos using physics-based SiC diode model," 2015 18th International Conference on Electrical Machines and Systems (ICEMS), 612–615, Oct. 25–28, 2015.
- Basu, S., S. A. Maas, and T. Itoh, "Quasi-periodic route to chaos in a microwave doubler," *IEEE Microwave Guided Wave Lett.*, Vol. 5, No. 7, 224–6, Jul. 1995.
- 10. Rollins, R. W. and E. R. Hunt, "Exactly solvable model of a physical system exhibiting universal chaotic behavior," *Phys. Rev. Lett.*, Vol. 49, 1295, 1982.
- 11. De Moraes, R. M. and S. M. Anlage, "Unified model and reverse recovery nonlinearities of the driven diode resonator," *Phys. Rev. E*, Vol. 68, 026201-1–026201-9, 2003.
- 12. Kim, C. M., C. H. Cho, C. S. Lee, J. H. Yim, J. Kim, and Y. Kim, "Period doubling and bifurcation in an electronic circuit with a fast-recovery diode and square-wave input," *Phys. Rev. A*, Vol. 38, 1645–1648, 1988.
- Corti, L., L. De Menna, G. Miano, and L. Verolino, "Chaotic dynamics in an infinite-dimensional electromagnetic system," *IEEE Trans. Circuits Syst. I, Fundamental Theory and Applications*, Vol. 41, No. 11, 730–736, 1994.
- 14. Blakely, J. N. and N. J. Corron, "Experimental observation of delay-induced radio frequency chaos in a transmission line oscillator," *Chaos*, Vol. 14, No. 4, 1035–41, Dec. 2004.
- 15. Demergis, V., A. Glasser, M. Miller, T. M. A. Jr, E. Ott, and S. M. Anlage, "Delayed feed-back and chaos on the driven diode-terminated transmission line," arXiv:nlin/0605037, May 2006.
- 16. Sharkovsky, A. N., Y. Maistrenko, P. Deregel, and L. O. Chua, "Dry turbulence from a time delayed Chua's circuit," J. Circuits, Syst. and Comput., Vol. 3, No. 2, 645–668, 1993.
- 17. Miano, G. and A. Maffucci, Transmission Lines and Lumped Circuits, Academic Press, 2001.
- Kawata, J., Y. Nishio, and A. Ushida, "Analysis of Chua's circuit with transmission line," *IEEE Trans. Circuits Syst. I, Fundamental Theory and Applications*, Vol. 44, No. 6, 556–558, 1997.