

BER Analysis in Non-Homogeneous Fading Environments with Impulsive Noise

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Abstract—In this paper, using binary phase-shift keying (BPSK) modulation, analytical expressions of bit-error-rate (BER) for various non-homogeneous fading environments (α - μ , η - μ and κ - μ) subjected to SaS noise are obtained. The derived results are expressed in terms of Meijer's G-function and Gamma function. These expressions are used to study the performance of other prominent fading models (like Nakagami- m , Rayleigh, Rician, and Hoyt) available in the technical literature. Further, it is shown that the effect of the impulsive index (α) over BER is much pronounced compared to the amount of fading (AF). Numerical results are provided for different impulsive settings. The results obtained agree with those from simulations.

1. INTRODUCTION

Fading and noise are two key phenomena which describe the statistical behavior of a wireless channel. Due to the presence of scatters within the propagation medium, the transmitted signal's characteristics get altered. Such a phenomenon, known as fading, severely affects the reception process [1]. The noise which may result from man-made or natural phenomenon also affects the signal reception [2]. Owing to simple analysis of additive white Gaussian noise, it has been extensively used to model noise in a wireless communication link [3]. However, there are various communications scenarios where the effect of non-Gaussian noise cannot be overlooked [4]. These include industrial environments [5], power line communication (PLC) [6], and underwater communication setups [7]. Hence, by considering the non-Gaussian nature of noise instead of classical Gaussian noise, the practical limits of a communication link can be analyzed. Several non-Gaussian noise models have been reported in the literature [8]. One of the important classes of non-Gaussian noise is the symmetric alpha-stable (SaS) noise [9–11]. It is a more general representation of additive white Gaussian noise (AWGN) since it includes Gaussian noise as a special case and satisfies generalized central limit theorem (GCLT) [9]. It is used to model atmospheric noise [12], acoustic noise [13], and noise in industrial wireless sensor network (IWSW) [14]. Moreover, the dynamic interference generated in a multi-user network, where different users are scattered in a spatial Poisson field, can be modeled by SaS distribution [15]. Hence, it can be concluded that SaS noise is a more general and practical representation of noise. To the best of our knowledge, there appears no publication which deals with the performance analysis of α - μ , η - μ or κ - μ fading environments with SaS noise. In this work, an attempt has been made to address this research gap. The motivation for selecting α - μ , η - μ or κ - μ fading models are (a) The two-parameter fading models give better fit of the signal variation as compared to a fading model with a single parameter. (b) Such fading distributions represent small-scale fading effects effectively. (c) Also, these fading distributions correspond to some well-know fading models as special cases. These include Nakagami- m , Rayleigh, Rician, Hoyt and Weibull fading models [16].

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In a practical wireless scenario, both fading and non-Gaussian noise may affect the communication link simultaneously and independently. For instance, in an industrial setup, the multi-path propagation due to highly reflective metal surfaces can be captured by small-scale fading models, while non-Gaussian noise models can effectively represent the excessive electromagnetic noise caused by the industrial equipment (like large motors). Hence, by incorporating the appropriate fading and non-Gaussian noise models, a practical insight of a typical communication link can be acquired.

The rest of this paper is structured as follows: Section 2 is devoted to BER performance over different non-Gaussian noise models available in the technical literature. Section 3 deals with statistics of S α S noise under consideration. The system model and performance analysis of BER over α - μ , η - μ , and κ - μ fading environments with S α S noise are given in Sections 4 and 5, respectively. Section 6 is dedicated to numerical results and analysis. This paper is concluded in Section 7.

2. BIT ERROR RATE (BER) ANALYSIS OVER DIFFERENT NON-GAUSSIAN NOISE MODELS

In [17], using BPSK, the impact of additive white generalized Laplacian noise (AWGLN) and generalized-K (GK) fading on the BER is evaluated. It is observed that there is a sharp increase in BER, as the non-Gaussianity parameter (v) approaches 0. In [18], BER performance of coherent modulation schemes subjected to generalized fading and generalized Gaussian noise (GGN) for different network topologies is evaluated. In [19], using MQAM modulation, the symbol-error-probability (SEP) of an information-bearing signal subjected to Gaussian and Middleton class A noise is evaluated. The effect of two critical parameters of Middleton class A noise, namely impulsive index ($A \leq 1$) and the ratio of Gaussian to impulsive noise power (Γ) is analyzed. For a fixed value of A , as Γ increase, the corresponding SEP approaches the SEP of Gaussian noise. On the other hand for a fixed value of Γ , as impulsive index A decreases, there is a significant increase in SEP. In [20] it is shown that the BER performance of a communication link affected by Rayleigh fading and Middleton Class A noise is improved by post-detection Combining (PDC) technique rather than maximal-ratio Combining (MRC), which works effectively in Gaussian noise. In [21], it is shown that BER can be further reduced by incorporating error control codes (ECCs) like (7, 4) Hamming code and (23, 12) Golay code over a channel subjected to Rician fading and Middleton Class A noise. In [22], using the MQAM modulation scheme, the BER performance of a communication link under the joint influence of Gaussian noise, gated noise, and Nakagami- m fading is evaluated. Two types of gated noise are considered: simple-gated and double-gated. For SNR greater than 10 dB, the BER of simple-gated noise is more than double-gated noise. This increase in BER occurs because simple-gated noise along with background Gaussian noise acts for a considerable time interval. Double-gated noise is added to the background Gaussian noise only when both the gating pulses are unitary. In [23], it is shown that by incorporating spatial diversity in a system affected by double-gated noise and η - μ fading, the BER can be reduced significantly. In [24], using MQAM modulation, bit-error-probability (BEP) is calculated for a communication channel subjected to S α S noise and Rayleigh fading. The S α S noise is characterized by the impulsive index (α), where $\alpha \in (0, 2]$. As α approaches 0, there is a significant increase in BEP. In [25], it is shown that the increase in BEP due to S α S noise can be reduced using different diversity techniques. It is shown that BEP obtained using selection combining (SC) is less than BEP obtained using MRC and equal-gain combining (EGC) for a strong impulsive channel. Recently Markov model was used to model impulsive noise. In [26], Markov chains is used to represent the bursty nature of noise since pulses in bursty noise are time-correlated while the Middleton model assumes that noise samples are independent and identically distributed (i.i.d) and it deals only with amplitude or envelope statistics. The number of states in a Markov model determines the dimension of a transition matrix. Hence, to decide the number of states which accurately model the bursty noise behavior is challenging. In [27], a two-state Markov chain model is used to study the effect of non-Gaussian noise and fading on BER performance. It takes into account gated additive white Gaussian noise (GAWGN) with η - μ fading as one state and double-gated additive white Gaussian noise (G²AWGN) with η - μ fading as another state. It is shown that BER increases with the increase in the constellation order (M). Against this background on the BER performance, we find that there is no work dealing with the BER performance of α - μ , η - μ and κ - μ fading models with S α S noise. In the next section, we discuss the statistics of S α S noise.

3. SYMMETRIC ALPHA-STABLE (S α S) NOISE

The probability density function (PDF) of S α S noise can be expressed as

$$f_{\alpha}(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(j\mu_{\alpha}t - \rho^{\alpha}|t|^{\alpha}) \exp(-jtz) dt, \tag{1}$$

The PDF of S α S noise is characterized by four parameters: $\alpha, \beta, \rho, \mu_{\alpha}$. $\alpha \in (0, 2]$ is the impulsive index which governs the impulsive nature of S α S noise. The impulsive behavior of S α S noise increase significantly as α approaches 0. Such behavior is shown in Figure 1. $\beta \in [0, 1]$ determines the skewness of S α S distribution. $\rho \in \mathcal{R}^+$ determines the spread, while $\mu_{\alpha} \in \mathcal{R}^+$ is associated with the position or location of S α S distribution. Equation (5), corresponds to Gaussian and Cauchy distribution for $\alpha = 2$ and $\alpha = 1$, respectively.

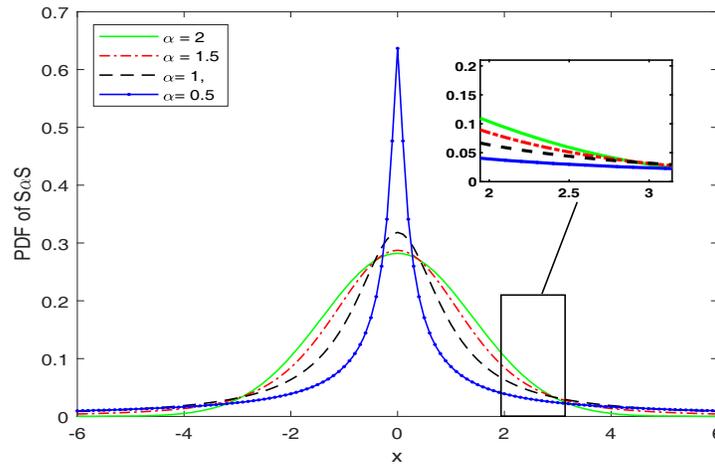


Figure 1. PDF of S α S noise.

3.1. Geometrical Signal-to-Noise Ratio

With the exception of Gaussian case ($\alpha = 2$), the variance of S α S noise is infinite, and thus the conventional definition of SNR cannot be used. The moments ($E\{|X|^p\}$) of S α S distribution exist only for $0 < p < \alpha$ [9, property 1.2.16]. Hence, geometric signal-to-noise ratio (SNR_g) is employed instead of conventional signal-to-noise ratio (SNR). Geometric power (S_0) for S α S noise is defined as [28]

$$S_0 = \frac{(C_g)^{1/\alpha} \rho}{C_g}. \tag{2}$$

The SNR_g is defined as

$$SNR_g = \frac{1}{2C_g} \left(\frac{A}{S_0} \right)^2, \tag{3}$$

where A is the amplitude of the signal and $C_g \approx 1.78$. The normalization constant ($1/2C_g$) maps S α S noise to Gaussian noise for $\alpha = 2$. For BPSK, $\frac{E_b}{N_o}$ can be expressed as

$$\frac{E_b}{N_o} = \frac{SNR_g}{2R_c}, \tag{4}$$

where R_c is the code rate. For uncoded BPSK system ($R_c = 1$) and $A = \pm 1$, Eq. (4) can be written as

$$\frac{E_b}{N_o} = \frac{SNR_g}{2} = \frac{1}{4C_g^{(\frac{2}{\alpha}-1)} \rho^2}. \tag{5}$$

3.2. Asymptotic Property of S α S Noise

The asymptotic property of a standard S α S random variable Y ($\alpha < 2$) can be expressed as [24]

$$\lim_{y \rightarrow \infty} P(Y > y) = \frac{C_\alpha}{y^\alpha}, \quad (6)$$

where $C_\alpha = \frac{1}{\pi} \Gamma(\alpha) \sin\left(\frac{\alpha\pi}{2}\right)$ and $\Gamma(\Phi) = \int_0^\infty t^{\Phi-1} e^{-t} dt$, is the Gamma function [29, Eq. (8.310.1)]. For S α S noise, Q-function is defined as

$$Q_\alpha(y) = \int_y^\infty f_\alpha(z) dz. \quad (7)$$

Therefore, Eq. (6) can be expressed as

$$\lim_{y \rightarrow \infty} Q_\alpha(y) = \frac{C_\alpha}{y^\alpha}, \quad (8)$$

The primary motivation for employing the asymptotic property of S α S random variable is the analytical tractability since its Cumulative Distribution Function (CDF) involves an integral [10, Theorem 1], which subsequently increases the mathematical complexity.

4. SYSTEM MODEL

Mathematically, the received signal can be expressed as [2]

$$r = Hx + z, \quad (9)$$

where $x \in \{\pm A\}$ corresponds to the symbols of binary phase-shift keying (BPSK) modulation. H is the channel gain with slow varying flat fading characteristics, and z can be Gaussian or non-Gaussian noise, whose real and imaginary components are independent and identically distributed (i.i.d). In Eq. (9), z follows S α S distribution. To distinguish the α appearing in α - μ fading model from the one appearing in S α S noise, we denote fading parameter by α_f and impulsive index by α .

5. PERFORMANCE ANALYSIS OF BER SUBJECTED TO FADING AND S α S NOISE

For an AWGN channel, $Q(y)$ is defined as $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty \exp(-t^2/2) dt$ [30]. As reported in [31] there exists a consistent mapping between $Q(y)$ and $Q_\alpha(y)$. Such a mapping is given as

$$Q(y) \rightarrow Q_\alpha \left(\sqrt{2C_g^{(\frac{2}{\alpha}-1)}} y \right). \quad (10)$$

Therefore, the probability of error for S α S noise with BPSK modulation can be expressed as

$$P_e^\alpha = Q \left(\sqrt{\frac{2E_b}{N_0}} \right) = Q_\alpha \left(\sqrt{\frac{4E_b C_g^{(\frac{2}{\alpha}-1)}}{N_0}} \right). \quad (11)$$

With the aid of such a mapping, the BER performance of a communication link perturbed by fading and S α S noise can be evaluated by its probability of error. It can be expressed as

$$P_e^\alpha = \int_0^\infty Q_\alpha \left(\sqrt{4C_g^{(\frac{2}{\alpha}-1)}} \gamma \right) f_\Upsilon(\gamma) d\gamma, \quad (12)$$

where $f_\Upsilon(\cdot)$ denotes the PDF of instantaneous signal-to-noise ratio (γ) per bit. An important performance metric of a wireless channel is the amount of fading (AF). It is defined as the measure of severity of fading. Mathematically, it is given as [1]

$$AF = \frac{E[\gamma^2]}{(E[\gamma])^2} - 1, \quad (13)$$

where $E[\cdot]$ is the statistical expectation operator. Higher values of AF indicate severe fading which subsequently leads to weaker reception of the signal. For Nakagami- m fading channel, AF is given as

$$AF = \frac{1}{m}, \tag{14}$$

where the fading parameter (m) ranges from $1/2$ to ∞ . Both α and AF dictate the BER performance of a communication link.

5.1. α - μ Fading Model

Using random variable transformation, the PDF of α - μ fading channel (γ) can be expressed as [32, 33]

$$f_{\Upsilon}(\gamma) = \frac{\alpha_f \mu^\mu \gamma^{\left(\frac{\alpha_f \mu}{2}\right)-1}}{2\Gamma(\mu) \bar{\gamma}^{\left(\frac{\alpha_f \mu}{2}\right)}} \exp\left(-\mu \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha_f}{2}}\right), \tag{15}$$

where $\alpha_f > 0$ and $\mu > 1/2$ represent the power exponent, and $\mu \in \mathcal{R}^+$ denotes the number of multipath clusters of the α - μ fading channel. $\bar{\gamma}$ is the average SNR and $\Gamma(\Phi) = \int_0^\infty t^{\Phi-1} e^{-t} dt$ is the Gamma function. With the aid of Eq. (8) and substituting Eq. (15) in Eq. (12), the error probability is given by

$$P_e^{\alpha-\mu} = \frac{\alpha_f \mu^\mu C_\alpha}{2\Gamma(\mu) \bar{\gamma}^{\left(\frac{\alpha_f \mu}{2}\right)} \left(4C_g^{\frac{2}{\alpha}-1}\right)^{\frac{\alpha}{2}}} \int_0^\infty \gamma^{\frac{\alpha_f \mu - \alpha}{2} - 1} \exp\left(-\mu \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha_f}{2}}\right) d\gamma. \tag{16}$$

Upon comparing Eq. (16) with [29, Eq. (3.326.2)], the above equation can be expressed as

$$P_e^{\alpha-\mu} = \frac{\mu^{\frac{\alpha}{\alpha_f}} C_\alpha \Gamma\left(\mu - \frac{\alpha}{\alpha_f}\right)}{\Gamma(\mu) \left(4C_g^{\frac{2}{\alpha}-1} \bar{\gamma}\right)^{\frac{\alpha}{2}}}. \tag{17}$$

Special case: The closed-form expression for probability of error over Rayleigh fading ($\alpha_f = 2$ and $\mu = 1$) with S α S noise can be expressed as

$$P_e^{Ray} = \frac{C_\alpha \Gamma(1 - \alpha/2)}{\left(4C_g^{\frac{\alpha}{2}-1} \bar{\gamma}\right)^{\frac{\alpha}{2}}}. \tag{18}$$

5.2. η - μ Fading Model

This model represents small-scaling fading in a wireless channel with non-line-of sight (NLOS) components. Using random variable transformation, the PDF of η - μ fading environment can be expressed as [34],

$$f_{\Upsilon}^{\eta-\mu}(\gamma) = \frac{2\sqrt{\pi} \mu^{\mu+\frac{1}{2}} h^\mu \gamma^{\mu-\frac{1}{2}}}{\Gamma(\mu) \mathcal{H}^{\mu-\frac{1}{2}} \bar{\gamma}^{\mu+\frac{1}{2}}} \exp\left(-\frac{2\mu\gamma h}{\bar{\gamma}}\right) I_{\mu-\frac{1}{2}}\left(\frac{2\mu\mathcal{H}\gamma}{\bar{\gamma}}\right), \tag{19}$$

where $I_v(\cdot)$ is the modified Bessel function of first kind and v th order. Using the channel parameter η , the parameters h and \mathcal{H} can be expressed in two formats [35]. With Format 1 settings, Eq. (23) corresponds to Nakagami- m fading ($\eta \rightarrow 1$ and $\mu = m/2$) and Hoyt fading ($\eta = q^2$ and $\mu = 0.5$). On comparing $e^{-x/2} I_\nu(x/2)$ with [36, Eq. (8.4.22.3)], (19) can be expressed as

$$f_{\Upsilon}^{\eta-\mu}(\gamma) = \frac{2\mu^{\mu+\frac{1}{2}} h^\mu \gamma^{\mu-\frac{1}{2}}}{\Gamma(\mu) \mathcal{H}^{\mu-\frac{1}{2}} \bar{\gamma}^{\mu+\frac{1}{2}}} \exp\left(-\frac{2\mu(h-\mathcal{H})\gamma}{\bar{\gamma}}\right) G_{12}^{11}\left(\frac{4\mu\mathcal{H}\gamma}{\bar{\gamma}} \middle| \begin{matrix} 1/2 \\ \mu-1/2, 1/2-\mu \end{matrix}\right). \tag{20}$$

With the aid of Eq. (8) and substituting Eq. (20) in Eq. (12), we get

$$P_e^{\eta-\mu} = \frac{2C_\alpha \left(4C_g^{\frac{2}{\alpha}-1}\right)^{-\frac{\alpha}{2}} \mu^{\mu+\frac{1}{2}} h^\mu}{\Gamma(\mu) \mathcal{H}^{\mu-\frac{1}{2}} \bar{\gamma}^{\mu+\frac{1}{2}}} \int_0^\infty \gamma^{-(\frac{\alpha}{2}-\mu+\frac{1}{2})} \exp\left(-\frac{2\mu(h-\mathcal{H})\gamma}{\gamma}\right) \times G_{12}^{11} \left(\frac{4\mu\mathcal{H}\gamma}{\bar{\gamma}} \middle| \begin{matrix} 1/2 \\ \mu-1/2, 1/2-\mu \end{matrix} \right) d\gamma. \quad (21)$$

Upon comparing Eq. (21) with the standard integral given in [29, Eq. (7.813.1)], closed form of Eq. (21) in terms of Meijer's G-function can be expressed as

$$P_e^{\eta-\mu} = \left(\frac{\mu}{C_1 \bar{\gamma}}\right)^{\frac{\alpha}{2}} \times \frac{2^{C_2} C_\alpha (h-\mathcal{H})^{C_3} h^\mu}{\Gamma(\mu) \mathcal{H}^{\mu-\frac{1}{2}}} G_{22}^{12} \left(\frac{2\mathcal{H}}{h-\mathcal{H}} \middle| \begin{matrix} C_2, 1/2 \\ \mu-1/2, 1/2-\mu \end{matrix} \right), \quad (22)$$

where $C_1 = 4C_g^{\frac{2}{\alpha}-1}$, $C_2 = (\frac{\alpha}{2} - \mu + \frac{1}{2})$, and $C_3 = (\frac{\alpha}{2} - \mu - \frac{1}{2})$.

5.3. κ - μ Fading Model

This model is used to represent small-scaling fading in a wireless channel with line-of-sight (LOS) components. Using random variable transformation, the PDF of κ - μ fading environment can be expressed as [34, 37]

$$f_Y^{\kappa-\mu}(\gamma) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}} \gamma^{\frac{\mu-1}{2}}}{\exp(\mu\kappa) \kappa^{\frac{\mu-1}{2}} \bar{\gamma}^{\frac{\mu+1}{2}}} \exp\left(-\frac{\mu(1+\kappa)\gamma}{\bar{\gamma}}\right) I_{\mu-1} \left(2\mu \sqrt{\frac{\kappa(1+\kappa)\gamma}{\bar{\gamma}}} \right), \quad (23)$$

where the parameter κ is defined as the ratio of power due to dominant components to the total power due to scattered components denotes the number of multi-path clusters within the fading environment. Equation (23) encompasses some of the prominent fading channels available in the technical literature, such as Nakagami- m ($\kappa \rightarrow 0$ and $\mu = m$) and Rician fading channel ($\kappa = K$ and $\mu = 1$). Expressing $I_\nu(\cdot)$ with infinite series representation [29, Eq. (8.445)], Eq. (23) can be expressed as

$$f_Y^{\kappa-\mu}(\gamma) = \sum_{n=0}^{\infty} \frac{\mu^{\mu+2n} (1+\kappa)^{\mu+n} \kappa^n \gamma^{\mu+n-1} \exp[-(\mu\gamma(1+\kappa))/\bar{\gamma}]}{\exp(\mu\kappa) \bar{\gamma}^{\mu+n} n! \Gamma(\mu+n)}. \quad (24)$$

With the aid of Eq. (8) and substituting Eq. (24) in Eq. (12), we get

$$P_e^{\kappa-\mu} = \sum_{n=0}^{\infty} \int_0^\infty \frac{C_\alpha (4C_g)^{\frac{2}{\alpha}-1} \mu^{\mu+2n} (1+\kappa)^{\mu+n} \kappa^n \gamma^{\mu+n-\frac{\alpha}{2}-1}}{\exp(\mu\kappa) \bar{\gamma}^{\mu+n} n! \Gamma(\mu+n)} \exp\left(-\frac{\mu(1+\kappa)\gamma}{\bar{\gamma}}\right) d\gamma. \quad (25)$$

Upon comparing Eq. (25) with [29, Eq. (3.326.2)], the above equation can be expressed as

$$P_e^{\kappa-\mu} = \frac{C_\alpha}{\exp(\mu\kappa)} \left(\frac{\mu(1+\kappa)}{C_1 \bar{\gamma}}\right)^{\alpha/2} \sum_{n=0}^{\infty} \frac{(\mu\kappa)^n \Gamma\left(\mu+n-\frac{\alpha}{2}\right)}{n! \Gamma(\mu+n)}, \quad (26)$$

where $C_1 = 4C_g^{\frac{2}{\alpha}-1}$. As shown in Appendix A, the infinite series given in Eq. (26) is convergent.

Equations (17), (22), and (26) are valid for arbitrary values of α , μ , and κ . These derived equations can be employed to assess the performance of other prominent fading environments (like Nakagami- m , Rayleigh and Weibull) available in the technical literature.

6. NUMERICAL RESULTS AND ANALYSIS

The amount of fading (AF) and impulsive index (α) are two key parameters which dictate the BER performance of the channel under consideration. As shown in Figures 2, 3, and 4, it is clear that BER varies significantly with respect to α . For a fixed impulsive index (α), as the AF is varied (by varying μ , η

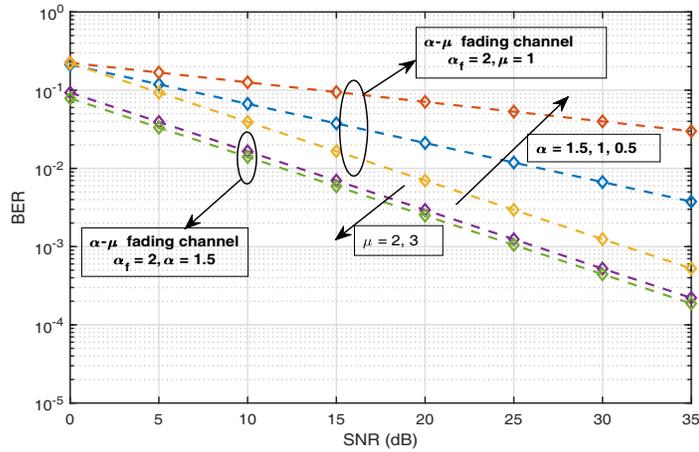


Figure 2. Effect of α (impulsive index) on the BER for BPSK modulation over α - μ fading channel with SaaS noise. Simulations: Diamond marker.

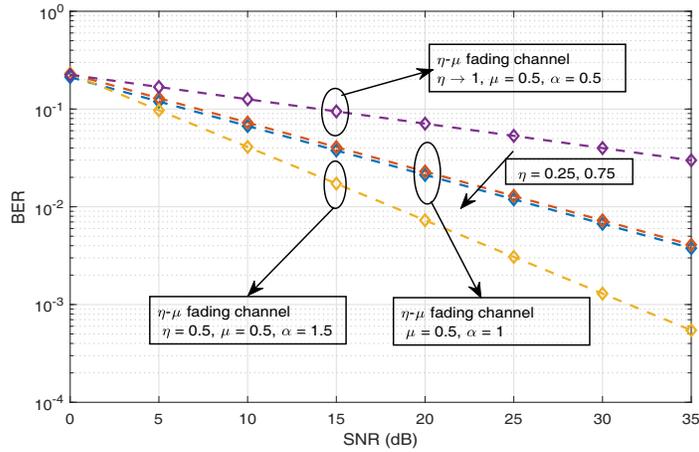


Figure 3. Effect of α (impulsive index) on the BER for BPSK modulation over η - μ fading channel (Format 1) with SaaS noise. Simulations: Diamond marker.

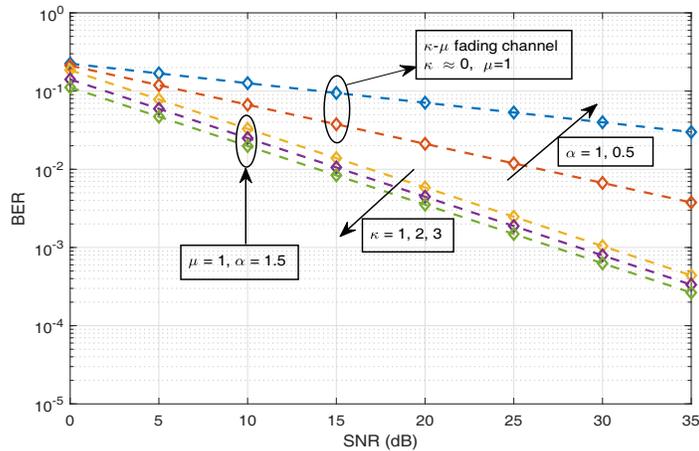


Figure 4. Effect of α (impulsive index) on the BER for BPSK modulation over κ - μ fading channels with SaaS noise. Simulations: Diamond marker.

and κ), the corresponding BER does not vary notably. Hence it is concluded that the effect of α is more pronounced than AF. In the case of κ - μ fading model with $\bar{\gamma} = 10$ dB, as the fading channel is perturbed from Rician channel ($K = \kappa = 1$, $\mu = 1$) to Rayleigh channel ($\mu = m = 1$, $\kappa \rightarrow 0$), the corresponding BER increases by 19.5% and 9.7% for $\alpha = 1.5$ and $\alpha = 1$ respectively. For η - μ fading model (Format 1) with $\bar{\gamma} = 10$ dB, as the fading dynamics change from Rayleigh ($\mu = m/2 = 1/2$, $\eta \rightarrow 1$) to Hoyt channel ($\mu = 0.5$, $\eta = 0.5 = q^2$), the corresponding BER increases by 4.15% and 2.45% for $\alpha = 1.5$ and $\alpha = 1$ respectively. Here K, m and q are different fading parameters corresponding to Rician, Nakagami- m and Hoyt fading environments, respectively.

7. CONCLUSION

In this work, analytical expressions of BER are obtained for various non-homogeneous fading environments (α - μ , η - μ and κ - μ) with SaS noise. The results reveal that BER increases significantly as α approaches 0. For different values of α_f , η , κ , and μ , the BER performance of various standard fading models (like Nakagami- m , Rayleigh, and Hoyt) are evaluated. From the interplay among different fading parameters and impulsive index (α), the results show that for a particular impulsive index (α), the BER variation due to different fading parameters (α_f , η , κ , and μ) is not so dominant. Hence, impulsive noise plays a crucial role in the BER performance of a wireless communication link.

APPENDIX A.

The infinite series given in Eq. (26) can be expressed as

$$\sum_{n=0}^{\infty} \frac{(\mu\kappa)^n \Gamma\left(\mu + n - \frac{\alpha}{2}\right)}{n! \Gamma(\mu + n)}. \quad (\text{A1})$$

The n th term of the above infinite series can be written as

$$u_n = \frac{(\mu\kappa)^n \Gamma\left(\mu + n - \frac{\alpha}{2}\right)}{n! \Gamma(\mu + n)}. \quad (\text{A2})$$

Employing D'Alembert's Ratio test [38] and $\Gamma(z + 1) = z\Gamma(z)$, we get

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \lim_{n \rightarrow \infty} \frac{(\mu\kappa) \left(\mu + n - \frac{\alpha}{2}\right)}{(1+n)(\mu+n)}. \quad (\text{A3})$$

The above equation can be written as

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \lim_{n \rightarrow \infty} \frac{\mu\kappa}{1+n} \left(1 - \frac{\alpha}{2(\mu+n)}\right). \quad (\text{A4})$$

From the above equation, it is clear that for finite values of μ and κ , $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} \rightarrow 0$ as $n \rightarrow +\infty$. Hence, the infinite series given in Eq. (26) is convergent by D'Alembert's Ratio test.

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