

Global Inductance Computation of a Multilayer Circular Air Coil with a Wire of Rectangular Cross Section: Case of a Uniform Current Distribution

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Abstract—In this paper we present a simple approach to compute quickly and accurately the global inductance of multilayer circular air coils with a wire of rectangular cross section. The case of the uniform current density distribution in the wire cross section is considered. The approach, implemented under GNU Octave, computes the inductance of the multilayer coil in three steps. First, the self-inductance of each coil turn is computed using the Maxwell's formula. Secondly, each wire section is subdivided into several negligible square or rectangular subsections to form a filiform turn, and then the mutual inductances between the turns are computed using Rosa's formula. The last step sums all obtained self-inductances and mutual inductances to deduce the global inductance of the multilayer coil. To verify the efficiency and accuracy of the proposed approach, the obtained equivalent inductance of each turn is compared to the computed one using finite element method implemented in FEMM open source. Furthermore, the global coil inductance is compared to the measured one. The proposed approach shows a good accuracy with a relative error less than 1% for all considered coils.

1. INTRODUCTION

Multilayer air coils with wire of rectangular cross section are used in several fields such as a winding of power transformers, electrical machines, inductors of induction heating systems, and in wireless power transfer. To study the internal behavior and to design these systems, the coil must be modeled. The circuit scheme is the most popular model. It represents the coil by distributed inductive, capacitive and resistive elements with several possible configurations [1–4]. Self and mutual inductances are the principal elements of these models.

The self-inductance of the coil turns with wire of rectangular cross section depends on the turn mean radius and wire cross section value. It can be computed using analytical or numerical method. All authors in [1, 5, 6] used the analytical Maxwell formula [7, 8] based on Geometric Mean Distances (GMD) theory.

Various methods can be used to compute mutual inductance between the turns of a multilayer air coil of rectangular cross section. The most used approaches are based on Maxwell formula [7, 8].

It is very complicated to compute the mutual inductance of a coil with an important turn number using the finite element method. This method can be only used to compute the equivalent or total inductance of each turn and the air coil global inductance.

The principal approaches used to compute the mutual inductance, found in the literature, are:

- Kim et al. [6] present the filamentary method to compute mutual inductance. In this approach, Maxwell's formula is used. The coil is divided into several coaxial filaments of the negligible cross section.

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- Akyel et al. [9] and Babic and Akyel [10, 11] use the elliptic integral-based solution for circular coils of rectangular cross section in air. All expressions are obtained by the complete elliptic integrals of the first and second kinds, and Heuman's lambda function is solved numerically.
- In [12] Ravaud et al. present a 3D semi-analytical expressions based on two numerical integrations to compute the mutual inductance and the force between two coaxial thick coils with rectangular cross-sections.
- Conway [13] presents a straightforward exact method for calculating the mutual inductance of two non-coaxial coils of rectangular cross section, which also gives the inductance of coaxial coils as a limiting case. The self inductance is given by taking two identical coils which coincide in space. The method expresses the inductance as a one-dimensional integral containing the product of Bessel and Struve functions.
- Holloway et al. [14] use a quasistatic Green's function to compute the magnetic fields inside a rectangular conductor. These magnetic field expressions are applied to formulate the internal inductance of the conductor. The numerical results and closed-form expressions are applied to the dc internal inductance.
- Pineda-Sanchez et al. [15] present the proper generalized decomposition (PGD) method applied to compute the DC inductance of a conductor with rectangular cross-section separated using singular value decomposition (SVD). PGD method is also extended to compute AC values of the resistance and internal inductance of a circular and a square conductor. Frequency is introduced in the formulation as an additional dimension.
- Parise et al. [16, 17] present an analytical expression to compute the mutual inductance of two thin-wire single and multi-turn coplanar round coils applied to two coplanar filamentary coil antennas. The Gegenbauer addition theorem and the Maclaurin expansion are used to express the double integral of the linkage flux as a sum of simpler integrals. The analytical solution of the latter allows to compute the mutual inductance.

In this paper, one proposes to use the Rosa's formula [18] in order to compute accurately and quickly the mutual inductance between two coaxial filamentary turns. The self-inductance of the turns of rectangular cross section is calculated using Maxwell's formula [7, 8]. The global inductance of the multilayer coil will be deduced by summing the self and mutual inductances of all turns.

2. SELF AND MUTUAL INDUCTANCES OF A COIL WITH TWO COAXIAL CIRCULAR TURNS OF RECTANGULAR CROSS SECTION

2.1. Self Inductance of One Circular Turn of Rectangular Cross Section

The self inductance L_u of a circular single turn u of rectangular cross section (Figure 1) is given by Maxwell's formula [7, 8]:

$$L_u = \mu R (\log 8R - \log \text{GMD} - 2) \quad (1)$$

where GMD is the Geometry Mean Distance. For rectangular form, GMD is given by the following equation [7, 8]:

$$\log \text{GMD} = \frac{1}{2} \log (a^2 + b^2) - \frac{1}{12} \frac{a^2}{b^2} \log \left(1 + \frac{b^2}{a^2} \right) - \frac{1}{12} \frac{b^2}{a^2} \log \left(1 + \frac{a^2}{b^2} \right) + \frac{2}{3} \frac{a}{b} \arctan \frac{b}{a} + \frac{2}{3} \frac{b}{a} \arctan \frac{a}{b} \quad (2)$$

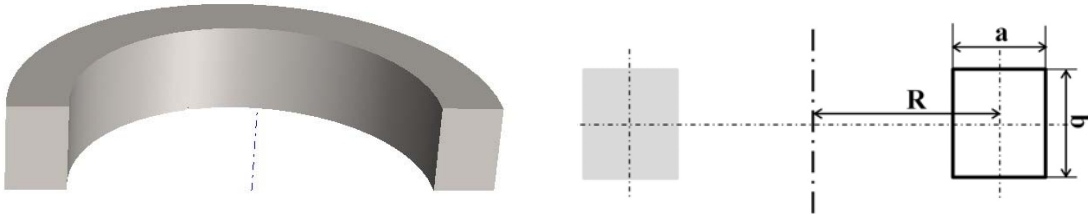


Figure 1. Circular single turn of rectangular cross section.

2.2. Mutual Inductance between Two Circular Coaxial Filaments

All referenced papers use Maxwell’s formula (Eq. (3)) to compute the mutual inductance between filamentary coaxial circular turns (Figure 2).

$$M_{12} = \mu_0 \frac{\sqrt{R_i(R_i + x)}}{k} \left[\left(1 - \frac{k^2}{2}\right) K(k) - E(k) \right] \tag{3}$$

where,

$$k^2 = \frac{4R_i(R_i + x)}{(2R_i + x)^2 + y^2} \tag{4}$$

K and E are the complete elliptic integrals of the second and the first kinds, respectively.

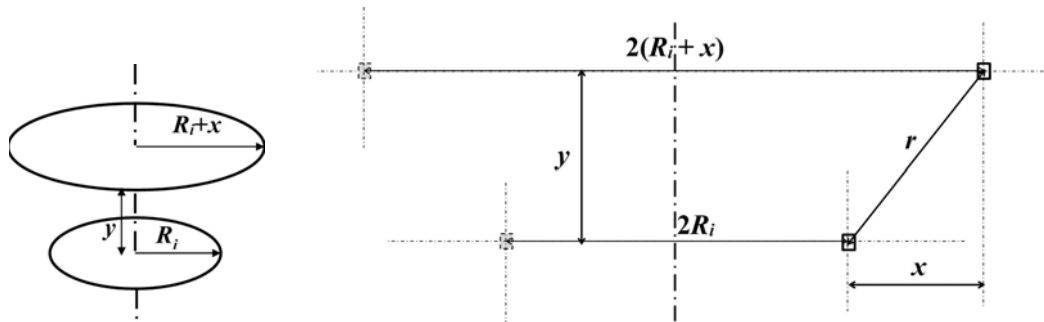


Figure 2. Circular coaxial filament turns.

In this paper, one proposes to use the Rosa’s formula bellow [5, 18] in order to compute accurately and quickly the mutual inductance between two coaxial filament turns (Figure 2):

$$M_{12} = \mu_0 R_i \left[\log \frac{8R_i}{r} \left(1 + \frac{y}{2R_i} + \frac{y^2 + 3x^2}{16R_i^2} - \frac{y^3 + 3yx^2}{32R_i^3} + \frac{17y^4 + 42x^2y^2 - 15x^4}{1024R_i^4} - \frac{19y^5 + 30y^3x^2 - 45yx^4}{2048R_i^5} \right) - 2 - \frac{y}{2R_i} + \frac{3y^2 - x^2}{16R_i^2} - \frac{y^3 - 6yx^2}{48R_i^3} - \frac{19y^4 + 534x^2y^2 - 93x^4}{6144R_i^4} + \frac{379y^5 + 3030y^3x^2 - 1845yx^4}{61440R_i^5} \right] \tag{5}$$

with

$$r = \sqrt{x^2 + y^2} \tag{6}$$

2.3. Mutual Inductance between Two Coaxial Circular Turns of Rectangular Cross Section

To compute the mutual inductance between two coaxial circular turns of rectangular cross section, each turn is subdivided into several filament turns (Figure 3) with negligible rectangular or square section. The computation formula is given by Eq. (7), and the algorithm that allows the calculation of the mutual inductance between the turns u and v which is illustrated by Figure 4. This algorithm and all other procedures used to compute the multilayer coil inductance are implemented in GNU Octave.

$$M_{uv} = \sum_{i=1}^N \sum_{j=1}^P \sum_{k=1}^n \sum_{s=1}^p M_{ik,js} \tag{7}$$

where ik is the subdivision filament of the turn u that consists of $N \times n$ filaments, and js is the subdivision filament of the turn v that consists of $P \times p$ filaments. $i = 1 \dots N$; $j = 1 \dots P$; $k = 1 \dots n$; $s = 1 \dots p$.

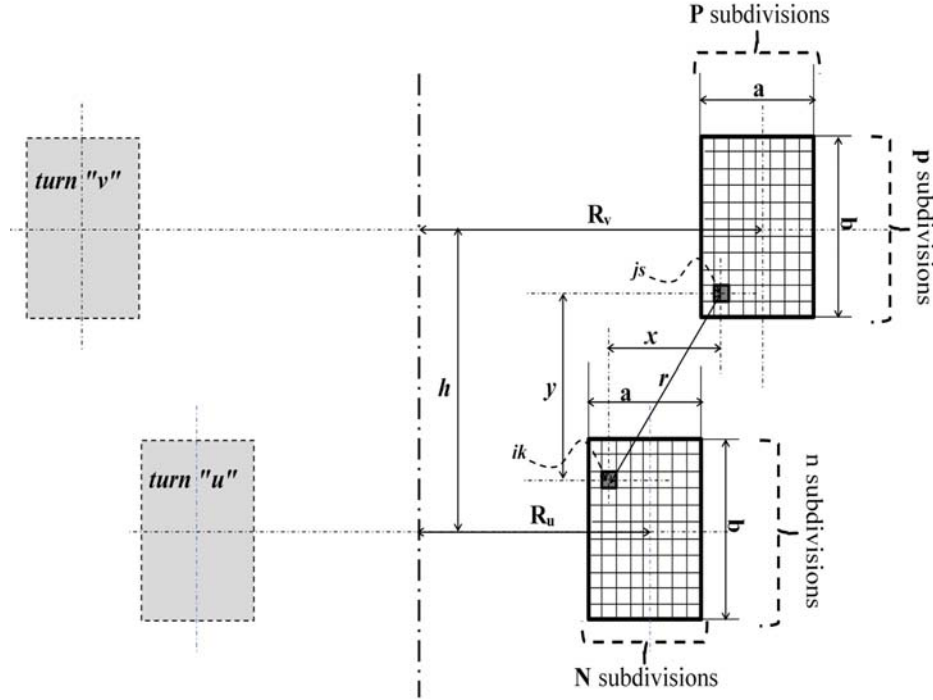


Figure 3. Subdivision of the two circular coaxial turns of rectangular cross section into several elementary turns.

2.4. Inductance of the Coil with Two Circular Turns of Rectangular Cross Section

The total inductance L_t of the coil with turns u and v is given as follows:

$$L_t = L_u + L_v + M_{uv} + M_{vu} = L_u + L_v + 2M_{uv} = L_{t_u} + L_{t_v} \tag{8}$$

where $M_{uv} = M_{vu}$ and L_{t_u}, L_{t_v} are the total inductance of each turn and given as:

$$\begin{cases} L_{t_u} = L_u + M_{uv} \\ L_{t_v} = L_v + M_{uv} \end{cases} \tag{9}$$

3. COMPUTATION OF A MULTILAYER COIL INDUCTANCE

Consider the 240-turn coil given in Figure 5. It is made up of 6 discs with 40 sections per disc.

Either the 240-turn coil given by the Figure 5, that is made up of 6 disks, where each disc has 40 sections.

The computation procedure of the mutual inductance between each turn and the other turns is given by the algorithm illustrated in Figure 6.

The total inductance of each turn is obtained by summing the self-inductance (Eq. (1)) and the mutual inductance between this turn and the others.

3.1. Computation Algorithm

3.2. Finite Element Computation of a Multilayer Coil Inductance

Finite element method (FEM) is a numerical technique applied to solve several engineering problems that arise in different fields. It is recognized as one of the most powerful numerical analysis tools ever devised to analyze complex problems in engineering. In this subsection, the inductance of each turn of the multilayer air coil is computed using the FEM implemented as 2D axisymmetric model in FEMM software [19]. Axisymmetric FEMM geometry and meshes for a 6-discs of 40-sections per disc are shown

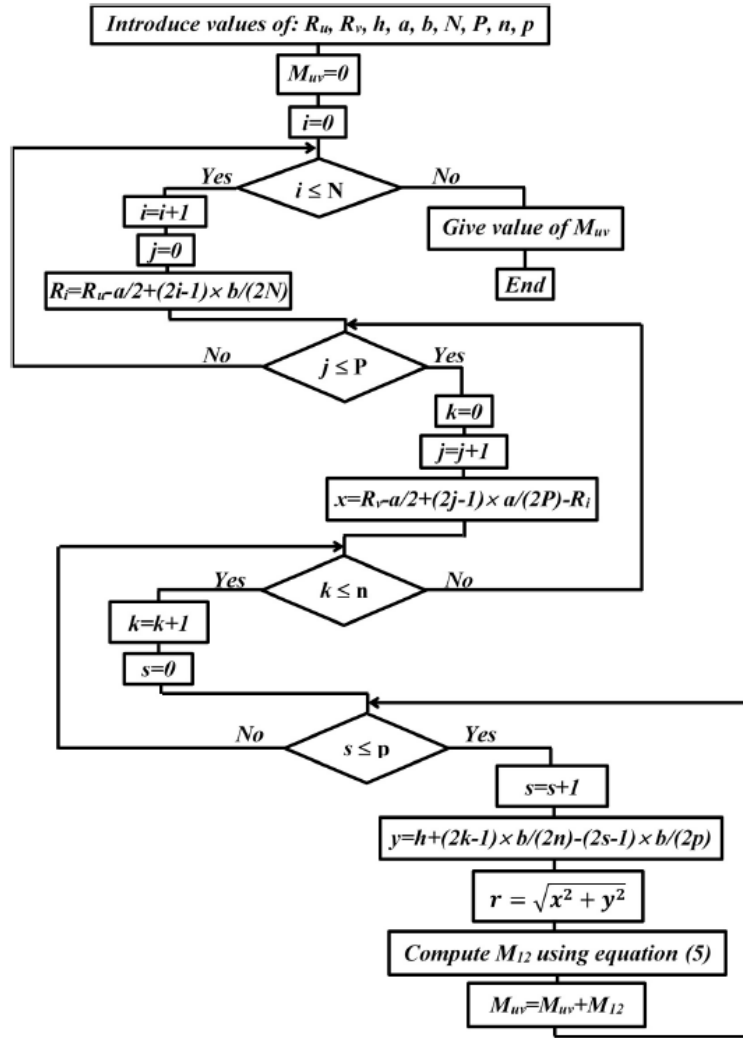


Figure 4. Mutuel inductance computation algorithm of two circular coaxial turns of rectangular cross section.

in Figure 7. The Isovalue lines of the magnetic vector potential and magnetic field density distributions are shown in Figure 8.

3.3. Measurement Setup

To verify the validity of the proposed analytical approach, the measurement of the multilayer coil inductance is done using a precision LCRmeter (Figure 9) at the frequency of 20 Hz. The geometric dimensions of the multilayer coil are given by Table 1.

Table 1. Geometric dimensions of a multilayed coil.

Coil inner radius (R_{in})	Axial distance between two discs (dis_y)	Radial distance between two sections (dis_x)	Wire high (b)	Wire larger (a)
42.2100 mm	0.2872 mm	1.6520 mm	5.0000 mm	1.5000 mm

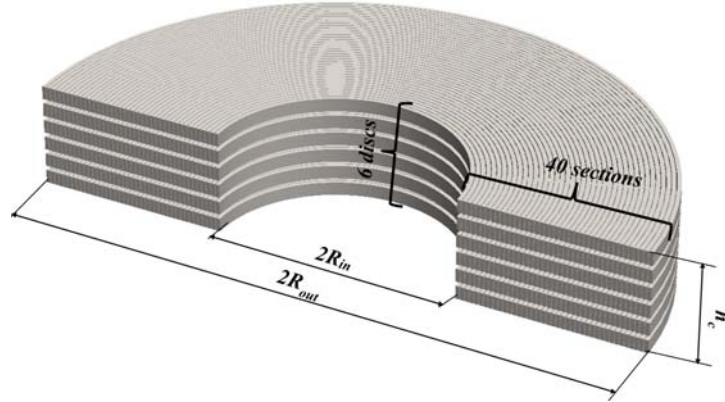


Figure 5. Inductance computation algorithm of a multilayer coil of wire of rectangular cross section.

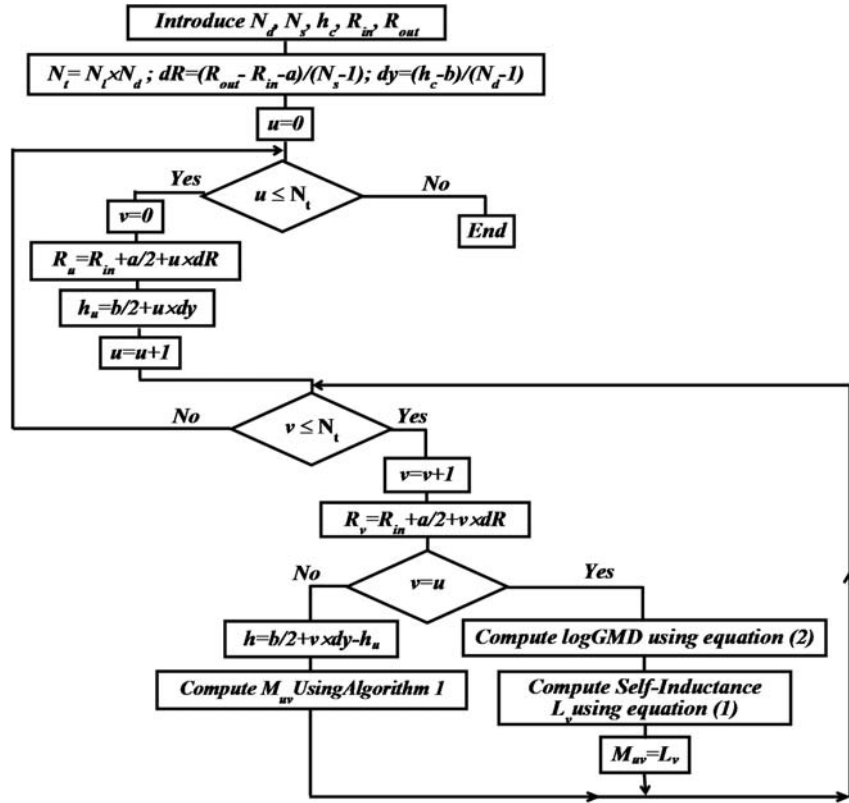


Figure 6. Inductance computation algorithm of a multilayer coil with wire of rectangular cross section.

4. RESULTS AND DISCUSSING

First the proposed analytical method is applied to compute the turn inductances of a 240-turns multilayer air coil with wire of rectangular cross section, shown in Figure 5. The geometry of the coil is implemented and simulated in FEMM software as an axisymmetric magneto-harmonic problem with computing the inductance at frequency of 20 Hz. The global inductance of the coil is obtained for both methods, by summing all turn inductances.

The computed inductances by both analytical and numerical methods are compared to the measured one obtained using a precision LCRmeter. The computations methods are also applied to a multilayer

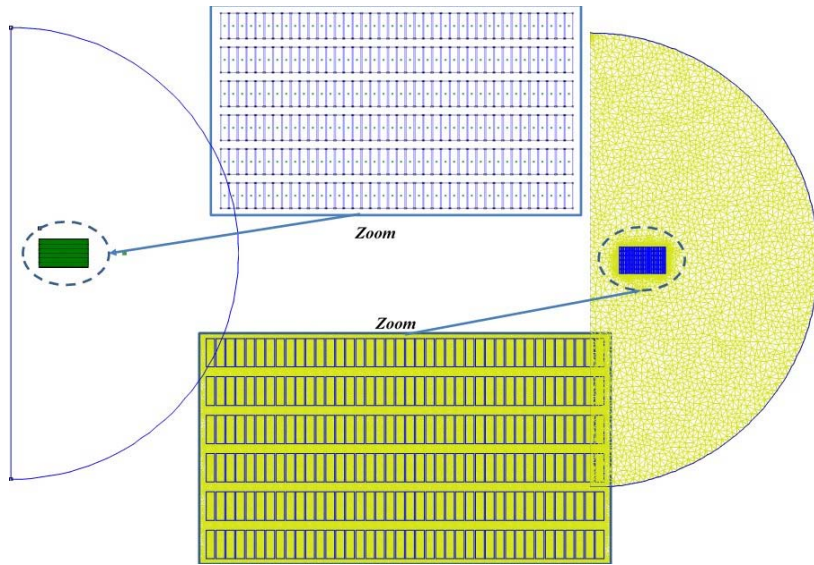


Figure 7. FEM geometry and mesh.

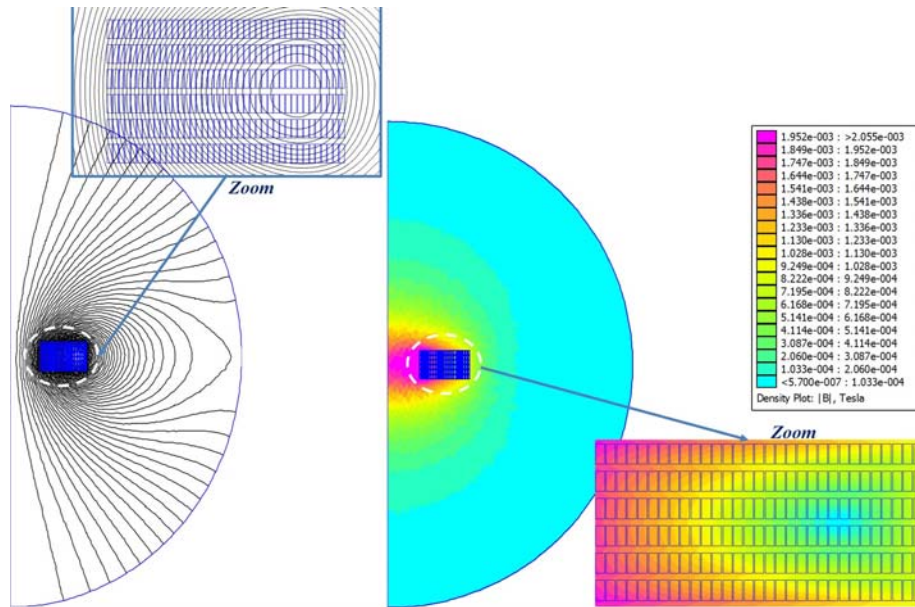


Figure 8. FEMM results — Isovalue Lines of the magnetic vector potential and the distribution of the magnetic field.



Figure 9. Experimental setup.

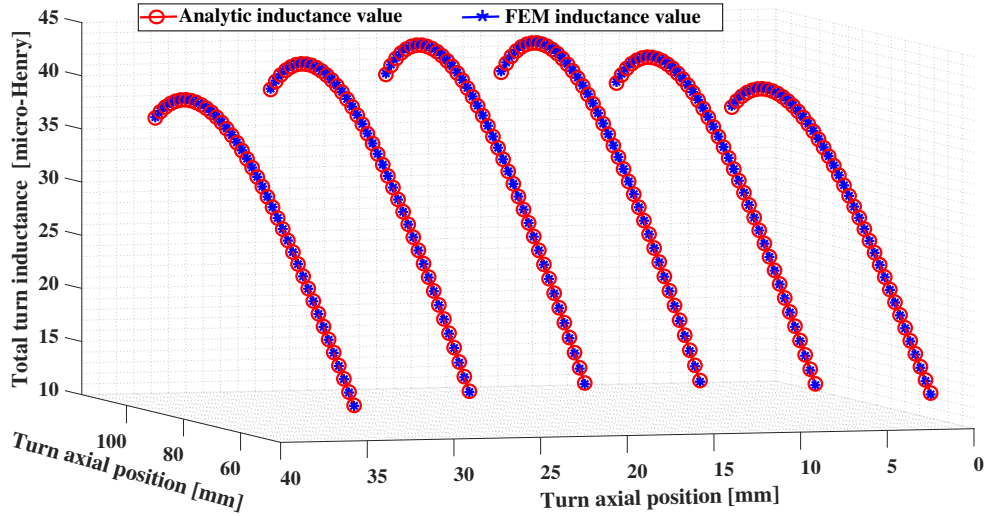


Figure 10. Total turn inductances of a multilayer 6-disc coil according to radial and axial turn positions.

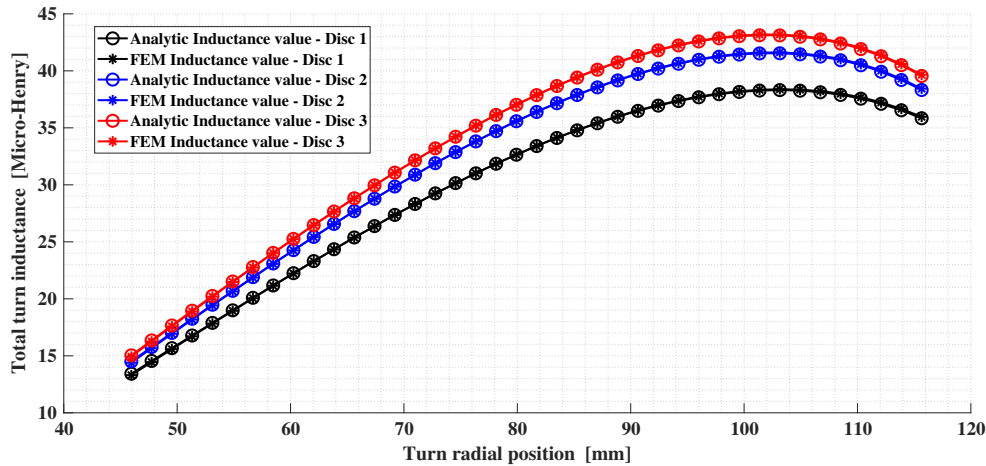


Figure 11. Total turn inductances of first 3 discs of a 6-disc multilayer coil, according to turn radial positions.

coil with two, three, four, and five discs with 40 sections per disc. Figure 10 shows the comparison of the proposed analytical approach to the finite element method. The total inductance values are computed for each multilayer coil turn according to their radial and axial positions. All relative differences between the obtained results using the two methods are less than 1%.

Figure 11 shows the curves of the turn total inductance variation for the first three disks according to the radial position of the turns. It should be noted that the curves of the last three disks are symmetrical to the first ones.

Table 2. Measured and computed inductance (mH) of a multilayer air coil of different disc number.

Coils with different number of discs	6 discs	5 discs	4 discs	3 discs	2 discs	1 disc
Proposed Analytical Method	7.7600	5.6239	3.7574	2.2132	1.0334	0.2726
Finite Element Method (FEMM)	7.7637	5.6179	3.7552	2.2103	1.0312	0.2726
Precision LCRmeter	7.7713	5.6243	3.7528	2.2043	1.0358	0.2761

Table 2 shows a comparison between the total inductances computed using analytical and numerical methods and the measured inductance. The results show a very good concordance for all considered cases.

5. CONCLUSION

In this paper, we have presented an analytical computation approach of inductance. It is applied to a multilayer circular air coil with a wire of rectangular cross section.

The approach first subdivides each wire turn into several filaments with negligible sections, in order to compute accurately the mutual inductance between turns by applying the proposed algorithm where Rosa's formula is used. The self-inductances of the turns are obtained using Maxwell's formula. The air coil global inductance is deduced by summing the mutual and self-inductances of all turns according to the proposed algorithm. For the high frequencies, several phenomena can appear, and one can quote the skin and proximity effects which make the distribution of the current density non-uniform. Mention may also be made of the capacitive effect.

We remind that the proposed approach in this paper, is only valid in the case of a uniform current density distribution in the wire cross section of the multilayer coil. The method will be improved and modified to take into account all the mentioned effects.

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