Integrated Waveform of Frequency Diversity Array Radar Communication Based on OFDM Random Frequency Offset Modulation

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Abstract—The integration of radar and communication has always been one of the cross-research hotspots in the field of radar and communication. In order to solve the problems of integration signal separation and angle-distance coupling, this paper proposes a radar and communication integrated waveform based on random Orthogonal Frequency Division Multiplexing (OFDM) frequency offset modulation for Frequency Diversity Array (FDA). This waveform directly loads OFDM symbols to the elements of FDA, and each element carries a complete OFDM symbol with different information. Random frequency offsets are added between the elements to separate different signals of different elements, which can solve the problem of signal separation and form decoupled radar beam. After transmitting and receiving a series of the waveform, the transmission of communication data and the positioning of radar targets can be completed at the same time. The simulation results show that the waveform not only solves the problem of separating and uncoupling the integrated signal, but also improves the frequency band utilization rate and information transmission rate of the radar communication integrated system.

1. INTRODUCTION

With the rapid development of science and technology and the increasing complexity of the electromagnetic environment, the modern combat platform has increasing requirements for the performance of the radar system. In some applications (such as bistatic and distributed radars), if some parameters of the radar (such as motion information) can be effectively transmitted to the receiver, it will be beneficial for improving the performance of the radar system, which will lead to the development of integrated radar-communication research.

Radar system and communication system are strictly distinguished because of their obvious differences in some aspects [1], but they also have similarities in many aspects and have the conditions to synthesize an integrated system. Refs. [2–5] analyzed, in detail, the feasibility of radar communication integration from theoretical principles and other aspects. At present, radar communication integration technology is mostly based on linear frequency modulation (LFM) signal and Orthogonal Frequency Division Multiplexing (OFDM), and completes the design of integrated signal model through modulation [6–9]. Therefore, radar waveform and communication waveform are generated independently and orthogonal, which also leads to the reduction of detection ability and low communication rate. At present, phased array with the advantage of high gain can not only improve the communication rate, but also realize the radar system and communication system to share the transceiver channel. As a special phased array, the frequency-controlled array (FDA) [10] has both the

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advantages of phased array and its own characteristics [11, 12]. Its beam pattern is doubly dependent and coupled in angle and distance, and the degree of freedom of transmitting beam energy gathering in space is improved. In essence, by adding different frequency differences between the transmitting units of the phased array, there are different beam directions at the same angle and different distances, which can solve the problem of distance ambiguity of the phased array. There are two main design methods of the existing FDA radar communication integrated waveform: One is based on the LFM signal, combining the digital baseband signal and FDA signal into an integrated signal with a certain modulation method [13–15], and the other is to link the frequency offset of FDA with OFDM to design integrated radar communication waveforms [16–18]. However, the above two integrated waveform design methods may have the problem of difficult separation of radar signal and communication signal, or occupy more radar pulse resources.

In response to the above problems, this paper proposes an integrated waveform of FDA radar communication based on random OFDM frequency offset modulation, which modulates the random communication signal to the frequency offset between the FDA array elements. This waveform is equivalent to modulating multi-bit information to the same array element, so that each array element of the FDA radar transmits OFDM symbols with the same length and different information, which can be easily demodulated at the communication receiver. This method not only does not affect the beam characteristics of DFA at the radar receiver, but also brings benefits for FDA target location because of the introduction of random communication signals. Moreover, the system can simultaneously realize radar target positioning and communication data transmission by transmitting a single pulse. Therefore, this technology not only solves the difficulty of signal separation in radar communication integration, but also solves the problem that the existing integrated waveform takes up more radar pulse resources.

The first section of this paper introduces the research background of FDA radar communication integration. The second section and third section establish the theoretical model of integrated waveform and analyze the new beam correlation characteristics. In the third section, the bit error rate, communication rate, spectrum efficiency and other parameters of the communication system in the integrated system are analyzed. The performance analysis of radar system target location is carried out in the fourth section. Finally, the fifth section summarizes the full text.

2. INTEGRATED RADAR COMMUNICATION SYSTEM

2.1. Application Background of Integrated Radar Communication System

FDA integrated radar communication system can be applied to bistatic radar. The integrated signal is sent by the launching platform and received by the receiving platform after being reflected by the target. Then, perform radar signal processing and communication signal processing on the echo signal to achieve target positioning and data communication. The integrated application model of radar communication is shown in Figure 1.

Take the FDA radar as the launch platform in Figure 1 ($0 \sim N - 1$ in the figure are the N launch elements of the radar). Take the first element of the launch platform as the reference element, and the distance from the reference element to the ground target is recorded as R_0 . The distance and angle information of the *n*-th transmitting array element to the ground target are R_n and θ_n . After the integrated signal is transmitted, it is reflected by the ground target and received by a single antenna (array element N in Figure 1) on another platform. The distance and angle from the receiving platform to the ground target are R_R and θ_R , respectively. Both transceiver platforms can transmit and receive integrated signals. When they fly in the air, they can only get their own speed, initial position, and other motion information. The launch platform loads its own motion information into the integrated signal. The signal is sent by the launch platform, and received by another platform after being reflected by the target. When transmitting a signal, the elements of the transmitting antenna transmit the signal at the same time, and each element transmits a complete and different OFDM symbol. Then, the echo signal is demodulated, the motion information of the transmitting platform obtained, and the target location realized at the same time.





2.2. The Processing Flow of the Integrated Radar Communication System

It can be seen from Section 2.1 that radar and communication systems can share transmitters, receivers, transceiver antennas, and other devices. Based on this condition, the flow of the FDA radar communication integration system in this paper is shown in Figure 2.



Figure 2. Flow chart of the integrated system.

The integrated system in Figure 2 uses a single-frequency transceiver mechanism, that is, using all antennas of the FDA radar to transmit signals, and a single antenna to receive echo signals. The formation process of the integrated signal is inferred in detail in Section 3.1. The integrated signal converts the binary raw data into M-ary communication data through M-ary pulse amplitude modulation

(M-PAM), and then converts them into OFDM symbol form through IFFT. A random frequency interval is added between the symbols to distinguish sub-carrier frequencies on different array elements. The integrated signal is transmitted by one platform and received by another platform after the reflection of the target. The signal processing unit of the receiving platform multiplies the echo signal with Ncarriers to perform coherent demodulation, and the carrier has the same frequency and phase as the FDA signal. The communication signal processing unit performs communication demodulation, first obtains N mutually independent and complete OFDM symbols, then obtains L * N baseband signals through FFT, and finally obtains communication data by sampling and judging the baseband signals to realize data communication. The radar signal processing unit demodulates the echo signal, performs the FDA target positioning algorithm on the N baseband signals obtained, and finally achieves target positioning.

3. INTEGRATED RADAR COMMUNICATION SIGNAL

3.1. Radar Communication Integrated Signal Model

In the integrated system of this paper, each element of FDA radar transmits OFDM symbols with the same length and different information. In order to meet the spectral interval requirements of each symbol, the frequency interval is increased between the array elements. However, increasing linear frequency intervals cannot solve the problem of angle-range coupling. Therefore, this paper proposes an OFDM-FDA radar and communication integrated waveform design method based on random frequency intervals. The integrated model is shown in Figure 3.



Figure 3. Integrated signal model of radar communication.

Assuming that the number of array elements required by the integrated signal system is N; the initial carrier frequency of transmission is f_0 ; c is the propagation speed of electromagnetic waves in space, wavelength $\lambda = c/f_0$, array element spacing $d = \lambda/2$. Original communication information is binary data, which can be converted into M-ary communication data A_{\ln} by M-PAM modulation, and the value of M satisfies $M = 2^k$, where k is any positive integer, and the value range of A_{\ln} is $A_{\ln} \in \{1, 2, \dots, M\}$, where A_{\ln} is an element in an l * n matrix, which refers to the communication information information loaded by the *l*th subcarrier on the *n*th array element. The number of sub-carriers of a complete OFDM is L, and the frequency interval between sub-carriers is Δf , then the frequency difference between the array elements is $L\Delta f$.

According to the generation principle of OFDM signal [19], the communication signal is loaded on the integrated signal waveform in the form of amplitude. The transmission signal of the lth subcarrier on the *n*th array element can be expressed as

$$S_{\rm ln}(t) = A_{\rm ln} \cdot e^{j2\pi f_{\rm ln}t} \quad l = 0, 1, ..., L - 1 \quad n = 0, 1, ..., N - 1 \tag{1}$$

where f_{ln} is the transmit frequency of the *l*th subcarrier on the *n*th array element:

$$f_{\rm ln} = f_0 + l\Delta f + b_n L\Delta f \quad l = 0, 1, ..., L - 1 \quad n = 0, 1, ..., N - 1$$
(2)

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The value range of b_n is $b_n \in \{1, 2, \dots, N-1\}$, which is random and not repeated.

Each array element transmits a complete OFDM symbol, then the transmitted signal of the n-th array element is:

$$s_n(t) = \sum_{l=0}^{L-1} A_{\ln} \cdot e^{j2\pi f_{\ln}t} \quad n = 0, 1, ..., N-1$$
(3)

Then the transmitted signal of the OFDM-FDA radar and communication integrated system based on random frequency offset is:

$$S(t) = \sum_{n=0}^{N-1} s_n(t) = \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} A_{ln} \cdot e^{j2\pi f_{ln}t} = \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} A_{ln} \cdot e^{j2\pi (f_0 + l\Delta f + b_n L\Delta f)t}$$
(4)

3.2. Radar Communication Integrated Beam Pattern

Due to the addition of random frequency offset, the transmission beam of the integrated signal model will break the traditional "S" shape of the FDA and form a spot beam pattern [20]. Assuming that there is an ideal observation point on the ground, its angle along the normal direction of the array ray is θ , and the slant distance to the reference array element is R. Then the transmission beam time delay of the *n*-th array element is:

$$\tau_n = \frac{R - nd\sin\theta}{c} \quad n = 0, 1, ..., N - 1 \tag{5}$$

Among them, d is the distance between the array elements, and c is the propagation velocity of electromagnetic waves. Then the beam pattern of the integrated signal transmission is as follows:

$$A(R,\theta,t) = \sum_{n=0}^{N-1} s_n(t-t_n) = \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} A_{ln} \cdot e^{j2\pi f_{ln}(t-t_n)} = \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} A_{ln} \cdot e^{j2\pi (f_0+l\Delta f+b_nLf)(t-t_n)}$$
(6)

From Equation (6), it can be known that after the communication signal is loaded on the transmission signal, the beam pattern is still related to time, angle, and distance information. Although the integrated beam still has time variability, the transmitted signal is a pulse signal, and the time variability of the beam will not affect the waveform of the integrated radar communication, so we will not discuss it for the time being. In addition, the transmitted beam pattern is related to not only the distance R, angle θ , time t, frequency offset Δf , the number of array elements N, and the spacing of the array elements d, but also the number L of communication information sub-carriers. According to Equation (6) and the periodicity of the array, it is easy to get: the communication signal is loaded on the frequency offset of the FDA radar. If the magnitude of $l\Delta f$ is much smaller than $b_n L\Delta f$, it can be ignored. When the angle and time are fixed, the period of the distance is $c/L\Delta f$; when the distance and angle are fixed, the period of time is $1/L\Delta f$.

Assuming that time $t = 0 \,\mu$ s, the number of array elements N = 64, the initial frequency $f_0 = 10 \,\text{GHz}$, the number of sub-carriers L = 8, the frequency interval $\Delta f = 1050 \,\text{Hz}$, and the array element interval $d = \lambda/2$. Without considering the antenna radiation, combined with Equation (6) to simulate the integrated signal transmission beam pattern, the integrated waveform transmit beam simulation is shown in Figure 4.

From Figure 4, it can be seen that the beam pattern of the OFDM-FDA radar and communication integrated signal based on random frequency offset is highly concentrated on the main lobe, forming a spot beam pattern with low sidelobe energy. It shows that the integrated signal can effectively suppress clutter and effectively solve the problem of target location caused by angle-range coupling of "S" beam signal transmitted by traditional frequency diversity array radar. The integrated waveform has application prospects in clutter suppression and fixed-point interference. Therefore, the design of the integrated waveform is feasible.



Figure 4. Beam pattern of integrated signal transmission.

4. COMMUNICATION PERFORMANCE ANALYSIS

The performance of communication system mainly includes three indicators: bit error rate, data rate, and band utilization. Considering that the bandwidth of the radar system is generally a fixed value, the integrated system bandwidth is:

$$B_N = N \cdot L \cdot \Delta f \tag{7}$$

Next, analyze the communication performance of the radar system:

(1) Bit error rate.

The integrated signal converts binary raw data into M-ary communication data through the MPAM modulation method, and then performs OFDM modulation to modulate the communication signal on the amplitude of the transmitted signal. At the receiving end, N groups of filters are used to detect the received echo signal, and output a signal with noise. Assuming that the noise is mutually independent narrow-band Gaussian white noise and obeys the Rayleigh distribution, the bit error rate of the communication system [21] is:

$$P_b = \frac{M-1}{M} \operatorname{erfc}\left(\sqrt{\frac{3}{M^2 - 1} \cdot k \cdot \gamma_b}\right) \tag{8}$$

where γ_b is the bit signal-to-noise ratio (bit SNR), and k is the number of bits contained in each piece of communication information. The relationship between bit SNR and bit error rate under different base numbers is verified through simulation, and the simulation result is shown in Figure 5.

It can be seen from Figure 5 that the bit error rate of the integrated system decreases with the increase of the bit SNR, which is inversely proportional. When the bit error rate is constant, the bit SNR increases with the increase of the value of the Base number M. That is, as M increases, in order to obtain the same bit error rate, it is necessary to increase the bit SNR; at the same time, it can be seen that when the bit SNR is constant, the communication performance gradually deteriorates with the increase of M. Therefore, it is necessary to select the corresponding M for modulation according to the requirements for system performance.

(2) Data rate.

In the OFDM-FDA radar and communication integrated system based on random frequency interval, the M-ary data is converted into OFDM symbol form through IFFT, and the number of subcarriers of an OFDM symbol is L, then the amount of data contained in each element of the integrated



Figure 5. The relationship between bit error rate and bit signal-to-noise ratio under different M values.

system is $L \cdot \log_2 M$ bit. For the integrated system with N array elements, the amount of information data of transmitting a pulse signal is $N \cdot L \cdot \log_2 M$ bit, then the data rate of the communication performance can be expressed as:

$$R_b = \frac{N \cdot L \cdot \log_2 M}{T_B} \tag{9}$$

Among them, T_B represents the duration of the code element, $\Delta f_{\min} = 1/T_B$.

It can be seen from the above formula that the data rate of the integrated system is related to the value of the number of array elements N, the number of sub-carriers L, the number of base M, and the frequency offset Δf , and is in a proportional relationship. However, when the system bandwidth is constant, B_N and Δf are fixed values. Increasing the base number M requires reducing the number of elements N, and reducing the number of elements N has a greater impact on the data rate than increasing the base number M. When the system bandwidth is variable, the data rate R_B increases with the increase of the number of array elements N, decreases with the increase of the duration of the code element T_B , and increases with the increase of the number of sub-carriers L.

(3) Frequency band utilization.

In the integrated system, the number of subcarriers of an OFDM symbol is L; the duration of the code element is T_B each subcarrier is modulated by M-ary data; the number of array elements in the transmitting array is N. Then the frequency bandwidth occupied by transmitting a single pulse is:

$$B = \frac{L+1}{T_B} \cdot N \,(\mathrm{Hz}) \tag{10}$$

The bit rate of transmission per unit bandwidth, that is, the frequency band utilization rate is:

$$\eta = \frac{R_b}{B} = \frac{N \cdot L \cdot \log_2 M}{T_B} \cdot \frac{T_B}{N \cdot (L+1)} = \frac{L}{L+1} \log_2 M(b/(s \cdot Hz))$$
(11)

When the value of L is very large, there is

$$\eta \approx \log_2 M(b/(s \cdot Hz))$$
 (12)

When the integrated system uses single carrier modulation for transmission, assuming that the modulation mode of each subcarrier is the same, and the transmission rate is the same as the OFDM symbol, the symbol duration of the single carrier transmission is T_B/L , and the bandwidth is $2L/T_B$. In this case, the band efficiency is:

$$\eta = \frac{L \cdot \log_2 M}{T_B} \cdot \frac{T_B}{2L} = \frac{1}{2} \log_2 M(b/(s \cdot Hz))$$
(13)

Simulate the variation relationship of the frequency band utilization rate with the base number M in the integrated system using OFDM modulation and the integrated system using single carrier modulation.

According to Figure 6 and comparing Equations (12) and (13), it can be found that the frequency band utilization rate of the FDA integrated radar system using OFDM is about twice that of the single carrier system under the same base number. And the frequency band utilization rate increases with the increase of the base number.



Figure 6. The relationship between the base number and the frequency band utilization.

5. RADAR PERFORMANCE ANALYSIS

5.1. Multi-Target Positioning of Radar System

The N array element signals emitted by the launching platform are received by the receiving platform after being reflected by the target. The expression of the echo signal received at this time is:

$$Y(t) = \sum_{n=0}^{N-1} y_n(t)$$
(14)

Among them, $y_n(t)$ represents the echo signal of the signal transmitted by the nth array element:

$$y_n(t) = \sum_{i=1}^{I} S_i(t) \cdot e^{j2\pi(f_c + c_n\Delta f + b_nM\Delta f)\left(t - \frac{R_i}{c} + \frac{nd\sin\theta_i}{c} - \frac{R_{Ri}}{c} + \frac{nd\sin\theta_{Ri}}{c}\right)} + n_n(t)$$
(15)

In Equation (15), $S_i(t)$ is the echo signal of the *i*-th target, and $n_n(t)$ is the additive noise. Assuming that the position of the reference element of the transmitting platform is the reference point, the position of the *i*-th target is expressed as (R_i, θ_i) , and the angle and distance information of the receiving platform relative to the *i*-th target is expressed as (R_{Ri}, θ_{Ri}) , where R_{Ri} and θ_{Ri} are respectively expressed as:

$$R_{Ri} = \sqrt{(R_i \cos \theta_i - R_R \cos \theta_R)^2 + (R_i \sin \theta_i - R_R \sin \theta_R)^2}$$
(16)

$$\theta_{Ri} = \arcsin\left(\frac{R_i \sin \theta_i - R_R \sin \theta_R}{R_{Ri}}\right) \tag{17}$$

In the formula, the position information of the receiving platform relative to the transmitting platform is (R_R, θ_R) . After the receiving platform receives the echo signal, it first needs to be filtered

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through N-channel filters. The center frequency of the N-channel filter corresponds to the transmission frequency of the N array element signals. Therefore, in the h-th sampling, the signal is expressed as a vector:

$$Y(h) = [y_0(h) \quad y_1(h) \quad \cdots \quad y_{N-1}(h)]^T$$
(18)

Among them, Y(h) is an N * 1 order matrix, next, and the multi-signal classification (MUSIC) algorithm can be used to achieve target positioning.

Next, the integrated signal multi-target positioning performance is analyzed and verified through simulation experiments. Mutual coupling between antenna elements is not considered. The simulation parameters are: the number of array elements N, the radar launch carrier frequency $f_0 = 10e9$ Hz, the frequency offset $\Delta f = 1050e3$ Hz, the scattering coefficient $\sigma_1 = \sigma_2 = 1$, the base number M = 8, the value range of the communication data $A_{\ln} \in \{0, 1, 2, 3, 4, 5, 6, 7\}$, the signal-to-noise ratio: SNR = 0 dB, SNR = 10 dB. Set multiple target positions: $(-60^\circ, 6 \text{ km}), (-30^\circ, 8 \text{ km}), (0^\circ, 10 \text{ km}), (30^\circ, 14 \text{ km}), (50^\circ, 12 \text{ km})$. The simulation result is shown in Figure 7. Figure 7(a) is the multi-target positioning result when SNR = 0 dB, and Figure 7(b) is the multi-target positioning result when SNR = 10 dB.

It can be obtained from Figure 7 that the spectral function forms the maximum gain at the target point. The imaging effect map of Figure 7(a) with a signal-to-noise ratio of 0 dB and the imaging effect map of Figure 7(b) with a signal-to-noise ratio of 10 dB have achieved good target positioning results, verifying the feasibility of integrated signal target positioning.



Figure 7. Multi-target positioning effect diagram. (a) SNR = 0 dB, (b) SNR = 10 dB.

5.2. Radar System Performance Analysis

The performance parameters of radar system mainly refer to range resolution and angle resolution. When the radar antenna is aimed at the target, if there are two targets in the same direction, the radar can identify the two targets, and the minimum distance between them is the range resolution. If two targets are at the same distance, when the radar can distinguish the two targets, the minimum angle between them is the angular resolution. The performance of the radar system will be analyzed from these two aspects as follows.

(1) Distance resolution.

FDA has range resolution due to frequency offset Δf . When $\Delta f = 0$, FDA becomes a phased array with only angle resolution. Therefore, the distance resolution of FDA depends on the value of Δf , and the distance resolution of the integrated system can be expressed as:

$$\rho_r = \frac{c}{2B_N} = \frac{c}{2N \cdot L \cdot \Delta f} \tag{19}$$

Among them, $B_N = N \cdot L \cdot \Delta f$ is the effective bandwidth of the signal transmitted by the integrated system. From Equation (20), the range resolution of the radar depends on the bandwidth of the integrated system. The bandwidth of the integrated system is proportional to the number of array elements N, the number of subcarriers L, and the frequency offset Δf . The range resolution of the radar system is inversely proportional to the bandwidth of the integrated system. Therefore, for an integrated system with an invariable bandwidth, if any parameter of N, L, Δf is changed, other parameters can be changed accordingly to keep the bandwidth fixed, and the radar range resolution will also be fixed with the bandwidth. From Equation (20), if any two parameters of N, L, and Δf are fixed, ρ_r decreases with the increase of the other parameter, and the range resolution of the radar is better. For an integrated system with variable bandwidth, ρ_r of the system can be improved by changing the bandwidth of the system.

(2) Angle resolution.

In the OFDM-FDA radar and communication integrated system based on random frequency intervals, the angular resolution of target points located at the same distance and different angles is:

$$\rho_{\theta} = \frac{c}{2Nf_0d} \tag{20}$$

It can be seen from Equation (21) that the angular resolution ρ_{θ} of the integrated system is inversely proportional to the number of array elements N, the center frequency Δf , and the distance between the array elements d.

Assuming that the values of the beam half-power point on θ are θ_+ and θ_- , combined with the radiation field vector sum formula, the angular resolution of the integrated system can be derived as:

$$\rho_{\theta} = 2\left(\theta_{+} - \theta_{-}\right) \approx \frac{0.64c}{Nf_{0}d\cos\theta_{0}} \tag{21}$$

It can be seen from the above formula that the angular resolution of the integrated system is not only related to f_0 , N, and d, but also related to the pointing angle of the beam θ_0 . The larger the angle is between the target point and the normal of the uniform linear array where the array element is located, the larger the beam width is, and the better the radar angle resolution is. When the target point is directly above the base station, Equation (22) is approximated to:

$$\rho_{\theta} = 2\left(\theta_{+} - \theta_{0}\right) \approx \frac{0.64c}{Nf_{0}d} \tag{22}$$

At this time, the pointing angle of the beam $\theta_0 = 0$, the angle resolution of the radar is the best.

6. CONCLUSION

Aiming at the integration of radar and communication, an OFDM-FDA integrated radar and communication waveform based on random frequency offset modulation is proposed. The waveform beam pattern is a spot beam, which can solve the angle and distance coupling problem. Moreover, its frequency offset in FDA can easily separate different channel information. From the analysis of the radar and communication performance, it can be seen that this integrated waveform can achieve multi-target positioning at the same time of communication and save the spectrum and pulse resources of the integrated system. In the actual application of the integrated model, the relevant performance can be adjusted according to the application scenarios.

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