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Polarimetric Parameters of Scattered Electromagnetic Waves in the Conductive Magnetized Plasma

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Abstract—Electromagnetic waves propagation in both homogeneous and random magnetized conductive plasma is considered including longitudinal, Pedersen and Hall's conductivities. The second-order statistical moments of scattered electromagnetic waves in the conductive turbulent magnetized plasma slab with electron density fluctuations are investigated on the bases of a set of stochastic differential equations. Refractive index and polarization coefficients of both the ordinary and extraordinary waves are calculated for the polar terrestrial ionosphere. Using new spectral method and the boundary conditions, transversal components of scattered electromagnetic waves are calculated. Experimentally observed Stokes parameters describing the depolarization effects are calculated for the arbitrary correlation function of electron density fluctuations. Coherent matrix describing polarization features of non-plane waves generalizing the Stokes parameters is obtained.

1. INTRODUCTION

At the present time the features of electromagnetic waves propagation in random media are well studied [1, 2]. Many articles and reviews are related to the statistical characteristics of scattered radiation and observations in the ionosphere. Randomness in the terrestrial atmosphere mainly is caused by electron density fluctuations having significant influence on key parameters of the wave leading to the depolarization of scattered radiation. Investigation of the statistical moments of small-amplitude electromagnetic waves propagating in the turbulent ionospheric plasma is very important in many practical applications associated with both natural and laboratory plasmas [3, 4].

In most papers isotropic irregularities have been considered. However, irregularities in the ionosphere are anisotropic and mainly elongated along the geomagnetic field. Statistical characteristics of the angular power spectrum (broadening and displacement of its maximum), scintillation effects, and the angle-of-arrival of scattered electromagnetic waves by turbulent anisotropic magnetized ionospheric plasma slab for both power-law and anisotropic Gaussian correlation functions of electron density fluctuations were investigated analytically in the complex geometrical optics approximation and numerically by statistical simulation using the Monte Carlo method [5, 6].

The problem of depolarization of electromagnetic waves in a turbulent magnetized plasma has attracted great attention. It is known that polarized characteristics of a space radio emission are caused by refraction and scattering on both density irregularities of space plasma and magnetic field bearing important information on physical conditions of a source and ionospheric plasma parameters over the path of wave propagation [7]. Depolarization of electromagnetic radiation and the Stokes parameters as a function of distance and one physical parameter characterizing the interstellar plasma in the parabolic approximation has been obtained in [8] for plane wave propagation.

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Linearly polarized wave in the earth's anisotropic turbulent ionosphere generates the ordinary and extraordinary waves traveling with slightly different phase velocities [3], and the geomagnetic field leads to the rotation of the polarization plane. Phase difference is proportional to the rotation angle (Faraday angle) of the polarization plane. Polarization characteristics of scattered radio signals provide important information about physical conditions in the localization of the sources and about the medium parameters on the path of wave propagation. Variances of the metric ordinary and extraordinary waves scattered by the magnetized plasma slab at different orientations of the receiving antennas, variance of the Faraday angle $\langle\theta_F^2\rangle$ and the features of broadening of the spatial spectrum of the scattered radiation in the inhomogeneous magnetized plasma were investigated analytically by the perturbation method [9, 10]. Depolarization effects in nonconductive turbulent plasma was considered in [10, 11].

In the present paper, second order statistical moments of a scattered field in the conductive turbulent magnetized plasma are investigated analytically taking account of Pedersen, Hall's and longitudinal conductivities. In Section 2, the attenuation of both the ordinary and extraordinary waves propagating in the homogeneous conductive magnetized plasma and rotation of the polarization plane — the Faraday angle is considered. Section 3 is devoted to the analytical calculations of both the refractive index and polarization coefficients of a scattered radiation in the polar terrestrial atmosphere using new spectral method [12] satisfying the boundary conditions. Second order statistical moments are obtained for the arbitrary correlation functions of electron density fluctuations. Section 4 is devoted to the analytical calculations of the statistical characteristics of scattered electromagnetic waves in the conductive collision magnetized plasma. Stokes parameters are calculated analytically for arbitrary correlation function of electron density fluctuations in Section 5. Application of the Stokes parameters allows to define polarization characteristics of scattered waves with a big accuracy in inhomogeneous conductive terrestrial atmosphere. The obtained results are valid for near and far zones with respect to plasma slab boundaries. Conclusion is given in Section 6.

2. FORMULATION OF THE PROBLEM

The initial set of the equations is:

$$\mathrm{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{J}_{cond} + \frac{4\pi}{c}eN\mathbf{V}, \quad \mathrm{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad \mathbf{D} = \hat{\varepsilon}\mathbf{E}, \quad \mathbf{J} = \hat{\sigma}\mathbf{E},$$
$$m\frac{d\mathbf{V}}{dt} + m\nu_{eff}\mathbf{V} = e\mathbf{E} + \frac{e}{c}\left[\mathbf{V}\mathbf{H}\right]. \tag{1}$$

where $\hat{\varepsilon}$ and $\hat{\sigma}$ are the permittivity and conductivity second rank tensors of the conductive collision magnetized plasma; c is the speed of light in vacuum; ν_{eff} is the collision frequency between plasma particles.

Current density $\mathbf{J} = \hat{\sigma}\mathbf{E}$ can be rewritten as [3]:

$$J_i = \left[(\sigma_{||} - \sigma_\perp)\tau_i\tau_j - \sigma_H\delta_{ij\,k}\tau_k + \sigma_\perp\delta_{ij}\right] = \sigma_{ij}e_j, \tag{2}$$

here $\boldsymbol{\tau}$ is the unit vector along the external magnetic field $\mathbf{H_0} = H_0\boldsymbol{\tau}$, and the antisymmetric tensor of the third rank $\delta_{ij\,k}$ (indices $i, k, l = 1, 2, 3$) has the following properties: $\delta_{ij\,k}$ is zero, if at least any two indexes are identical; is equal to $+1$, if shift of indexes i, k, l is even; and -1, if shift of indexes is odd. Existence of the magnetic field leads to the anisotropy of the conductivity collision plasma characterizing by: the longitudinal (parallel to the magnetic field) conductivity $\sigma_{||}$, transversal (perpendicular to H) as the Pedersen conductivity $\sigma_\perp$, and the Hall's conductivity σ_H, transversal to the both electric and magnetic fields:

$$\sigma_\perp = e^2N_e\left(\frac{\nu_e}{m_e\left(\nu_e^2+\omega_e^2\right)} + \frac{\nu_i}{m_i\left(\nu_i^2+\omega_i^2\right)}\right), \quad \sigma_H = e^2N_e\left(\frac{\omega_e}{m_e\left(\nu_e^2+\omega_e^2\right)} - \frac{\omega_i}{m_i\left(\nu_{in}^2+\omega_i^2\right)}\right),$$
$$\sigma_{||} = e^2N_e\left(\frac{1}{m_e\nu_e} + \frac{1}{m_m\nu_i}\right), \tag{3}$$

$N_e(\mathbf{r})$ is the electron density which is a random function of the spatial coordinates; e and m_e are the charge and mass of an electron; $\nu_{e,i}$ is the electron or ion collision frequency with the neutral molecules;

ω_e and ω_i are the angular gyro frequencies of an electron and ion, respectively. At high frequencies the influence of ions can be neglected.

Dielectric permittivity tensor of the collision plasma in general case can be written as:

$$\varepsilon_{ij} = \left(1 - \frac{\mathrm{v}g}{g^2 - u}\right)\delta_{ij} + \frac{\mathrm{v}}{g^2 - u}\left(i\sqrt{u}\delta_{ij\,k}\tau_k + \frac{u}{g}\tau_i\tau_j\right). \tag{4}$$

where $g = 1 - is$, $s = \nu_{eff}/\omega$; $v(\mathbf{r}) = \omega_p^2(\mathbf{r})/\omega^2$ and $u = (eH_0/m_e c\omega)^2$ are nondimensional magneto-ionic parameters of the ionospheric plasma; $\omega_p(\mathbf{r}) = [4\pi N_e(\mathbf{r})e^2/m_e]^{1/2}$ is the plasma frequency.

Calculating the velocity $\mathbf{V}$ from the set of Equation (1), we obtain:

$$\mathbf{V} = -\frac{ie}{m\omega}\frac{g}{g^2 - u}\left\{\mathbf{E} - i\frac{\sqrt{u}}{g}[\mathbf{E}\boldsymbol{\tau}] - \frac{u}{g^2}(\mathbf{E}\boldsymbol{\tau})\boldsymbol{\tau}\right\}, \tag{5}$$

Substituting Equation (5) into Equation (1) yields:

$$\nabla \times \nabla \times \mathbf{E}_i - k_0^2 \sum_{i=1}^{3} \hat{\varepsilon}_{ij*}\mathbf{E}_j = 0, \tag{6}$$

here $\hat{\varepsilon}_*\mathbf{E} = \mathbf{E} - \frac{\mathrm{v}g}{g^2-u}\{\mathbf{E} - i\frac{\sqrt{u}}{g}[\mathbf{E}\boldsymbol{\tau}] - \frac{u}{g^2}(\mathbf{E}\boldsymbol{\tau})\boldsymbol{\tau}\} - i\hat{\tilde{\sigma}}\mathbf{E}$; $\hat{\tilde{\sigma}}$ is the second rank conductivity tensor; $\hat{\varepsilon}_* = \hat{\varepsilon} - i\hat{\tilde{\sigma}}$, $\tilde{\sigma} = 4\pi\hat{\sigma}/k_0 c$, $\mathbf{k_0}$ is the wave vector of an incident wave.

3. WAVES PROPAGATION IN THE HOMOGENEOUS CONDUCTIVE MAGNETIZED PLASMA

In this section, we consider wave propagation in the absorptive homogeneous magnetized plasma. Components of the wave vector $\mathbf{k}$ of an oblique incident refractive electromagnetic wave can be written as [13]:

$$k_x = k_0 N \sin\theta \sin\varphi \equiv k_0\tau_1, \quad k_y = k_0 N \sin\theta\cos\varphi \equiv k_0\tau_2, \quad k_z = k_0 N\cos\theta \equiv k_0\tau_3, \quad \tau_1^2 + \tau_2^2 + \tau_3^2 = N^2$$

where θ is an angle between the wave vector $\mathbf{k}$ and Z axis; φ is the angle between the projection of the wave vector $\mathbf{k}$ on the YOZ plane and X-axis of the Cartesian coordinate system. External magnetic field in the polar ionosphere is in the main (YOZ) plane $\mathbf{H_0}||Z$.

Components of the second rank complex permittivity tensor of the conductive collision magnetized plasma are [3]:

$$\tilde{\varepsilon}_{xx} = \tilde{\varepsilon}_{yy} = \tilde{a}_1 - i(sa_1' + \tilde{\sigma}_\perp), \quad \tilde{\varepsilon}_{xy} = -\tilde{\varepsilon}_{yx} = -i(\tilde{a}_2 + \tilde{\sigma}_H) + s\tilde{a}_2 a_2', \quad \tilde{\varepsilon}_{zz} = \tilde{a}_6 - i(sa_6' + \tilde{\sigma}_{||});$$
$$\tilde{\varepsilon}_{xz} = \tilde{\varepsilon}_{zx} = \tilde{\varepsilon}_{yz} = \tilde{\varepsilon}_{zy} = 0. \tag{7}$$

where $\tilde{a}_1 = 1 - p_0$, $\tilde{a}_2 = p_0\sqrt{u}$, $\tilde{a}_6 = 1 - \mathrm{v}$, $a_1' = p_0(1+u)/(1-u)$, $a_2' = 2/(1-u)$, $a_6' = \mathrm{v}$, $p_0 = \mathrm{v}/(1-u)$. Numerical estimations are carried out for 3 MHz incident wave; magneto-ionospheric parameters: $\mathrm{v} = 0.28$, $u = 0.22$).

Solving Equation (6), determinant of the set of equation can be written as:

$$\Delta(x) = (\tilde{\varepsilon}_{xx}\sin^2\theta + \tilde{\varepsilon}_{zz}\cos^2\theta)x^4 - \left[(1+\cos^2\theta)\tilde{\varepsilon}_{xx}\tilde{\varepsilon}_{zz} + \sin^2\theta(\tilde{\varepsilon}_{xx}^2 + \tilde{\varepsilon}_{xy}^2)\right]x^2 + \tilde{\varepsilon}_{zz}(\tilde{\varepsilon}_{xx}^2 + \tilde{\varepsilon}_{xy}^2) = 0, \tag{8}$$

here $x = k_z/k_0$.

In the most interesting case: $s \ll \tilde{\varepsilon}_{ij}, \tilde{\sigma}_{ij}$ and $s^2 \ll (1 - \sqrt{u})^2$ we analyze the wave propagation in the homogeneous conductive magnetized plasma. When the direction of wave propagation coincides with the direction of an external magnetic field $\mathbf{H_0}||Z$ $(\theta = 0)$ — longitudinal propagation, we have:

$$\left(\frac{k_z}{k_0}\right)^2_{||} = [\tilde{a}_1 \pm (\tilde{a}_2 + \tilde{\sigma}_H)] - i\tilde{\sigma}_\perp,$$

at transversal propagation $\mathbf{k}\perp\mathbf{H_0}$ $(\theta = \pi/2)$ we obtain:

$$\left(\frac{k_z}{k_0}\right)^2_{\perp 1} = 2(\tilde{a}_6 - i\tilde{\sigma}_{||}), \quad \left(\frac{k_z}{k_0}\right)^2_{\perp 2} = \frac{2}{\tilde{a}_1}\left\{\left[\tilde{a}_1 - (\tilde{a}_2 + \tilde{\sigma}_H)^2 + \tilde{\sigma}_\perp\right] - i\tilde{a}_1\tilde{\sigma}_\perp\right\}.$$

Upper sign corresponds to the ordinary wave k_{zI} and lower sign to the extraordinary wave k_{zII}, respectively.

For weakly attenuate monochromatic plane wave $E \sim \exp[i(\omega t - \mathbf{kr})]$ ($\mathbf{k} = \mathbf{k}' + i\mathbf{k}''$) attenuation coefficient of the wave amplitude $\alpha \sim 1/k''$. Numerical calculation for an incident wave with frequency 3 MHz shows that at longitudinal propagation $\alpha_I \sim 30\,\text{m}$, $\alpha_{II} \sim 25\,\text{m}$; at transversal propagation the ordinary wave does not attenuate, for the extraordinary wave $\alpha_{II} \sim 20\,\text{m}$. In both cases $k_{zI} > k_{zII}$.

When a linearly polarized electromagnetic wave propagates through a region of magnetized plasma, its plane of polarization will rotate — Faraday rotation [1]. Rotation is clockwise. Faraday angle is equal to:

$$\theta_F = \frac{k_0}{2}\left[\sqrt{1 - \frac{\text{v}}{1+u}} - \sqrt{1 - \frac{\text{v}}{1-u}}\right]. \tag{9}$$

Numerical calculations show that in the considered case $\theta_F \approx 3 \cdot 10^{-3}$. Electromagnetic radiation travelling through a turbulent magnetized plasma becomes less and less polarized. This depolarization has a particular interest in the development of laser light and the possibility of polarization modulation of signals. Now we consider wave propagation through a medium containing random irregularities.

4. REFRACTIVE INDEX AND POLARIZATION COEFFICIENTS OF A SCATTERED RADIATION IN THE POLAR TERRESTRIAL IONOSPHERE

Let us consider the second-order statistical moments of scattered electromagnetic waves in the conductive turbulent magnetized plasma slab with electron density fluctuations. Each of the terms in Equation (6) can be presented as the sum of the mean value and small fluctuating terms, which are random functions of the spatial coordinates: $\mathbf{E} = \langle \mathbf{E} \rangle + \mathbf{e}$, $\mathbf{H_0} = \langle \mathbf{H_0} \rangle$, $N = \langle N \rangle + n$. The angular brackets indicate the statistical average.

Wave vector $\mathbf{k}$ of a refractive plane electromagnetic wave in the absorptive random medium is in the YOZ plane (main plane) of the Cartesian coordinate system, $\mathbf{H_0}||Z$.

In the case, at $s \ll \tilde{\varepsilon}_{ij}, \tilde{\sigma}_{ij}$ and $s^2 \ll (1-\sqrt{u})^2$ from Equation (6), we obtain the refractive index of the conductive turbulent magnetized plasma:

$$\tilde{N}_j^2 = 1 - \frac{2}{\tilde{\Delta}_1^2 + \tilde{\Delta}_2^2}(\tilde{\Phi}_{1j} - i\tilde{\Phi}_{2j}) = 1 - (\tilde{N}_{1j}^2 + i\tilde{N}_{2j}^2), \tag{10}$$

where index j is devoted to the ordinary and extraordinary waves:

$$\begin{aligned}
\tilde{\Phi}_1 &= \tilde{B}_1\tilde{\Delta}_1 - \tilde{B}_2\tilde{\Delta}_2, \quad \tilde{\Phi}_2 = \tilde{B}_1\tilde{\Delta}_2 + \tilde{B}_2\tilde{\Delta}_1, \quad \tilde{B}_1 = \tilde{\Lambda}_0 - \tilde{\Lambda}_2 + \tilde{\Lambda}_4, \quad \tilde{B}_2 = \tilde{\Lambda}_1 - \tilde{\Lambda}_3 + \tilde{\Lambda}_5,\\
\tilde{\Delta}_1 &= 2\tilde{\Lambda}_0 - \tilde{\Lambda}_2 \pm \tilde{G}_1, \quad \tilde{\Delta}_2 = -2\tilde{\Lambda}_1 + \tilde{\Lambda}_3 \pm \tilde{G}_2, \quad \tilde{G}_1 = \frac{1}{\sqrt{2}}\left\{\left[\tilde{D}_1^2 + D_1'^2\right]^{1/2} + \tilde{D}_1\right\}^{1/2},\\
\tilde{G}_2 &= \tilde{G}_1(\tilde{D}_1 \to -\tilde{D}_1),\\
\tilde{D}_1 &= \tilde{\Lambda}_2^2 - \tilde{\Lambda}_3^2 - 4(\tilde{\Lambda}_0\tilde{\Lambda}_4 - \tilde{\Lambda}_1\tilde{\Lambda}_5), \quad \tilde{D'}_1 = 2\left[-\tilde{\Lambda}_2\tilde{\Lambda}_3 + 2(\tilde{\Lambda}_0\tilde{\Lambda}_5 + \tilde{\Lambda}_1\tilde{\Lambda}_4)\right], \quad \tilde{\Lambda}_0 = \tilde{a}_1\sin^2\theta + \tilde{a}_6\cos^2\theta,\\
\tilde{\Lambda}_2 &= \left[\tilde{a}_1\tilde{a}_6(1+\cos^2\theta) + (\tilde{a}_1^2 - \tilde{a}_2^2)\sin^2\theta\right] - \left[\tilde{\sigma}_\perp\tilde{\sigma}_{||}(1+\cos^2\theta) + (\tilde{\sigma}_\perp^2 + \tilde{\sigma}_H^2 + 2\tilde{a}_2\tilde{\sigma}_H)\sin^2\theta\right],\\
\tilde{\Lambda}_3 &= (\tilde{a}_1\tilde{\sigma}_{||} + \tilde{a}_6\tilde{\sigma}_\perp)(1+\cos^2\theta) + 2\tilde{a}_1\tilde{\sigma}_\perp\sin^2\theta, \quad \tilde{\Lambda}_4 = \tilde{a}_6\left[(\tilde{a}_1^2 - \tilde{a}_2^2) - \tilde{a}_6(\tilde{\sigma}_\perp^2 + \tilde{\sigma}_H^2 + 2\tilde{a}_2\tilde{\sigma}_H)\right],
\end{aligned}$$

$\tilde{\Lambda}_5 = \tilde{\sigma}_{||}\left[\tilde{a}_1^2 - \tilde{\sigma}_\perp^2 - (\tilde{a}_2^2 + \tilde{\sigma}_H)^2\right] + 2\tilde{a}_1\tilde{a}_6\tilde{\sigma}_\perp$, $\tilde{\Lambda}_1 = \tilde{\sigma}_\perp\sin^2\theta + \tilde{\sigma}_{||}\cos^2\theta$, signs "+" and "−" refer to the ordinary and extraordinary waves, respectively; θ is an angle between the wave vector of a refracted wave $\mathbf{k}$ ($\mathbf{k} \in YOZ$) and the external magnetic field $\mathbf{B_0}||Z$.

For the collisionless plasma, $s = 0$ and $\tilde{\sigma}_{ij} = 0$, we obtain the well-known result [3].

If $\mathbf{k_0}||Z$ and $\mathbf{B_0} \in YOZ$, at $s \ll \tilde{\varepsilon}_{ij}, \tilde{\sigma}_{ij}$ polarization coefficients of the conductive magnetized plasma are:

$$\text{P}_{1,2} = \frac{\langle E_y\rangle}{\langle E_x\rangle} = \frac{\left\{\left[(p_0 - N_1^2)\tilde{a}_4 - \tilde{a}_2\tilde{a}_3\right] - (q_1 + q_3)\right\} - iq_2}{\left[(q_6 - q_4) - i(q_7 - q_5)\right] + i\left[\tilde{a}_3(1 - \tilde{a}_5 - N_1^2) - \tilde{a}_2\tilde{a}_4\right]} \equiv P'_{1,2} - iP''_{1,2}, \tag{11}$$

where $q_1 = (N_2^2 - \tilde{\sigma}_\perp)(\tilde{\sigma}_{||} - \tilde{\sigma}_\perp)\sin\theta\cos\theta$, $q_2 = (p_0 - N_1^2)(\tilde{\sigma}_{||} - \tilde{\sigma}_\perp)\sin\theta\cos\theta + \tilde{a}_4(N_2^2 - \tilde{\sigma}_\perp)$, $q_3 = \tilde{\sigma}_H(\tilde{a}_2\sin\theta + \tilde{a}_3\cos\theta) + \tilde{\sigma}_H^2\sin\theta\cos\theta$, $q_4 = (\tilde{a}_2 + \tilde{\sigma}_H\cos\theta)(\tilde{\sigma}_{||} - \tilde{\sigma}_\perp)\sin\theta\cos\theta$, $q_5 = \tilde{a}_4\tilde{\sigma}_H\cos\theta$, $q_6 = (\tilde{a}_3 + \tilde{\sigma}_H\sin\theta)\left[N_2^2 - (\tilde{\sigma}_\perp\cos^2\theta + \tilde{\sigma}_{||}\sin^2\theta)\right]$, $q_7 = \tilde{\sigma}_H\sin\theta(1 - \tilde{a}_5 - N_1^2)$, $\tilde{a}_2 = p_0\sqrt{u_L}$, $\tilde{a}_3 = p_0\sqrt{u_T}$, $\tilde{a}_4 = p_0\sqrt{u_L u_T}$, $\tilde{a}_5 = 1 - p_0(1 - u_T)$, $u_T = u\sin^2\theta$, $u_L = u\cos^2\theta$.

$$G_{1,2} = \frac{\langle E_z\rangle}{\langle E_x\rangle} = G'_{1,2} + iG''_{1,2}, \tag{12}$$
$$G'_{1,2} = -(\Psi_1 + \Psi_3 P'_{1,2} + \Psi_4 P''_{1,2}), \quad G''_{1,2} = \Psi_2 + \Psi_3 P''_{1,2} - \Psi_4 P'_{1,2},$$

where $\Psi_1 = \frac{(\tilde{a}_3 + \tilde{\sigma}_H\sin\theta)\Omega_0}{\Delta_1}$, $\Psi_2 = \frac{\tilde{a}_6(\tilde{a}_3 + \tilde{\sigma}_H\sin\theta)}{\Delta_1}$, $\Psi_3 = \frac{\tilde{a}_4\tilde{a}_6 + \Omega_0\Omega_1}{\Delta_1}$, $\Omega_0 = \tilde{\sigma}_{||}\cos^2\theta + \tilde{\sigma}_\perp\sin^2\theta$, $\Omega_1 = (\tilde{\sigma}_{||} - \tilde{\sigma}_\perp)\sin\theta\cos\theta$, $\Delta_1 = \tilde{a}_6^2(\tilde{\sigma}_{||}\cos^2\theta + \tilde{\sigma}_\perp\sin^2\theta)^2$, $\Psi_4 = \frac{\tilde{a}_4\Omega_0 - \tilde{a}_6\Omega_1}{\Delta_1}$, $\tilde{a}_6 = 1 - p_0(1 - u_L)$. If $\theta = 0^0$ i.e., $\mathbf{k_0}||\mathbf{B_0}||Z$ we have: $\mathrm{P}_{1,2} = \pm i$, $G_{1,2} = 0$. Polarization coefficients at $\mathbf{k_0} \in YOZ$ and $\mathbf{B_0} \in YOZ$ were calculated in.

5. STATISTICAL CHARACTERISTICS OF SCATTERED ELECTROMAGNETIC WAVES IN THE CONDUCTIVE COLLISION MAGNETIZED PLASMA

Applying the perturbation method submits all terms as the sum of a constant mean and fluctuating terms:

$$\mathbf{E}(\mathbf{r}) = \langle\mathbf{E}\rangle + \mathbf{e}(\mathbf{r}), \quad N(\mathbf{r}) = N_0 + n_1(\mathbf{r}), \quad \hat{\varepsilon}(\mathbf{r}) = \hat{\tilde{\varepsilon}} + \hat{\varepsilon}' n_1(\mathbf{r}), \quad \hat{\tilde{\sigma}}(\mathbf{r}) = \hat{\tilde{\sigma}}\left[N_0 + n_1(\mathbf{r})\right]. \tag{13}$$

Second terms are random functions of the spatial coordinates.

Differential equation of the fluctuating scattered field in the collision conductive magnetized plasma can be written as:

$$\mathrm{grad\ div}\,\mathbf{e} - \Delta\mathbf{e} - k_0^2\left[\left(1 - \frac{\mathrm{v}g}{g^2 - u} - i\hat{\tilde{\sigma}}\right)\mathbf{e} + \frac{\mathrm{v}}{g^2 - u}\left\{i\sqrt{u}\left[\mathbf{e}\boldsymbol{\tau}\right] + \frac{u}{g}(\mathbf{e}\boldsymbol{\tau})\boldsymbol{\tau}\right\}\right] = \mathbf{j}, \tag{14}$$

Current density is proportional to the electron density fluctuations:

$$\mathbf{j} = k_0^2\left[\left(\frac{\mathrm{v}g}{g^2 - u} + i\hat{\tilde{\sigma}}\right)\langle\mathbf{E}\rangle - \frac{\mathrm{v}}{g^2 - u}\left\{i\sqrt{u}\left[\langle\mathbf{E}\rangle\boldsymbol{\tau}\right] + \frac{u}{g}(\langle\mathbf{E}\rangle\boldsymbol{\tau})\boldsymbol{\tau}\right\}\right] n_1. \tag{15}$$

Current density is proportional to the electron density fluctuations.

Fourier transformations of a scattered field and current density are:

$$\mathbf{e}(\mathbf{r}) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \tilde{\mathbf{e}}(k_x, k_y, z)\exp[i(k_x x + k_y y)],$$
$$\mathbf{j}(\mathbf{r}) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \mathbf{g}(k_x, k_y, z)\exp[i(k_x x + k_y y)]. \tag{16}$$

If the time dependence is $\exp(i\omega t)$, fluctuating scattered field satisfies the set of stochastic differential equations:

$$\tilde{e}_x'' + a_1\tilde{e}_z' + b_1\tilde{e}_x + c_1\tilde{e}_y = f_1, \quad \tilde{e}_y'' + a_2\tilde{e}_z' + b_2\tilde{e}_x + c_2\tilde{e}_y = f_2, \quad \tilde{e}_x' + a_3\tilde{e}_y' + d_3\tilde{e}_z = f_3, \tag{17}$$

where $a_1 = -ik_x$, $b_1 = (k_0^2\tilde{a}_1 - k_y^2) - ik_0^2\tilde{\sigma}_\perp$, $c_1 = k_x k_y + ik_0^2(\tilde{a}_2 + \tilde{\sigma}_H)$, $a_2 = -ik_y$, $a_3 = k_y/k_x$, $b_2 = k_x k_y - ik_0^2(\tilde{a}_2 + \tilde{\sigma}_H)$, $c_2 = (k_0^2\tilde{a}_1 - k_x^2) - ik_0^2\tilde{\sigma}_\perp$, $d_3 = [k_0^2\tilde{\sigma}_{||} - i(k_x^2 + k_y^2 - k_0^2\tilde{a}_6)]/k_x$, $f_1 = -k_0^2 g_x$, $f_2 = -k_0^2 g_y$, $f_3 = -i(k_0^2/k_x)g_z$.

Derivatives of the Fourier spectral functions $\tilde{\mathbf{e}}_i$ in the set of Equation (16) are carried out with respect to the coordinate z. Let XOY-plane coincide with the lower boundary of a slab. The boundary conditions are: at $z \geq L$ (L is a thickness of an inhomogeneous plasma slab) waves propagating in negative direction must be absent, at $z \leq 0$ — in the positive direction.

We solve the set of Equation (16) using the spectral method [12]

$$\begin{bmatrix} \tilde{e}_x(z) \\ \tilde{e}_y(z) \\ \tilde{e}_z(z) \end{bmatrix} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt \begin{bmatrix} A(t) \\ B(t) \\ C(t) \end{bmatrix} \exp[-i(L-z)t],$$

$$\begin{bmatrix} f_1(z) \\ f_2(z) \\ f_3(z) \end{bmatrix} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt \begin{bmatrix} F_1(z) \\ F_2(z) \\ F_3(z) \end{bmatrix} \exp[-i(L-z)t]. \tag{18}$$

The set of stochastic differential Equation (17) can be transformed into the set of algebraic equations:

$$\alpha_1(t)A(t) + c_1B(t) + \alpha_2(t)C(t) = F_1(t), \quad b_2A(t) + \beta_1(t)B(t) + \beta_2(t)C(t) = F_2(t),$$
$$\gamma_1(t)A(t) + \gamma_2(t)B(t) + d_3C(t) = F_3(t), \tag{19}$$

where:

$$A(t) = \frac{1}{\Delta(t)}\left\{[d_3\beta_1(t) - \beta_2(t)\gamma_2(t)]F_1(t) + [\alpha_2(t)\gamma_2(t) - c_1d_3]F_2(t) + \frac{i}{k_x}[c_1\beta_2(t) - \beta_1(t)\alpha_2(t)]F_3(t)\right\},$$

$$B(t) = \frac{1}{\Delta(t)}\left\{[\beta_2(t)\gamma_1(t) - b_2d_3]\,F_1(t) + [d_3\alpha_1(t) - \alpha_2(t)\gamma_1(t)]\,F_2(t) + [b_2\alpha_2(t) - \alpha_1(t)\beta_2(t)]\,F_3(t)\right\},$$

$$C(t) = \frac{1}{\Delta(t)}\left\{[b_2\gamma_2(t) - \gamma_1(t)\beta_1(t)]\,F_1(t) + [c_1\gamma_1(t) - \alpha_1(t)\gamma_2(t)]\,F_2(t) + [\alpha_1(t)\beta_1(t) - b_2c_1]\,F_3(t)\right\},$$

$$\alpha_1(t) = b_1 - t^2, \quad \alpha_2(t) = ia_1t, \quad \beta_1(t) = c_2 - t^2, \quad \beta_2(t) = ia_2t, \quad \gamma_1(t) = it, \quad \gamma_2(t) = ia_3t.$$

Determinant set of Equation (19) is:

$$\Delta(x) = i\frac{k_0^5}{\gamma_x}\left[(\tilde{a}_6 - i\tilde{\sigma}_{||})x^4 + (h_0 + ih_1)x^2 - (h_2 + ih_3)\right] = 0. \tag{20}$$

here:

$$h_0 = 2(\tilde{\sigma}_{||}\tilde{\sigma}_{\perp} - \tilde{a}_1\tilde{a}_6) + (\tilde{a}_1 + \tilde{a}_6)\gamma^2, \quad h_1 = 2(\tilde{\sigma}_{||}\tilde{a}_1 + \tilde{\sigma}_{\perp}\tilde{a}_6) - (\tilde{\sigma}_{||} + \tilde{\sigma}_{\perp})\gamma^2, \quad \gamma^2 = \gamma_x^2 + \gamma_y^2,$$
$$\gamma_x = k_x/k_0, \quad h_3 = \left\{\tilde{\sigma}_{||}\left[\tilde{a}_1^2 - \tilde{\sigma}_{\perp}^2 - (\tilde{a}_2 + \tilde{\sigma}_H)^2\right] + 2\tilde{\sigma}_{\perp}\tilde{a}_1\tilde{a}_6\right\} - \left[\tilde{\sigma}_{||}\tilde{a}_2 + \tilde{\sigma}_{\perp}(2\tilde{a}_2 + \tilde{a}_6)\right]\gamma^2,$$
$$\gamma_y = k_y/k_0, \quad h_2 = \left\{2\tilde{a}_1\tilde{\sigma}_{||}\tilde{\sigma}_{\perp} - \tilde{a}_6\left[\tilde{a}_1^2 - \tilde{\sigma}_{\perp}^2 - (\tilde{a}_2 + \tilde{\sigma}_H)^2\right]\right\} - \left[\tilde{\sigma}_{||}\tilde{\sigma}_{\perp} - \tilde{a}_1^2 + \tilde{\sigma}_{\perp}^2 + (\tilde{a}_2 + \tilde{\sigma}_H)^2 - \tilde{a}_1\tilde{a}_6\right]\gamma^2,$$
$$x = k_z/k_0.$$

Applying the Cauchy and residue theory to Equation (19), Fourier spectral function of a scattered field for X-component can be written as:

$$e_x(\boldsymbol{\kappa}, L) = \frac{ik_0\langle E_x\rangle}{\tilde{a}_6^2 + \tilde{\sigma}_{||}^2}\sum_{i=1}^{4}\frac{1}{\delta_i}\mathrm{T}_0(x_i)\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\exp\left[-i(L-z')k_0x_i\right], \tag{21}$$

where $\mathrm{T}_0(x_i) = (\tilde{a}_6 + i\tilde{\sigma}_{||})\sum_{i=1}^{4}\Upsilon_i(x_i)(J_i' + iJ_i'')$, components of the current density contain polarization coefficients and at $s \ll \varepsilon_{ij}, \tilde{\sigma}_{ij}$ can be written as:

$$\sum_{i=1}^{4}\frac{\varepsilon_{||} + i\tilde{\sigma}_{||}}{\delta_i}(J' + iJ_i'') = \sum_{i=1}^{4}\left[(E_i + iE_i')\Upsilon_i(x_1) - (Q_i + iQ_i')\Upsilon_i(x_2)\right], \tag{22}$$

where:

$$J_x' + iJ_x'' = \left[p_0 - (\tilde{\sigma}_H - \tilde{a}_2)\mathrm{P}_{1,2}''\right] + i\left[(\tilde{\sigma}_H - \tilde{a}_2)\mathrm{P}_{1,2}' - \tilde{\sigma}_{\perp}\right]$$
$$J_y' + iJ_y'' = (p_0\mathrm{P}_{1,2}' + \tilde{\sigma}_{\perp}\mathrm{P}_{1,2}'') + i\left[(\tilde{a}_2 - \tilde{\sigma}_H)\mathrm{P}_{1,2}' + p_0\mathrm{P}_{1,2}''\right]$$
$$J_z' + iJ_z'' = \left[p_0(1-u) + G_{1,2}''\tilde{\sigma}_{||}\right] + i\left[p_0(1-u)G_{1,2}'' - G_1'\tilde{\sigma}_{||}\right],$$
$$E_i = \tilde{\delta}'_1J_i' - \tilde{\delta}_1''J_i'', \quad E_i' = \tilde{\delta}'_1J_i'' + \tilde{\delta}_1''J_i', \quad Q_i = \tilde{\delta}'_2J_i' - \tilde{\delta}_2''J_i'', \quad Q_i' = \tilde{\delta}'_2J_i'' + \tilde{\delta}_2''J_i',$$
$$\delta_1' = 2(\Omega_3\Lambda_1 - \Omega_4\Lambda_2),$$
$$\delta_1'' = 2(\Omega_3\Lambda_2 + \Omega_4\Lambda_1), \quad \delta_2' = 2(\Omega_5\Lambda_1 - \Omega_6\Lambda_2), \quad \delta_2'' = 2(\Omega_5\Lambda_2 + \Omega_6\Lambda_1),$$
$$\Lambda_1 = (\Omega_3^2 - \Omega_5^2) - (\Omega_4^2 - \Omega_6^2),$$

$\Lambda_2 = 2(\Omega_3\Omega_4 - \Omega_5\Omega_6)$; $\Omega_3 \ldots \Omega_5$ are the roots of the dispersion Equation (20).

$\Upsilon_1(x_1) = (\mu_0 - i\mu_0') + (\mu_1 + i\mu_1')\gamma_x^2 + (\mu_2 + i\mu_2')\gamma_y^2, \quad \Upsilon_2(x_1) = \nu_0 + (\nu_1 + i\nu_1')\gamma_x\gamma_y - i\nu_2'\gamma^2,$

$\Upsilon_3(x_1) = (\zeta_1 - i\zeta_1')\gamma_x - (\zeta_2 + i\zeta_2')\gamma_y; \quad \mu_0 = \tilde{a}_6(a_1 - \tilde{a}_1), \quad \mu_0' = \sigma_{||}(a_1 - \tilde{a}_1), \quad \mu_1' = \tilde{a}_6 g_2 - \sigma_{||}(1 - a_1 g_1),$

$\mu_1 = \tilde{a}_6(1 - a_1 g_1) + \tilde{a}_1 - a_1 + \sigma_{||} g_2, \quad \mu_2 = \tilde{a}_1 - \varepsilon_{||} a_1 g_1 + \sigma_{||} g_2, \quad \mu_2' = \tilde{a}_6 g_2 - \sigma_\perp + \sigma_{||} a_1 g_1;$

$\nu_0 = \sigma_{||}(\tilde{a}_2 + \sigma_H), \quad \nu_1 = \tilde{a}_6 - a_1, \quad \nu_1' = \sigma_\perp - \sigma_{||} - (\tilde{a}_2 + \sigma_H)\tilde{a}_6, \quad \nu_2' = (\tilde{a}_2 + \sigma_H)\gamma^2;$

$\zeta_1 = (\varepsilon_\perp - a_1)\Omega_3, \quad \zeta_1' = (\varepsilon_\perp - a_1)\Omega_4, \quad \zeta_2 = (æ + \sigma_H)\Omega_3, \quad \zeta_2' = (æ + \sigma_H)\Omega_4,$

$g_1 = \dfrac{c_1 a_1 + \sigma_\perp c_2}{2a_1(\tilde{a}_6^2 + \tilde{\sigma}_{||}^2)}, \quad g_2 = \dfrac{\sigma_\perp c_1 - a_1 c_2}{2(\tilde{a}_6^2 + \tilde{\sigma}_{||}^2)}, \quad c_1 = \dfrac{\alpha_1 a_1 - \alpha_2\tilde{\sigma}_\perp}{a_1^2 + \tilde{\sigma}_\perp^2}, \quad c_2 = \dfrac{\alpha_1\tilde{\sigma}_\perp + \alpha_2 a_1}{a_1^2 + \tilde{\sigma}_\perp^2},$

$\alpha_1 = \tilde{a}_6(\tilde{a}_6 + \tilde{a}_1 + \tilde{a}_2 + \tilde{\sigma}_H),$

$a_1 = \tilde{a}_1 + \tilde{a}_2 + \tilde{\sigma}_H$, $\alpha_2 = \tilde{\sigma}_{||}(\tilde{a}_1 + \tilde{a}_2 + \tilde{\sigma}_H) - \tilde{a}_6\tilde{\sigma}_\perp$. Parameters $\Upsilon_1(x_2)$ can be easily obtained having replaced $a_1 \to a_2$, $g_1 \to g_3$, $g_2 \to g_4$, $\Omega_3 \to \Omega_5$, $\Omega_4 \to \Omega_6$. As a result, we obtain

$$\begin{aligned} e_x(\boldsymbol{\kappa}, L) = & -\frac{2k_0\langle E_x\rangle}{\tilde{a}_6^2 + \tilde{\sigma}_{||}^2}\Bigg\{V_1\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\sin\left[(L - z')k_0 x_1\right] + \left[(V_2 + V_2') - iV_1'\right] \\ & \cdot\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\cos\left[(L - z')k_0 x_1\right] - V_3\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\sin\left[(L - z')k_0 x_2\right] - \left[(V_4 + V_4') - iV_3'\right] \\ & \cdot\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\cos\left[(L - z')k_0 x_2\right]\Bigg\}, \end{aligned} \tag{23}$$

where:

$$\begin{aligned} V_1 &= E_1(\mu_0 + \mu_1\gamma_x^2 + \mu_2\gamma_y^2) + E_1'(\mu_0' - \mu_1'\gamma_x^2 - \mu_2'\gamma_y^2) + E_2(\nu_0 + \nu_1\gamma_x\gamma_y) - E_2'(\nu_1'\gamma_x\gamma_y - \nu_2'\gamma^2), \\ V_1' &= E_3(\zeta_1\gamma_x - \zeta_2\gamma_y) - E_3'(\zeta_1'\gamma_x + \zeta_2'\gamma_y); \\ V_2 &= -E_1(\mu_0' - \mu_1'\gamma_x^2 - \mu_2'\gamma_y^2) + E_1'(\mu_0 + \mu_1\gamma_x^2 + \mu_2\gamma_y^2) + E_2(\nu_1'\gamma_x\gamma_y - \nu_2'\gamma^2) + E_2'(\nu_0 + \nu_1\gamma_x\gamma_y), \\ V_2' &= -E_3(\zeta_1'\gamma_x + \zeta_2'\gamma_y) + E_3'(\zeta_1\gamma_x - \zeta_2\gamma_y); \\ V_3 &= Q_1(\mu_3 + \mu_4\gamma_x^2 + \mu_5\gamma_y^2) + Q_1'(\mu_3' - \mu_4'\gamma_x^2 - \mu_5'\gamma_y^2) + Q_2(\nu_3 + \nu_4\gamma_x\gamma_y) + Q_2'(-\nu_4'\gamma_x\gamma_y + \nu_5'\gamma^2), \\ Q_3' &= Q_3(\zeta_3\gamma_x - \zeta_4\gamma_y) + Q_3'(\zeta_3'\gamma_x + \zeta_4'\gamma_y), \\ V_4 &= Q_1(-\mu_3' + \mu_4'\gamma_x^2 + \mu_5'\gamma_y^2) + Q_1'(\mu_3 + \mu_4\gamma_x^2 + \mu_5\gamma_y^2) + Q_2(\nu_4'\gamma_x\gamma_y - \nu_5'\gamma^2) + Q_2'(\nu_3 + \nu_4\gamma_x\gamma_y), \\ V_4' &= -Q_3(\zeta_3'\gamma_x + \zeta_4'\gamma_y) + Q_3'(\zeta_3\gamma_x - \zeta_4\gamma_y); \end{aligned}$$

Similar calculations can be carried out for the next component of scattered electromagnetic wave:

$$\begin{aligned} e_y(\boldsymbol{\kappa}, L) = & -\frac{2k_0\langle E_x\rangle}{\tilde{a}_6^2 + \tilde{\sigma}_{||}^2}\Bigg\{Z_1\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\sin\left[(L - z')k_0 x_1\right] + \left[(Z_2 + Z_2') - iZ_1'\right] \\ & \cdot\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\cos\left[(L - z')k_0 x_1\right] - Z_3\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\sin\left[(L - z')k_0 x_2\right] - \left[(Z_4 + Z_4') - iZ_3'\right] \\ & \cdot\int_0^L dz' n_1(\boldsymbol{\kappa}, z')\cos\left[(L - z')k_0 x_2\right]\Bigg\}, \end{aligned} \tag{24}$$

where:

$$\begin{aligned} Z_1 &= E_1(\eta_0 + \eta_1\gamma_x\gamma_y) - E_1'(\eta_0' + \eta_1'\gamma_x\gamma_y - \eta_2'\gamma^2) + E_2(\rho_0 + \rho_1\gamma_x^2 + \rho_2\gamma_y^2) + -E_2'(\rho_0' + \rho_1'\gamma_x^2 + \rho_2'\gamma_y^2), \\ Z_1' &= E_3(-\omega_0\gamma_x + \omega_1\gamma_y) - E_3'(-\omega_0'\gamma_x + \omega_1'\gamma_y), \\ Z_2 &= E_1(\eta_0' + \eta_1'\gamma_x\gamma_y - \eta_2'\gamma^2) + E_1'(\eta_0 + \eta_1\gamma_x\gamma_y) + E_2(\rho_0' + \rho_1'\gamma_x^2 + \rho_2'\gamma_y^2) + E_2'(\rho_0 + \rho_1\gamma_x^2 + \rho_2\gamma_y^2), \end{aligned}$$

$$
\begin{aligned}
Z_3 &= Q_1(\eta_3+\eta_4\gamma_x\gamma_y)+Q'_1(\eta'_3+\eta'_4\gamma_x\gamma_y-\eta'_5\gamma^2)+Q_2(\rho_3+\rho_4\gamma_x^2+\rho_5\gamma_y^2)+Q'_2(\rho'_3+\rho'_4\gamma_x^2+\rho'_5\gamma_y^2),\\
Z'_3 &= Q_3(-\omega_2\gamma_x+\omega_3\gamma_y)-Q'_3(-\omega'_2\gamma_x+\omega'_3\gamma_y),\\
Z_4 &= Q_1(\eta'_3+\eta'_4\gamma_x\gamma_y-\eta'_5\gamma^2)+Q'_1(\eta_3+\eta_4\gamma_x^2)+Q_2(\rho'_3+\rho'_4\gamma_x^2+\rho'_5\gamma_y^2)+P'_2(\rho_3+\rho_4\gamma_x^2+\rho_5\gamma_y^2),\\
Z'_4 &= Q_3(-\omega'_2\gamma_x+\omega'_3\gamma_y)+Q'_3(-\omega_2\gamma_x+\omega_3\gamma_y),\quad Z'_2=E_3(-\omega'_0\gamma_x+\omega'_1\gamma_y)+E'_3(-\omega\gamma_x+\omega_1\gamma_y).
\end{aligned}
$$

6. THE STOKES PARAMETERS

The field at the antenna exit with isotropic polarization is split on two orthogonal components, and at the exit of the device all four Stokes parameters are allocated.

Knowledge of the correlation functions of scattered radiation in the conductive magnetized plasma fields allows to calculate the Stokes parameters:

$$I=\langle e_x e_x^*\rangle+\langle e_y e_y^*\rangle,\quad Q=\langle e_x e_x^*\rangle-\langle e_y e_y^*\rangle,\quad U=2\mathrm{Re}(\langle e_x e_y^*\rangle),\quad V=2\mathrm{Im}(\langle e_x e_y^*\rangle). \tag{25}$$

here e_x and e_y are orthogonal components of a scattered field. Stokes I is the total intensity of the wave. Stokes Q and U are measures of the linear polarization of the wave, and Stokes V is a measure of the circular polarization of the wave. The first Stokes parameter is invariant with respect to the choice of the orthogonal basis while other parameters depend on such choice. In general, the set of these parameters describes elliptically polarized wave. Inclination angle of the main axis ϑ and ellipticity degree E of the polarized ellipse are determined by formulae:

$$\vartheta=\frac{1}{2}\mathrm{arctg}\frac{U}{Q},\quad E=tg\left[\frac{1}{2}\arcsin\frac{V}{(Q^2+U^2+V^2)^{1/2}}\right]. \tag{26}$$

Depolarization degree is the ratio of the unpolarized energy to the polarized energy

$$d=\frac{I-(Q^2+U^2+V^2)^{1/2}}{I}.$$

For a completely polarized wave, $I^2=Q^2+U^2+V^2$ and polarization fluctuations are absent. Note that depolarization effect and fluctuations of the angle-of-arrival are in the same order.

Still the following three measures are more interesting than the Stokes parameters. The total polarization P, the degree of ellipticity E, and the degree of linear polarization L are defined by:

$$P=\frac{(Q^2+U^2+V^2)^{1/2}}{I},\quad E=\frac{V}{(Q^2+U^2+V^2)^{1/2}},\quad L=\frac{(Q^2+U^2)^{1/2}}{(Q^2+U^2+V^2)^{1/2}}. \tag{27}$$

These parameters are of interest and have wide application because they are invariant under Lorentz transformation, which is not true of the Stokes parameters.

Electromagnetic wave with fluctuating polarization can be represented as: $\langle E_x\rangle=A_x\exp(i\varphi_x)$ and $\langle E_y\rangle=A_y\exp(i\varphi_y)$ in a general case are random functions of the spatial coordinate and time. Sometimes, i is convenient, instead of orientation angle ϑ and the ratio of the polarization ellipse axis, to consider module M and the phase φ_0 of the polarization coefficient P determined as:

$$P=\frac{\langle E_y\rangle}{\langle E_x\rangle}=p\exp(i\varphi_0), \tag{28}$$

where $p=A_y/A_x$, $\varphi_0=\varphi_y-\varphi_x$. These parameters can be expressed via the Stokes parameters:

$$\varphi_0=\mathrm{arctg}\frac{V}{U},\quad p=\left(\frac{I_0+Q}{I_0-Q}\right)^{1/2}, \tag{29}$$

where $I_0=(Q^2+U^2+V^2)^{1/2}$.

The Stokes parameters are related with the correlation of an interferometer. The response of an interferometer is a linear combination of two Stokes parameters. Observing different combinations of polarizations, all the Stokes parameters can be determined and the complex state of the polarization of the wave found. Parameter I is always positive (ignoring noise, errors), whereas other parameters:

Q, U and V may be positive or negatively depending on the polarization position angle, or sense of rotation.

The knowledge of spectral functions of a scattered field in the conductive collision magnetized plasma allows to calculate Stokes parameters:

$$\langle e_x(x+\rho_x, y+\rho_y, L)e_x^*(x,y,L)\rangle = \frac{2k_0^2 L\langle E_x\rangle^2}{(\tilde{a}_6^2+\sigma_{||}^2)^2}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \exp(ik_x\rho_x + ik_y\rho_y)\int_{-\infty}^{\infty} d\rho_z W_n(k_x,k_y,\rho_z)$$

$$\left\{-V_1^2\left[\frac{\sin(2k_0Lx_1)}{2k_0Lx_1} - \cos(k_0\rho_z x_1)\right] + \left[V_1'^2 + (V_2+V_2')^2\right]\left[\frac{\sin(2k_0Lx_1)}{2k_0Lx_1} + \cos(k_0\rho_z x_1)\right] + \right.$$
$$-V_3^2\left[\frac{\sin(2k_0Lx_2)}{2k_0Lx_2} - \cos(k_0\rho_z x_2)\right] + \left[V_3'^2 + (V_4+V_4')^2\right]\left[\frac{\sin(2k_0Lx_2)}{2k_0Lx_2} + \cos(k_0\rho_z x_2)\right]$$
$$-2V_1(V_2+V_2')\left[\frac{\cos(2k_0Lx_1)}{2k_0Lx_1} + \sin(k_0\rho_z x_1)\right] - 2V_3(V_4+V_4')\left[\frac{\cos(2k_0Lx_2)}{2k_0Lx_2} + \sin(k_0\rho_z x_2)\right]$$
$$+2\left[V_1V_3 - V_1'V_3' - (V_2+V_2')(V_4+V_4')\right]\left[b_0\sin\left(\frac{t}{2}k_0\rho_z\right) + b_1\cos\left(\frac{t}{2}k_0\rho_z\right) + b_2\sin\left(\frac{y}{2}k_0\rho_z\right)\right.$$
$$\left.+b_3\cos\left(\frac{y}{2}k_0\rho_z\right)\right] - 2\left[V_1(V_4+V_4') + V_3(V_2+V_2')\right]\left[b_0\cos\left(\frac{t}{2}k_0\rho_z\right) - b_1\sin\left(\frac{t}{2}k_0\rho_z\right)\right]$$
$$\left.-2\left[V_1(V_4+V_4') - V_3(V_2+V_2')\right]\left[b_2\cos\left(\frac{y}{2}k_0\rho_z\right) - b_3\sin\left(\frac{y}{2}k_0\rho_z\right)\right]\right\}. \tag{30}$$

where: $b_0 = \frac{1-\cos(k_0Ly)}{k_0Ly}$, $b_1 = \frac{\sin(k_0Ly)}{k_0Ly}$, $b_2 = \frac{1-\cos(k_0Lt)}{k_0Lt}$, $b_3 = \frac{\sin(k_0Lt)}{k_0Lt}$;

$$\langle e_y(x+\rho_x, y+\rho_y, L)e_y^*(x,y,L)\rangle = \frac{2k_0^2 L\langle E_x\rangle^2}{(\tilde{a}_6^2+\sigma_{||}^2)^2}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \exp(ik_x\rho_x + ik_y\rho_y)\int_{-\infty}^{\infty} d\rho_z W_n(k_x,k_y,\rho_z)$$

$$\left\{-Z_1^2\left[\frac{\sin(2k_0Lx_1)}{2k_0Lx_1} - \cos(k_0\rho_z x_1)\right] + \left[Z_1^2 + (Z_2+Z_2')^2\right]\left[\frac{\sin(2k_0Lx_1)}{2k_0Lx_1} + \cos(k_0\rho_z x_1)\right]\right.$$
$$+\left[Z_3'^2 + (Z_4+Z_4')^2\right]\left[\frac{\sin(2k_0Lx_2)}{2k_0Lx_2} + \cos(k_0\rho_z x_2)\right] - Z_3^2\left[\frac{\sin(2k_0Lx_2)}{2k_0Lx_2} - \cos(k_0\rho_z x_2)\right]$$
$$-2Z_1(Z_2+Z_2')\left[\frac{\cos(2k_0Lx_1)}{2k_0Lx_1} + \sin(k_0\rho_z x_1)\right] - 2Z_3(Z_4+Z_4')\left[\frac{\cos(2k_0Lx_2)}{2k_0Lx_2} + \sin(k_0\rho_z x_2)\right]$$
$$+2\left[Z_1Z_3 - Z_1'Z_3' - (Z_2+Z_2')(Z_4+Z_4')\right]\left[b_0\sin\left(\frac{t}{2}k_0\rho_z\right) + b_1\cos\left(\frac{t}{2}k_0\rho_z\right) + b_2\sin\left(\frac{y}{2}k_0\rho_z\right)\right.$$
$$\left.+b_3\cos\left(\frac{y}{2}k_0\rho_z\right)\right] - 2\left[Z_1(Z_4+Z_4') + Z_3(Z_2+Z_2')\right]\left[b_0\cos\left(\frac{t}{2}k_0\rho_z\right) - b_1\sin\left(\frac{t}{2}k_0\rho_z\right)\right]$$
$$\left.-2\left[Z_1(Z_4+Z_4') - Z_3(Z_2+Z_2')\right]\left[b_2\cos\left(\frac{y}{2}k_0\rho_z\right) - b_3\sin\left(\frac{y}{2}k_0\rho_z\right)\right]\right\}. \tag{31}$$

$$\mathrm{Re}\langle e_x(x+\rho_x, y+\rho_y, L)e_y^*(x,y,L)\rangle = \frac{2k_0^2 L\langle E_x\rangle^2}{(\tilde{a}_6^2+\sigma_{||}^2)^2}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \exp(ik_x\rho_x + ik_y\rho_y)$$

$$\cdot\int_{-\infty}^{\infty} d\rho_z W_n(k_x,k_y,\rho_z)\left\{(Q_1 - V_1Z_1)\frac{\sin(2k_0Lx_1)}{2k_0Lx_1} + (Q_1+V_1Z_1)\cos(k_0\rho_z x_1) + (Q_2+V_3Z_3)\frac{\sin(2k_0Lx_2)}{2k_0Lx_2}\right.$$

$$+(Q_2+V_3Z_3)\cos(k_0\rho_z x_2)-Q_3\left[\frac{\cos(2k_0Lx_1)}{2k_0Lx_1}+\sin(k_0\rho_z x_1)\right]-Q_8\left[\frac{\cos(2k_0Lx_2)}{2k_0Lx_2}+\sin(k_0\rho_z x_2)\right]$$
$$-\left[(Q_4+Q_7)b_0+(Q_6-Q_5)b_1\right]\cos\left(\frac{t}{2}k_0\rho_z\right)+\left[(Q_7-Q_4)b_2+(Q_5-Q_6)b_3\right]\cos\left(\frac{y}{2}k_0\rho_z\right)$$
$$+\left[(Q_5-Q_6)b_2+(Q_4-Q_7)b_3\right]\sin\left(\frac{y}{2}k_0\rho_z\right)\Big\}, \tag{32}$$

where:

$$Q_1=(V_2+V_2')(Z_2+Z_2')+V_1'Z_1',\quad Q_2=(V_4+V_4')(Z_4+Z_4')+V_3'Z_3',$$
$$Q_3=V_1(Z_2+Z_2')+Z_1(V_2+V_2'),\quad Q_4=V_1(Z_4+Z_4')+Z_1(V_4+V_4'),\quad Q_5=V_1Z_3+Z_1V_3,$$
$$Q_6=(V_2+V_2')(Z_4+Z_4')+V_1'Z_3'+(V_4+V_4')(Z_2+Z_2')+Z_1'V_3',$$
$$Q_7=V_3(Z_2+Z_2')+Z_3(V_2+V_2'),\quad Q_8=V_3(Z_4+Z_4')+Z_3(V_4+V_4')$$

$$\mathrm{Im}\langle e_x(x+\rho_x,y+\rho_y,L)e_y^*(x,y,L)\rangle=\frac{2k_0^2L\langle E_x\rangle^2}{(\tilde{a}_6^2+\sigma_{||}^2)^2}\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\exp(ik_x\rho_x+ik_y\rho_y)$$
$$\cdot\int_{-\infty}^{\infty}d\rho_z W_n(k_x,k_y,\rho_z)\Big\{Q_1'\left[\frac{\sin(2k_0Lx_1)}{2k_0Lx_1}+\cos(k_0\rho_z x_1)\right]+Q_2'\left[\frac{\sin(2k_0Lx_2)}{2k_0Lx_2}+\cos(k_0\rho_z x_2)\right]$$
$$-Q_3'\left[\frac{\cos(2k_0Lx_1)}{2k_0Lx_1}+\sin(k_0\rho_z x_1)\right]+\left[(Q_4'+Q_7')b_0+Q_6'b_1\right]\cos\left(\frac{t}{2}k_0\rho_z\right)$$
$$+\left[(Q_4'+Q_7')b_0+Q_6'b_1\right]\cos\left(\frac{t}{2}k_0\rho_z\right)+\left[Q_6'b_0-(Q_4'+Q_7')b_1\right]\sin\left(\frac{t}{2}k_0\rho_z\right)$$
$$+\left[(Q_4'-Q_7')b_2+Q_6'b_3\right]\cos\left(\frac{y}{2}k_0\rho_z\right)+\left[(Q_7'-Q_4')b_3+Q_6'b_2\right]\sin\left(\frac{y}{2}k_0\rho_z\right)\Big\}, \tag{33}$$

where:

$$Q_1' = (V_2+V_2')Z_1'-(Z_2+Z_2')V_1',\quad Q_2'=(V_4+V_4')Z_3'-(Z_4+Z_4')V_3',$$
$$Q_3' = V_1Z_1'-Z_1V_1',\quad Q_4'=Z_1V_3'-V_1Z_3',\quad Q_7'=Z_3V_1'-V_3Z_1',\quad Q_8'=V_3Z_3'-Z_3V_3',$$
$$Q_6' = V_1'(Z_4+Z_4')+Z_3'(V_2+V_2')+V_3'(Z_2+Z_2')-Z_1'(V_4+V_4').$$

The analysis of non-plane waves needs the generalization of Stokes parameters analyzing polarization phenomena. For the covariant description of polarization features of plane waves, it is convenient to use the coherence matrix

$$M_2=\left\|\begin{matrix}\langle e_xe_x^*\rangle & \langle e_xe_y^*\rangle\\ \langle e_ye_x^*\rangle & \langle e_ye_y^*\rangle\end{matrix}\right\|.$$

Unlike a plane wave where there are only two transversal components of an electric field e_x and e_y, in a general case of a non-plane wave the longitudinal component e_z of a scattered field should be taken into account. In this case instead of a square coherent matrix describing a plane wave, we will have:

$$M_3=\left\|\begin{matrix}\langle e_xe_x^*\rangle & \langle e_xe_y^*\rangle & \langle e_xe_z^*\rangle\\ \langle e_ye_x^*\rangle & \langle e_ye_y^*\rangle & \langle e_ye_y^*\rangle\\ \langle e_ze_x^*\rangle & \langle e_ze_y^*\rangle & \langle e_ze_z^*\rangle\end{matrix}\right\|.$$

They are generalized Stokes parameters. These Stokes parameters in general case are independent and form a full system. They will have different amplitudes and phases. Longitudinal component e_z is allocated. Generally, a full description of the polarization characteristics of quasi-monochromatic non-plane wave needs experimental measurements of all nine Stokes parameters. Unlike plane waves, where polarization measurements are very simple, in this case, the problem becomes more complicated. Particularly in measurements of a low-frequency band, the approximation of a plane wave becomes unacceptable. In this case, addition information of a source can be obtained in comparison with the polarization measurements for a plane wave.

7. CONCLUSION

Electromagnetic waves propagation in both homogeneous and turbulent conductive magnetized plasma is considered including longitudinal, Pedersen and Hall's conductivities. Attenuation coefficients of the wave amplitude are calculated at waves propagation along and perpendicular directions with respect to the external magnetic field. Rotation of the polarization plane and the Faraday angle is calculated for both ordinary and extraordinary waves. The second-order statistical moments of scattered electromagnetic waves in the conductive turbulent magnetized plasma slab with electron density fluctuations are investigated on the bases of the stochastic wave equation in the polar terrestrial ionosphere (the external magnetic field is directed vertically upward). Refractive index and polarization coefficients of the ordinary and extraordinary waves are calculated containing magnetoionic parameters and the angle between the wave vector of an incident wave and external magnetic field. The set of stochastic differential equations of a scattered field is obtained. Solving these equations using a new spectral method satisfying the boundary conditions, transversal (with respect to the external magnetic field) components of scattered electromagnetic waves are calculated. Experimentally observed Stokes parameters describing the depolarization effects are valid for the arbitrary correlation function of electron density fluctuations. Coherent matrix is presented for the description of polarization features of low-frequency non-plane waves generalizing the Stokes parameters of a plane wave.

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