

# Compact Permittivity Tuning Using Reconfigurable Substrate Block for Microwave Tuning Design

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**Abstract**—In this work, a permittivity tuning method using a reconfigurable substrate block is presented. The ratio of two substrate blocks with different permittivities is proved to construct a new permittivity level. This method is validated on a microstrip line, where the theory and simulation show a good agreement with a maximal permittivity calculation difference of less than 5%. In the implementation, only two pieces of substrate blocks with high and low permittivity levels respectively are needed, and it can be utilized for future flexible microwave tuning design.

## 1. INTRODUCTION

In the current microwave design, the dielectric filling is very widely used in various components and devices, such as coaxial connectors, microstrip lines, substrate integrated waveguides, etc. Therefore, the property of substrate or dielectric, particularly the permittivity, becomes critical in all kinds of device design. However, with the rapid development of various microwave RF (Radio Frequency) applications, the requirement of different permittivities becomes very common. The development of a substrate with various permittivity values or permittivity tuning methods becomes urgent. The traditional solution is mostly based on the complex manufacturing process regulation with different material component ratios; however, once the microwave device or substrate was manufactured, it cannot be tuned in the actual service used, and for a tunable microwave device, the development of flexible permittivity tuning technology becomes promising.

Previous researches investigated various effective medium models for calculating the permittivity under different substrate combination situations. For example, [1] proposed different analytical models for evaluating effective substrate parameters of multilayer materials. [2] created four equivalent models to determine the effective permittivity by using effective dielectric theory and applied it to the simulation of layered dielectric substrates. [3] developed the application of equivalent medium models in carbon fiber composites and proposed three models for the electromagnetic properties. [4] applied effective dielectric theory to simulate materials of multiple layers, created an equivalent model, and then determined the effective permittivity of the two layers of the medium. [5] compared layered mediums to uniform mediums with effective permittivity and loss tangent, and investigated the effect on printed circuit board impedance. Based on the conformal mapping method, [6] derived an expression of effective permittivity for microstrip with a multi-layered substrate in the quasi-TEM (Transverse Electromagnetic) assumption. Additionally, [7] demonstrated that the geometry relations of transmission lines in conformal transformation would not change with the permittivity of a multilayer medium. In [8], a conformal mapping method is used to get the equivalent permittivity in quasi-TEM mode.

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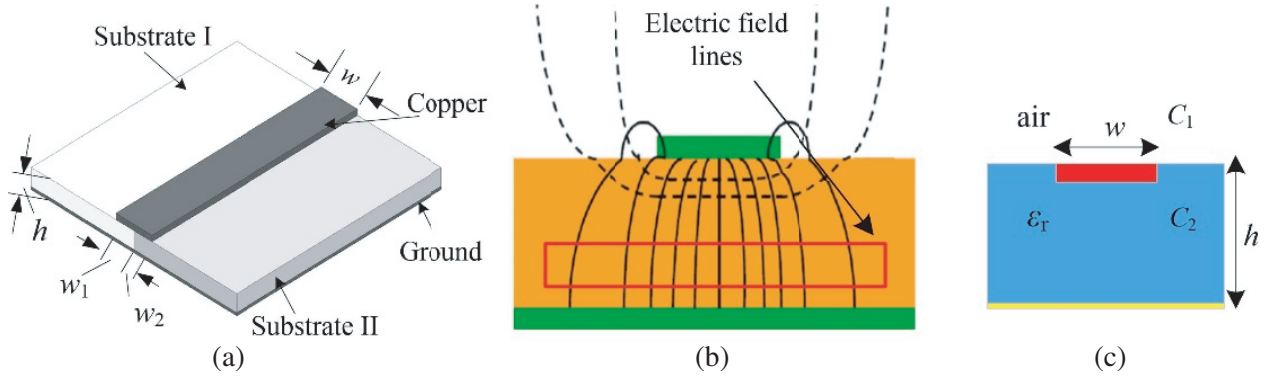
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Based on the previous publications, in order to further the flexible permittivity tuning methodology, this work proposes a simple reconfiguration method to get arbitrary permittivity for microwave design. Based on the dispersion relationship of the medium filling, the demonstration based on a microstrip line is provided, wherein the conformal mapping method is used to get an analytical solution for a mixed substrate condition, which matches the electromagnetic simulations and proves the proposed method.

## 2. THEORETICAL MODELING

In the following, a normal microstrip line model is used to prove the proposed permittivity tuning method. As shown in Fig. 1(a), two kinds of mediums are filled in the lower layer of the microstrip conductor, to form the basic tuning structure. Normally, the field distribution on the microstrip line is shown in Fig. 1(b), where the electromagnetic field is mainly distributed between the conductor and ground [9], while some part is also distributed in the air. Finally, as a result, the field distribution in microstrip line is similar to TEM mode, called the quasi-TEM mode. In this work, the permittivity tuning at substrate structure is demonstrated, for the analysis of microstrip line, this work is to solve the equivalent permittivity by following the traditional solution steps of the capacitance in relevant structure, and the model considering capacitance distribution is as shown in Fig. 1(c).



**Figure 1.** (a) Microstrip line structure by two substrates. (b) Electric field distribution of microstrip line. (c) Capacitive distribution in microstrip.

The capacitance of the air region is assumed to be  $C_1$ ; the capacitance inside the dielectric substrate region is  $C_2$  [10]; the total capacitance  $C$  is,

$$C = C_1 + C_2 = C_{\text{air}} + (\epsilon_r - 1)C_2 \quad (1)$$

where  $C_{\text{air}}$  means a line capacitance by replacing the substrate with air, and  $\epsilon_r$  is the relative permittivity of the substrate. Then the effective permittivity  $\epsilon_e$  is defined as,

$$\epsilon_e = \frac{C}{C_{\text{air}}} \quad (2)$$

Further, the capacitance value is substituted into the permittivity formula to obtain the final permittivity of the microstrip line as.

$$\epsilon_e = 1 + (\epsilon_r - 1) \frac{C_2}{C_{\text{air}}} \quad (3)$$

In [11], which provided an accurate closed-form Hammerstad-equation for calculating the permittivity of the microstrip line as

$$\epsilon_e = \frac{1}{2} [(\epsilon_r + 1) + (\epsilon_r - 1) \cdot F] \quad (4)$$

where  $F$  is the factor determined by the detailed section configuration of microstrip line.

$$F = \begin{cases} (1 + 12h/w)^{-1/2} + 0.04(1 - w/h)^2 & w/h \leq 1 \\ (1 + 12h/w)^{-1/2} & w/h > 1 \end{cases} \quad (5)$$

Similarly in [12], the characteristic impedance of the microstrip line is,

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\epsilon_e}} \cdot n \left( \frac{8h}{w} + \frac{w}{4h} \right) & w/h \leq 1 \\ \frac{120\pi}{\sqrt{\epsilon_e} \cdot \frac{w}{h} + 1.393 + \frac{2}{3} \ln \left( \frac{w}{h} + 1.444 \right)} & w/h > 1 \end{cases} \quad (6)$$

Based on the proven theory and following the modeling procedure above, the permittivity reconfiguration demo case is shown as follows. In this reconfiguration, firstly two different substrates with different permittivities are filled under the upper conductor from left to right. For this structure, the microstrip line substrate can be divided into two regions. The permittivity of region I is  $\epsilon_1$ , and the permittivity of region II is  $\epsilon_2$ . The solution to the capacitances inside the microstrip is used to analyze the model.

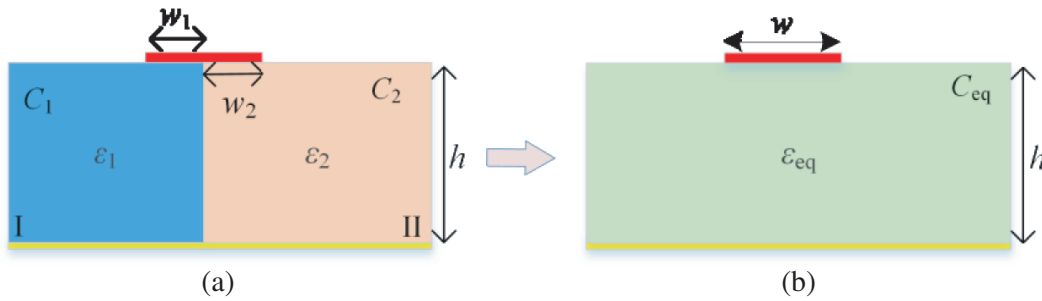


Figure 2. Equivalent model of parallel capacitance.

As shown in Fig. 2, referring to the center axis of the section view,  $w_1$  and  $w_2$  are the actual widths of microstrip conductors that the two mediums occupied respectively. As the field distributed in the air cannot be ignored, the effective width  $w'$  of the microstrip conductor can be expressed as the calculated width with the correction item as

$$w'' = w' + \Delta w \quad (7)$$

where  $\Delta w$  is the width correction for the microstrip conductor, which is determined by the leaky field distribution in the air. For the microstrip section, using conformal mapping method, the complex plane  $z = xi + yj$  is mapped into the bounded plane  $w = ui + vj$ . The variation formula is expressed as  $dw/dz$ , and  $z$  and  $w$  are the corresponding coordinate points in the mapping [12].

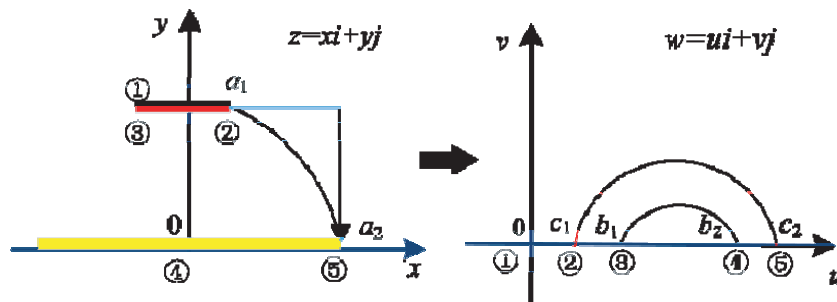


Figure 3. Conformal mapping of dielectric region.

Using the conformal mapping relation shown in Fig. 3, we can get,

$$\frac{dz}{dw} = \frac{w^2 - c_1^2}{w^2 - \Delta b^2} \quad (8)$$

$$z = w - \operatorname{arctanh} \left( \frac{\Delta b}{w} \right)$$

Further, the coordinate points can be expressed as,

$$\begin{aligned} a_1 &= c_1 - \arctan h \left( \frac{\Delta b}{c_1} \right) \\ 2\operatorname{arctanh} \left( \frac{\Delta b}{c_1} \right) &= \operatorname{arcsinh} (c_1) = \ln \left( \frac{\Delta b + c_1}{\Delta b - c_1} \right) \end{aligned} \quad (9)$$

where  $a_1$  and  $c_1$  are the mapping points, and  $\Delta b$  is the difference between the positions of  $b_1$  and  $b_2$ . The actual width of the transmission line is  $a_1$  in  $z$  plane, with a correction factor of  $a'_1$  after conformal transformation, so the effective width  $w_e$  is,

$$w_e = a_1 + a'_1 \quad (10)$$

The corresponding coordinate point is,

$$2\Delta b - 1 = 2a_1 + 1 + 2 \ln [2a_1 + 1 + 2 \ln (4\pi - 1)] \quad (11)$$

The effective width of region I is  $w'_1$ ; the effective width of region II is  $w'_2$ ; and the sum of the effective widths of the two regions  $w'$  is a constant value.

$$w' = w'_1 + w'_2 \quad (12)$$

Based on the electric field distributions of the two capacitors, the capacitors are considered to parallel together, and the equivalent structure is shown in Fig. 2. According to the principle of parallel capacitance relation [13], the unit capacitance of the whole equivalent structure  $C_{eq}$  is,

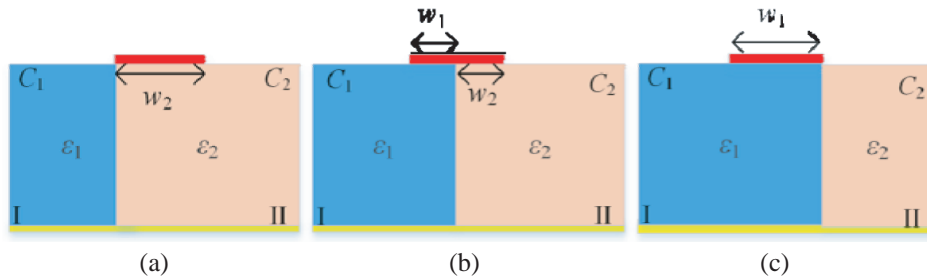
$$C_{eq} = \frac{w'_1 \cdot C_1 + w'_2 \cdot C_2}{w'_1 + w'_2} = \frac{w'_1 \cdot C_1 + w'_2 \cdot C_2}{w'} \quad (13)$$

then based on the microstrip capacitor's circuit theory and parallel plate capacitor's principle, the equivalent permittivity,  $\varepsilon_{eq}$ , of the uniform medium is given as

$$\begin{aligned} \varepsilon_{eq} &= \frac{\{w_1 + 1 + \ln [2w_1 + 1 + 2 \ln (4\pi - 1)]\} \cdot \varepsilon_1}{w + 2 + \ln [2w_1 + 1 + 2 \ln (4\pi - 1)] + \ln [2w_2 + 1 + 2 \ln (4\pi - 1)]} \\ &+ \frac{\{w_2 + 1 + \ln [2w_2 + 1 + 2 \ln (4\pi - 1)]\} \cdot \varepsilon_2}{w + 2 + \ln [2w_1 + 1 + 2 \ln (4\pi - 1)] + \ln [2w_2 + 1 + 2 \ln (4\pi - 1)]} \end{aligned} \quad (14)$$

### 3. SIMULATION DEMONSTRATION

In the demo of permittivity reconfiguration, a simple microstrip line model is used; the tuning operation of permittivity tuning is realized by changing the ratio of the left substrate to the right substrate, and the left substrate and right substrate have different permittivities. Fig. 4 shows a schematic view of the substrate tuning progress with different statuses, and the equivalent conductor width of the upper conductor changes with different substrate combinations.

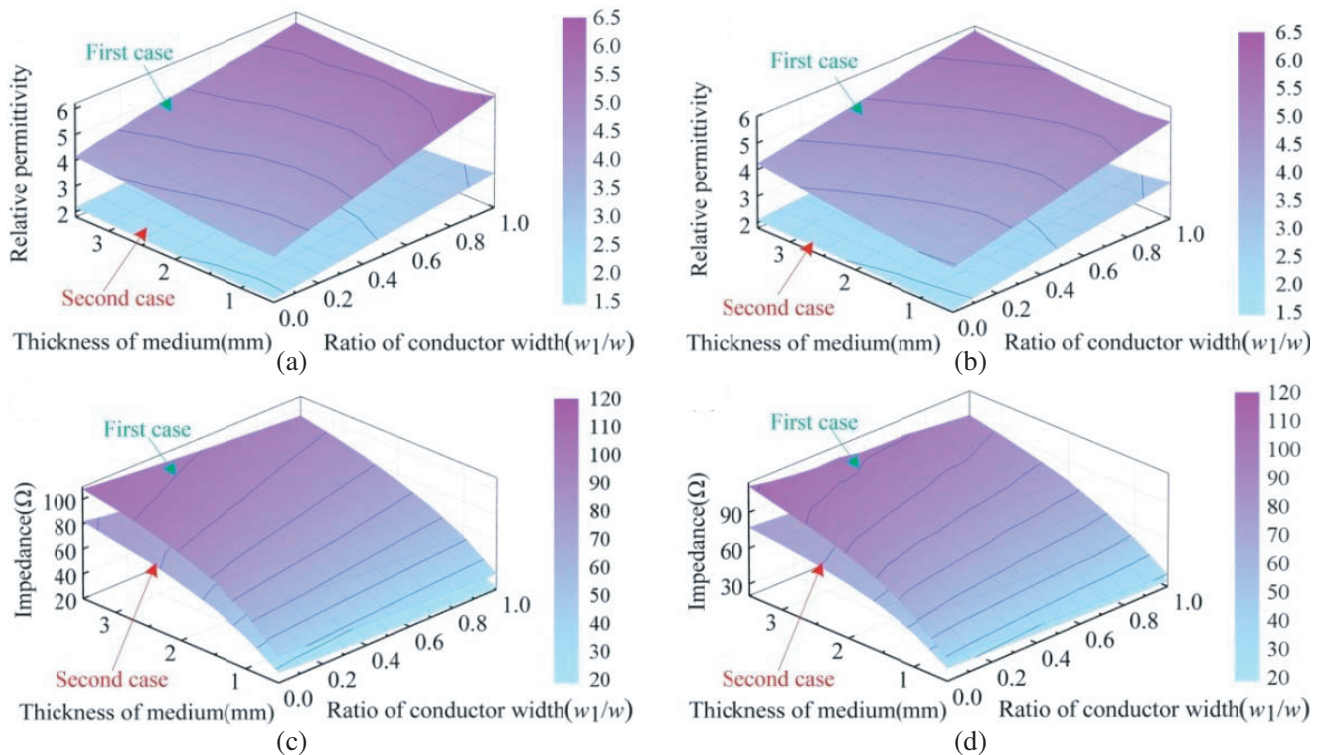


**Figure 4.** The structural changes in microstrip line.

As illustrated in Fig. 4, with the upper conductor moves from right to left, the calculation situation can be as follows.

$$(a) \ w_1 = 0, \quad w_2 = w; \quad (b) \ w_1 + w_2 = w; \quad (c) \ w_1 = w, \quad w_2 = 0 \quad (15)$$

It is obvious that for the configuration as shown in Fig. 4, if the permittivity  $\epsilon_1$  of region I is larger than the permittivity  $\epsilon_2$  of region II, when the upper conductor moves from right to left, the effective permittivity of microstrip line will gradually increase, and the characteristic impedance will gradually decrease or vice versa. In the demonstration, for the contrast group, the microstrip line structure of the multilayer medium is created in electromagnetic simulation software, while the conformal mapping is used to validate further. The preconditions are that the width of the microstrip line conductor  $w$  is assumed to be 2 mm; the value of  $w_1/w$  ranges between 0 and 1 with a step of 0.1; and the thickness of the dielectric substrate  $h$  varies between 0.5 mm and 3.5 mm with a step of 0.5 mm. The corresponding permittivity values calculated by the conformal mapping method are compared in Fig. 5. Two groups of the medium are compared. (1) First case:  $\epsilon_1 = 4.4$  (FR4)  $\epsilon_2 = 2.1$  (Teflon), (2) Second case:  $\epsilon_1 = 8.8$  (AlN)  $\epsilon_2 = 4.4$  (FR4). For these two cases, it is proved that the max permittivity difference between the conformal mapping and finite element simulation is less than 5%, while the impedance difference (solved by substituting the calculated permittivity into Eq. (6)) is less than 6%. In the first case, the error is minimal at  $w = h$ , and this difference is potentially caused by the non-uniform dispersion of the electric field in the air. In the second case, the difference trend is relatively small, which may be caused by the decreased leaky field as the enlarged equivalent permittivity focuses more in the field inside the substrate.



**Figure 5.** The permittivity and characteristic impedance comparison, (a) theoretical calculated permittivity values, (b) simulated permittivity values, (c) theoretical calculated characteristic impedance values by Eq. (6), (d) simulated characteristic impedance values.

From the comparison in Fig. 5, it is clear that the calculated and simulated permittivity and impedance values are highly consistent, confirming the correctness of the theoretical derivation. For the case here, it is proved by changing the substrate position, and the permittivity can be effectively tuned. In the case, the method only needs two sets of substrate blocks with different permittivities, and the permittivity values by two substrates are the permittivity tuning range. It can be expected that by using more pieces of substrate block, the tuning of permittivity can be more flexible. Compared with the traditional microwave design that needs specially customized substrate material, the proposed

method by flexibly combining different substrate blocks is much more cost-efficient, while for the future tunable microwave device, the proven method can also be an option to realize smart microwave tuning.

For the design of tunable microwave devices, the flexible permittivity tuning technology has a broad prospect. Firstly, the variable permittivity for different microwave circuit devices is realizable, which can save the cost in the manufacture of different laminates. The proposed solution is simple and efficient. Only two kinds of substrates are needed, and a wide range of permittivity tuning can be realized by using a flexible design strategy. Additionally, such a technology can also be widely used in the field of antenna technology [14–16]. For a reconfigurable antenna, by changing the material characteristics of the antenna, the antenna working frequency can be changed. Further, the reconfigurable permittivity can also be used for impedance matching of the antenna, by tuning permittivity and characteristic impedance; the impedance can be matched; and a good return loss can also be achieved. Further, this efficient strategy can be used on other tunable devices or circuit designs, such as tunable filter, load-pull, and phase shifter.

#### 4. CONCLUSION

This work has demonstrated a permittivity reconfiguration method by reconfiguring different dielectric substrate blocks. It proves that permittivity and characteristic impedance can both be achieved with a smart combination of different dielectric mediums. As the proposed method does not need any special requirements about the material with customized permittivity, the permittivity can be effectively regulated. For the microwave designs need tuning application cases, such as reconfigurable antenna, tunable filter, load-pull, and phase shifter, the proposed method here is cost-effective and has excellent flexible tuning ability.

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