

# Quantification of Combat Team Survivability with High Power RF Directed Energy Weapons

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**Abstract**—Modern combat teams face an increasingly complex battlefield, where threats may arise from a number of different sources. Examples include not only conventional attacks through rocket propelled grenades but also improvised explosive devices and weaponised unmanned aerial vehicles. Combat teams can now be equipped with sophisticated surveillance and reconnaissance capability, as well as automatically activated defences. The focus of this paper is to consider the utility of collaborative active protection systems, which are designed to provide an active defence against threats to a combat team. Specifically, a general statistical framework for the analysis of such systems is introduced, with a particular focus on high power radio frequency directed energy weapon countermeasures. The mathematical model allows for a subset of the combat team to be responsible for target detection and tracking, and a time-varying subset of team members with suitable countermeasures to be specified separately. The overall probability of threat defeat and team survivability is then derived. Some examples are provided to investigate the utility of such systems.

## 1. INTRODUCTION

Modern combat teams can be equipped with sophisticated electronic devices to counter a wide variety of threats. Here a combat team is assumed to consist of a series of land vehicles, each with appropriate personnel. Some of these vehicles will be equipped with sensors while others with automatically activated threat defense. It is also permissible that a given vehicle can be equipped with both technological attributes. The team can work collaboratively, in terms of coordinated defense, or the entire system can be automated. The types of threats such a team may encounter include rocket propelled grenades (RPGs), improvised explosive devices (IEDs), terrorists on a suicide mission in a vehicle-borne IED and weaponised unmanned aerial vehicles (UAVs). This paper introduces a novel mathematical framework in which collaborative active protection system performance can be quantified, and its military utility examined. Such systems are designed to provide a coordinated team defense against threats as discussed above. Towards this objective, the paper defines a number of stochastic quantities associated with the combat scene under consideration. Based upon this, metrics for team survivability and defeat can then be produced. Throughout the paper team survivability is defined to be the likelihood that each of the vehicles survives the attack. These mathematical expressions are then specialised to the case where area surveillance is provided by an X-band radar, and the threat countermeasure is provided by a high power radio frequency (HPRF) directed energy weapon (DEW). The latter has been shown to provide military utility against pertinent threats as discussed above [1]. Specific examples of HPRF DEW applications are vehicle motor disruption [2], radio frequency amplifier and digital modulation scheme attack [3] and UAV defeat [4, 5]. An overview of the usage of pulsed HPRF to disruption of electronic components in systems is provided in the context of intentional electromagnetic interference (IEMI) in [6, 7], while [8] provides a useful summary of developments in IEMI.

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HPRF DEWs apply pulsed radio frequency (RF) energy towards targets, with the objective to induce electrical currents within the target. These currents can then couple into the target through vulnerabilities such as antennae, sensors, guidance and navigation control and wires, to achieve varying degrees of damage. The wavelength of the transmitted signal must be smaller than the target's radar cross section to achieve coupling. Hence larger wavelengths may be applied to vehicles, while UAVs may require significantly smaller wavelengths. HPRF effectors can operate at either narrowband or wideband. The advantage of the latter is that since a wide range of frequencies are transmitted toward the target, a resonant frequency may be matched and thus cause coupling. The disadvantage is that the resultant power is distributed over the frequency spectrum, unlike a narrowband source, and so significantly less power at each frequency is applied. For the narrowband effector to cause disruption, it must be matched to the target's antenna receive frequency, for example. Sufficiently intense electromagnetic signals in the frequency range of 200 MHz to 5 GHz are known to cause disruption in systems [9]. In the time domain these signals have a very fast rise time, in the order of nanoseconds. Required power levels to achieve coupling in targets have been documented in the literature. Computers have been shown to malfunction when 500 MHz pulsed waveforms are applied, with a field intensity as low as 500 volts per meter. Modern vehicle's motor control can also be disrupted with the same field intensity, but at 1.3 GHz. When a field strength of 24 kV per meter is applied at 2.86 GHz to a vehicle, permanent motor control damage can occur [6]. In the context of this paper, an incoming threat can be disabled by an HPRF DEW in a number of ways. Firstly, if the threat is delivered on a motorised platform, then the HPRF DEW can disable its motor. If the threat is using automatic guidance and control, then the HPRF DEW may affect this and cause the threat to veer off course. Another possibility is that the induced currents on the threat may trigger an earlier detonation of explosives, destroying the threat.

Since the purpose of this paper is to examine the performance of a combat team with collaborative active protection system defense provided by HPRF DEWs, it is important to note that in the battlefield there is the danger that electromagnetic radiation may have an adverse impact on enemy soldiers. The potential impact of non-ionising radiation on humans have been examined extensively [10]. However, since the coordinated defense is focused on disrupting a low-altitude missile, the energy of the HPRF DEWs will be directed toward the projectile, and not intentionally towards soldiers. Additionally, recent studies have examined the issue of collateral damage caused by HPRF DEWs and it has been reported that these risks and effects can be minimised [11].

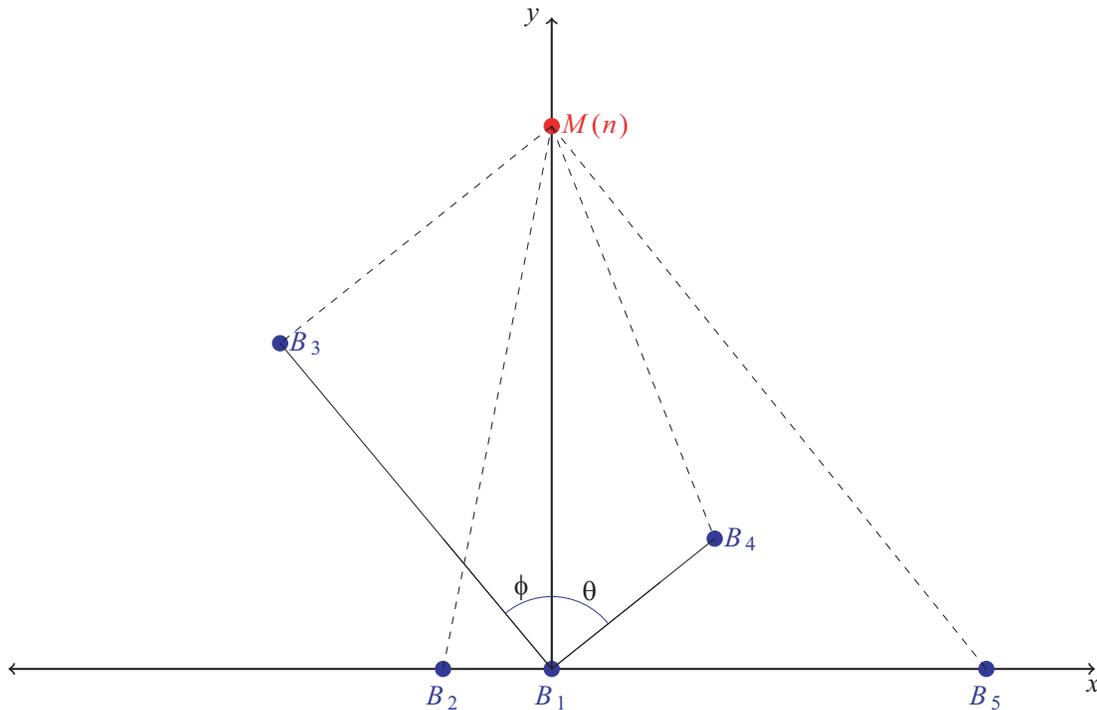
There are several novel contributions introduced in this paper. Firstly, a new stochastic model to quantify combat team survivability is derived, and it is shown how it can provide a probability metric that is a function of the probability of detection and disruption of the threat. Although the probability of detection is provided by the well-known Marcum Q-function, the probability of HPRF DEW effect on a target is new and it is demonstrated how it can be linked to the target's radar cross section (RCS). It is also shown how the RCS from a radar and HPRF DEW effector can be linked to facilitate the analysis. The paper is organised as follows. Section 2 develops the model in terms of stochastic characteristics of the scenario, while Section 3 provides some comments on the geometry of the combat scene. Section 4 discusses the specifics of radar, and introduces a suitable expression for the probability of detection of threats in a land environment. It also discusses the specification of the HPRF DEWs and derives their probability of disruption of a target. Finally, Section 5 provides tangible examples to illustrate the concepts introduced.

## 2. MEASURING SURVIVABILITY STOCHASTICALLY

The combat team, referred to as Blue Force throughout, consists of a series of  $N$  subsystems, which can either be interpreted as vehicles in a mechanized combat team or a dismounted combat team with the necessary portable functionality to detect, track and counter an incoming threat. For brevity, these subsystems will be referred to as vehicles throughout. These vehicles are stationary and their relative positions are known to each member of the combat team. The missile has been initiated by Red Force, and is assumed to be a close combat weapon such as an RPG launched by an insurgent. As such, it will be assumed that the missile travels in an almost linear path to its intended target.

Figure 1 provides a schematic of the combat scene under investigation. A two-dimensional perspective of the engagement is adopted throughout. Here there are five members in Blue Force,

and the missile is fired toward  $B_1$ . The geometry in Figure 1 is oriented so that the trajectory of the missile is along the  $y$ -axis. Team members  $B_2$  and  $B_5$  are on the same horizontal line as  $B_1$  (the  $x$ -axis) while  $B_3$  and  $B_4$  are at angles  $\phi$  and  $\theta$  respectively to the  $y$ -axis as shown. Define  $T$  to be the time, in seconds, it takes the missile to reach its target if it is not disrupted. Throughout this analysis time will be discretised to facilitate the analysis. If the missile is not disabled it will strike its target at time  $T$ , and potentially effect other vehicles within its blast range. Hence let  $d_j$ , for  $j \in \{1, 2, \dots, N\}$  be the probability that vehicle  $j$ , denoted  $B_j$  henceforth, is not within the missile's blast range. This probability can also be interpreted as the survivability of a team member in the event that the threat is not neutralised. At time  $n$  the missile is located at  $M(n)$  as illustrated in Figure 1. The assumptions adopted above permit the derivation of the distance of the missile, at time  $n$ , from each of the vehicles in Blue Force; this will be clarified subsequently.



**Figure 1.** An illustration of the geometry in the combat scene. The Red Force threat is travelling along the  $y$ -axis as indicated by  $M(n)$ , and directed towards  $B_1$ . Blue Force members  $B_2$  and  $B_5$  are on the same axis as  $B_1$ , while  $B_3$  and  $B_4$  are at angles  $\phi$  and  $\theta$  as shown.

As discussed previously, some of the members will have a radar detection and tracking capability, while others will have a countermeasure. It is also possible that a vehicle has both technological capabilities. Hence define

$$\mathcal{D} = \{B_j : B_j \text{ equipped with detection and tracking capability}\}.$$

Since each vehicle has its own communication capability information will be shared within the combat team to members equipped with a countermeasure. Thus define

$$\mathcal{S}(n) = \{B_j : B_j \text{ potential disruptor of missile at time } n\}.$$

The operating protocol applied within the combat team is that at time  $n$  if a member of  $\mathcal{D}$  detects the target, then it informs the relevant members of  $\mathcal{S}(n)$ , which then activate their countermeasure.

Let  $p_j(n)$  be the probability that  $B_j \in \mathcal{D}$  detects the missile at time  $n$ , and similarly define  $q_j(n)$  to be the probability that  $B_j \in \mathcal{S}(n)$  defeats the threat at time  $n$ . The probability that the missile is

detected by the combat team at time  $n$  is hence

$$\begin{aligned}
 P_D(n) &= 1 - \mathbb{P}(\text{missile not detected at time } n) \\
 &= 1 - \mathbb{P}(\text{each } B_j \in \mathcal{D} \text{ does not detect it at time } n) \\
 &= 1 - \prod_{\{j: B_j \in \mathcal{D}\}} [1 - p_j(n)], \tag{1}
 \end{aligned}$$

where it is assumed that each vehicle with a detection capability does so independently, although the combat team has the capacity to notify each member of the imminent threat. Next the probability that the missile is countered at time  $n$  is given by

$$\begin{aligned}
 P_C(n) &= \mathbb{P}(\text{at least one of the } B_j \in \mathcal{S}(n) \text{ defeat the missile at time } n) \\
 &= 1 - \mathbb{P}(\text{none of the } B_j \in \mathcal{S}(n) \text{ defeat the missile at time } n) \\
 &= 1 - \prod_{\{j: B_j \in \mathcal{S}(n)\}} [1 - q_j(n)], \tag{2}
 \end{aligned}$$

and it is supposed that each countermeasure acts independently, although they can be fired simultaneously. It is worth noting that Eq. (2) is actually a conditional probability, where the latter is based upon the event that the threat is detected by a member of  $\mathcal{D}$  at time  $n$ .

Team survivability is a function of whether the missile is defeated, at some time  $n$ , and also whether a particular Blue Force team member is not within the missile's blast range. To determine the probability of both individual team member as well as full team survivability, the probability that none defeat the threat is determined. This is given by

$$\mathbb{P}(\text{None defeat the threat}) = \prod_{n=1}^{T-1} \mathbb{P}(\text{None defeat the threat at time } n). \tag{3}$$

In Eq. (3), this probability has been determined from the fact that the event that none defeat the threat is equivalent to the event that the threat is not defeated over each time epoch, and by applying an independence assumption.

Throughout the following it will be assumed that detection of targets and firing of a countermeasure happen in the same time interval. It is possible to introduce a time delay to the firing of the countermeasure, if required. However, since the countermeasure to be examined will be a HPRF DEW, one can assume speed of light engagement, providing a justification for this assumption. In addition to this, it is also assumed that detection is performed independently over different time intervals, and that HPRF DEW disruption effects are not cumulative. These assumptions have been adopted to produce a first order approximation to the problem under consideration.

At discrete time  $n$  none of the team members defeat the threat under two conditions. Firstly, this can happen if none detect the threat (which occurs with probability  $1 - P_D(n)$  based upon Eq. (1)) or if at least one detects it (with probability  $P_D(n)$ ) and no member of  $\mathcal{S}(n)$  neutralise the threat (which occurs with probability  $1 - P_C(n)$  in view of Eq. (2)). Applying these considerations to Eq. (3) it follows that

$$\begin{aligned}
 \mathbb{P}(\text{None defeat the threat}) &= \prod_{n=1}^{T-1} [(1 - P_D(n)) + P_D(n)(1 - P_C(n))] \\
 &= \prod_{n=1}^{T-1} [1 - P_D(n)P_C(n)]. \tag{4}
 \end{aligned}$$

Suppose that we are interested in the survivability of team member  $B_j$ . The probability of this is the probability that at least one defeats the threat, which is the complement of (4), or the probability

that none defeat the threat and  $B_j$  is not in the missile's effective disruption range. Since these are mutually exclusive events, it follows that

$$\begin{aligned} \mathbb{P}(B_j \text{ survives the attack}) &= 1 - \mathbb{P}(\text{None defeat the threat}) + \mathbb{P}(\text{None defeat the threat})d_j \\ &= 1 - (1 - d_j) \prod_{n=1}^{T-1} [1 - P_D(n)P_C(n)]. \end{aligned} \quad (5)$$

Hence Eq. (5) provides a probability metric for the survivability of an individual combat team member.

The probability that the entire combat team survives the attack is now derived. The survivability of the combat team can be interpreted as the event that each individual member survives the attack. The required probability consists of two mutually exclusive components. The team will survive the attack if the missile is defeated, or if the latter does not occur, then the survivability of the team is then a function of the probabilities that each member is not within the effective range of the missile. Thus it follows that

$$\begin{aligned} \mathbb{P}(\text{Blue Team survives the attack}) &= 1 - \mathbb{P}(\text{None defeat the threat}) + \mathbb{P}(\text{None defeat the threat}) \prod_{j=1}^N d_j \\ &= 1 - \prod_{n=1}^{T-1} [1 - P_D(n)P_C(n)] \left[ 1 - \prod_{j=1}^N d_j \right], \end{aligned} \quad (6)$$

where the probability that none defeat the threat is given by Eq. (4), and has been applied to produce Eq. (6). Therefore, Eq. (6) is a probability metric for the survivability of the combat team.

### 3. GEOMETRIC CONSIDERATIONS

This section discusses the geometry in the combat scene, as illustrated in Figure 1, as well as some important quantities required in the analysis. Recall that the missile's trajectory is described by  $M(n)$  at time  $n$ . Then  $B_j \in \mathcal{S}(n)$  at some time  $n$  if the distance of the missile at time  $n$  from the vehicle  $B_j$  is smaller than the maximum effective range of the countermeasure. This is expressed as

$$R_j(n) := d(M(n), B_j) < \epsilon_j, \quad (7)$$

where  $d$  is the Euclidean distance, and  $\epsilon_j$  is the maximum effective range of the countermeasure. This expression indicates how members of the set  $\mathcal{S}(n)$  can be determined adaptively.

The combat team members are assumed to be stationary throughout the engagement. Hence the distances  $d(R_i, R_j)$  between any two combat team members is fixed and known. The relative positions of each vehicle are also assumed known and fixed. Without loss of generality, suppose the missile is fired toward  $B_1$ , as in Figure 1. As it is assumed that the missile's trajectory is linear, one can orient the combat scene geometrically based upon axes consisting of the path from the missile to its target, and a line perpendicular to this path. Then triangles have one side the distance  $R_1(n)$ , another the distance  $d(R_1, R_j)$ , and the third side  $R_j(n)$ , with the angle between the first two edges  $\theta$  fixed and not varying with time. Based upon the Cosine Rule, one can determine the edge

$$R_j(n) = \sqrt{R_1(n)^2 + d^2(B_1, B_j) - 2R_1(n)d(B_1, B_j)\cos(\theta)}. \quad (8)$$

The above expression is not only required for Eq. (7) but will also be an integral component in the expressions for the probability of detection as well as HPRF DEW disruption of the target. Determination of the distance  $R_1(n)$  can be based upon the assumed motion of the missile. A radar can also provide an estimate of its speed once detected. Recall it is assumed that it takes  $T$  seconds for the missile to strike its intended target. Suppose that it is travelling at a time-varying speed of  $\nu(t)$  meters per second. Then its displacement from its starting point is given by

$$x(t) = \int_0^t \nu(\omega) d\omega. \quad (9)$$

Then it follows that  $d(B_1, M(1)) = x(T)$  and consequently

$$R_1(t) = \int_t^T \nu(\omega) d\omega, \quad (10)$$

where continuous time has been used for convenience. To illustrate, if the missile is travelling at a constant speed  $\nu$  meters per second, then Eq. (10) reduces to  $R_1(n) = \nu(T - n)$ , while if the speed is given by  $\nu(t) = 2\kappa t$ , for some fixed  $\kappa > 0$  then  $R_1(n) = \kappa[T^2 - n^2]$ . Hence Eq. (10) can be applied to Eq. (8) to determine the distance, at discrete time  $n$ , of the missile from  $B_j$ . This information can be applied to Eq. (7).

The results derived in this section will be required in the examples of performance in Section 5.

#### 4. RADAR AND HPRF DEW COUNTERMEASURE SPECIFICATIONS

The purpose of this section is to provide some expressions for the probabilities of detection and disruption defined in Section 2. Here it is assumed that the incoming threat appears at a low grazing angle relative to the radar, and is in the radar's line of sight. Target detection and tracking is provided by either a vehicle-mounted or portable X-band surveillance radar. Given the radar is line of sight the Rayleigh amplitude assumption is valid for backscattering in the absence of a target. In a land radar context, and at low grazing angles, this can be validated, especially when the radar signals are vertically polarised [12, 13]. Since the incoming threat is moving, it is supposed that the target's RCS fluctuates. In particular, it is assumed that it is also Rayleigh distributed in amplitude, or exponentially distributed in intensity, but with a different mean parameter. The Rayleigh assumption is equivalent to the supposing that the target follows a Swerling I distributional model [14]. For simplicity of the analysis, the radar bases its detection on a series of  $M$  pulses, which are non-coherently integrated in response. Due to these conditions, the probability of detection of the target is given by the Marcum Q-function [15], defined by

$$p = Q_M(\sqrt{2M\zeta}, \sqrt{2\nu}), \quad (11)$$

where

$$Q_M(a, b) = \frac{1}{a^{M-1}} \int_b^\infty x^M \exp\left(-\frac{(x^2 + a^2)}{2}\right) I_{M-1}(ax) dx \quad (12)$$

$I_M$  is the modified Bessel function of the first kind of order  $M$ ,  $\zeta$  the signal to clutter ratio (SCR), and  $\nu$  the detection threshold, given by the solution to the equation

$$P_{\text{FA}} = Q_M\left(0, \sqrt{2\nu}\right) \quad (13)$$

where  $P_{\text{FA}}$  is the probability of false alarm. The SCR can be specified as a function of radar characteristics through

$$\zeta = \frac{P_R G_R^2 \lambda_R \mathbb{E}(\sigma_R)}{(4\pi)^3 R^4 2\theta}, \quad (14)$$

where  $P_R$  is the power radiated by the radar (in Watts),  $G_R$  the radar antenna's gain (in dBi),  $R$  the distance to the target (in metres),  $\lambda_R$  the wavelength (in metres) of the radar signal,  $\mathbb{E}(\sigma_R)$  the mean of the RCS  $\sigma_R$  of the target (in square metres), and  $\theta$  the variance of the compound Gaussian model in the underlying assumption of Rayleigh amplitude statistics. The probability Eq. (11) can be applied directly to Eq. (1) as required.

Next the characteristics of the HPRF DEWs are specified. Each applies a pulsed RF waveform toward the target, with source transmission power  $P_D$  through a directive antenna with effective area  $A_D$  and gain  $G_D$  with a radiated signal with wavelength  $\lambda_D$ . The units for each of these are the same as for their radar counterparts. Then the power density on the target is given by

$$S_1 = \frac{P_D A_D G_D}{R^2 \lambda_D^2}, \quad (15)$$

where  $R$  is the distance from the effector to the target. Power density is measured in Watts per square metre, but in the numerical analysis to follow the field intensity is instead considered, which is in volts per metre. The relationship between Eq. (15) and field strength is given by

$$S_1 = \frac{E_F^2}{120\pi}, \quad (16)$$

where  $E_F$  is the field intensity, and the factor of  $120\pi$  is due to the characteristic impedance of free space.

When RF energy impinges on the target, it causes surface currents to be induced, and it is well known in radar that a component of the surface is likely to reradiate electromagnetic energy unidirectionally. The area with this potential to radiate this energy is the target RCS, as discussed in the description of the probability of detection. In the case of an HPRF source, a percentage of this area will potentially couple into the targeted system. Hence we can refer to the target coupling cross section as the fraction of area on the target which has the potential to induce currents that couple into a system.

Since the HPRF effector will operate at a different frequency from the radar, the RCS for the HPRF DEW will be different from that of the radar. Suppose that  $\sigma_D$  is the RCS from the DEW's perspective, and define a random variable  $\kappa$  to be the percentage of surface area of  $\sigma_D$  that has induced currents which have the potential to couple into the target. Then  $\kappa$  takes values in the unit interval, with zero corresponding to no potential currents being induced, and unity to the situation where all the power impinged on the RCS having the potential to couple. Since it is very difficult to determine the actual distribution of  $\kappa$  in practice, here it is assumed that  $\kappa$  has a continuous uniform distribution on the unit interval. In view of this, the power density on the target that has potential to cause coupling is therefore given by

$$S_2 = \frac{P_D A_D G_D \sigma_D \kappa}{R^2 \lambda_D^2}, \quad (17)$$

based upon Eq. (15).

At the system level, a target will be affected by the HPRF DEW if the power delivered to the target is sufficiently high; this can be interpreted as the event that the resultant power density on the target exceeds a power threshold level. If this threshold is denoted  $\tau$  for a given target, then coupling will occur in the case when  $S_2 > \tau$ . System level vulnerability thresholds can be determined by consulting the well-known literature on IEMI [6, 7, 9]. Although in this study the power density level on a target is used to quantify HPRF DEW effects, it is worth noting that other factors will also have a significant effect on the success of an HPRF DEW to disrupt a target. These include the waveform and its characteristics. Specifically, such waveforms must have a very fast rise time to their peak power level, so are essentially short pulses. The pulse repetition rate is another significant factor, which will be examined in subsequent studies. It is also worth noting that waveforms from the HPRF DEW will also be attenuated by the environment, which can also have a significant impact on its performance.

Hence if  $q$  is the probability of HPRF DEW disruption of a target with such a vulnerability threshold, then

$$q = \mathbb{P}(S_2 > \tau) = \int_0^1 \mathbb{P}\left(\sigma_D > \frac{\tau}{\kappa} \left(\frac{R^2 \lambda_D^2}{P_D A_D G_D}\right)\right) d\kappa, \quad (18)$$

where conditioning with respect to  $\kappa$  has been applied, as well as the fact that the latter is assumed to be uniformly distributed. Note that the probability in Eq. (18) is exactly the complementary distribution function of  $\sigma_D$ . Since in the analysis of radar detection it is assumed that the target RCS has a Swerling I fluctuation, it is appropriate to adopt this assumption in the analysis here. Hence it is assumed that  $\sigma_D$  has an exponential distribution with parameter  $\mu$ . Then, Eq. (18) reduces to

$$q = \int_0^1 e^{-\frac{\tau}{\kappa} \left(\frac{R^2 \lambda_D^2}{P_D A_D G_D}\right) \mu} d\kappa. \quad (19)$$

Thus, Eq. (19) provides an expression which can then be applied to Eq. (2).

It is informative to derive the maximum effective range of an HPRF DEW, since this is required for Eq. (7) to determine membership of the set  $\mathcal{S}(n)$ . To do this the analysis will be based upon Eq. (15)

to determine the distance in ideal situations. It can be shown, through manipulation of Eq. (15) and with an application of Eq. (16), that the maximum effective range of an HPRF effector is given by

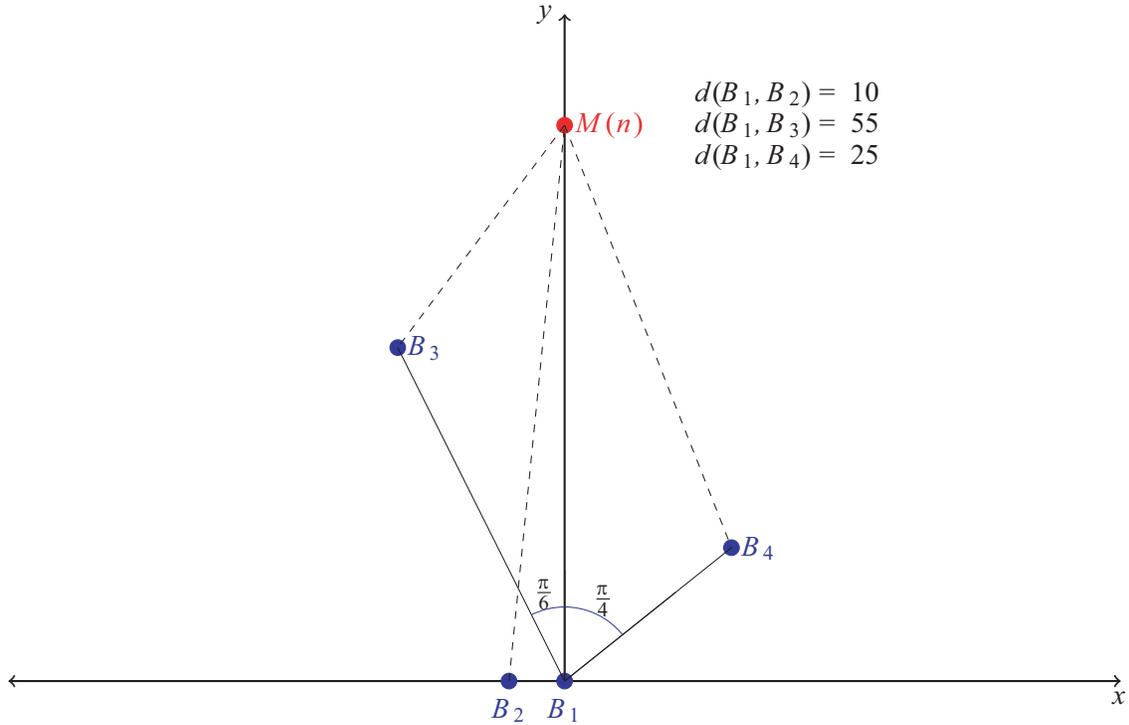
$$\epsilon_j = \sqrt{\frac{120\pi P_D A_D G_D}{\lambda_D^2} \frac{1}{E_F}}. \quad (20)$$

This can be applied to Eq. (7) when required.

The next section provides a tangible example to illustrate the concepts derived in this paper.

## 5. A SPECIFIC EXAMPLE

To examine the ideas introduced in the previous sections, consider the configuration illustrated in Figure 2. Blue Force consists of four vehicles, with  $d(B_1, B_2) = 10$ ,  $d(B_1, B_3) = 55$ , and  $d(B_1, B_4) = 25$  metres. As illustrated, the angle between the missile trajectory and  $B_3$  is  $\frac{\pi}{6}$  radians, while it is  $\frac{\pi}{4}$  radians relative to  $B_4$ . Vehicles  $B_1$  and  $B_2$  are located on the same axis as shown, and the missile is fired toward  $B_1$ . Two configurations of technology capability will be examined. Firstly it will be assumed that  $B_1$  and  $B_3$  operate radars, while  $B_2$  and  $B_4$  operate the HPRF DEWs. In the second configuration, it will be supposed that  $B_2$  and  $B_3$  operate the radars, while  $B_1$  and  $B_4$  have the HPRF DEWs. These two operating configurations will provide insights into team survivability when the technology roles assigned to  $B_1$  are switched.



**Figure 2.** Geometry of the combat scene under examination. The first configuration assumes that radars are operated by vehicles  $B_1$  and  $B_3$ , while HPRF DEWs are used by  $B_2$  and  $B_4$ . In the second configuration radars are applied in vehicles  $B_2$  and  $B_3$  and the HPRF DEWs used in vehicles  $B_1$  and  $B_4$ .

The constraint of  $T = 120$  with  $\nu = 5$  will be imposed, which implies that the missile is fired toward  $B_1$  at a distance of 600 metres.

Selection of probabilities  $d_1$ ,  $d_2$ ,  $d_3$ , and  $d_4$  is based upon the following considerations. Firstly,  $d_1$  is assumed to be determined by the vehicle's resilience to the missile strike. Since this is platform

dependent, for the purposes of the example here it is assumed that  $d_1 = 0.4$ . The remaining  $d_j$  are also dependent on each platform's ability to survive the attack and are also dependent on the distance between each  $B_j$  and the target of the missile, namely  $B_1$ . If it is assumed that each platform has the same resilience to a direct strike, then each  $d_j$  can be defined to be the product of the vehicle's capability to withstand the strike and its normalised distance from  $B_1$ . Therefore for each  $j \in \{2, 3, 4\}$

$$d_j := \min \left( 1, d_1 \left[ 1 + \frac{d(B_1, B_j)}{d(B_1, B_2) + d(B_1, B_3) + d(B_1, B_4)} \right] \right), \quad (21)$$

where this selection has been made so that  $d_1$  is minimal. The reason for including a minimum in Eq. (21) is to ensure that  $d_j$  are bounded by unity. With an application of Eq. (21), it is not difficult to show that  $d_2 = 0.4444$ ,  $d_3 = 0.6444$ , and  $d_4 = 0.5111$ .

The radar is X-band operating with a wavelength of 3 cm (frequency 10 GHz), with an output power of  $10^6$  Watts and an antenna gain of 10 dBi. The underlying compound Gaussian model has a variance of 0.1. This value was selected to indicate that in the line of sight there is only a small variation in radar backscatter. The radar integrates  $M = 4$  pulses and applies a probability of false alarm of  $10^{-4}$ , with a detection threshold of 15.9138, as determined through the solution to Eq. (13).

The HPRF DEW operates with a wavelength of 10 cm (frequency of 3 GHz), with a output power of  $10^4$  Watts with an antenna with effective area of 5 square metres and gain of 10 dBi. Based upon this, it follows that  $\epsilon = \frac{\sqrt{15\pi} \times 2 \times 10^4}{E_F}$  from Eq. (20). It will be assumed that the HPRF DEWs have a field intensity of 1500 volts per metre in both configurations, and all radars and HPRF DEWs operate with identical characteristics. For this field intensity the above implies an effective range for the HPRF DEW of  $\frac{40}{3} \sqrt{15\pi} \approx 91.53$  metres. A smaller field intensity will increase the effective range of the HPRF DEW; however in the current scenario,  $B_2$  is in a very good tactical situation to defeat the threat and protect  $B_1$  if necessary.

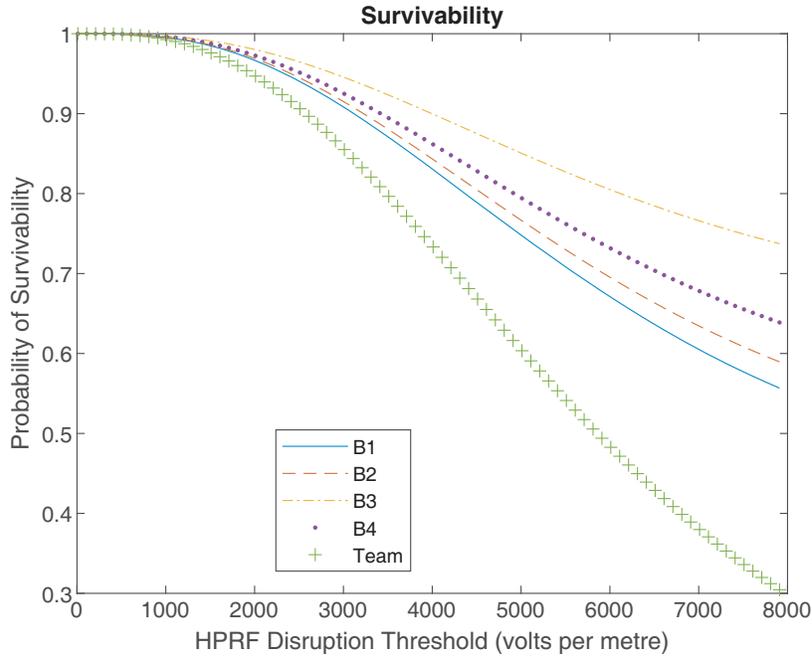
It is required to produce a measurement of the average RCS of the missile, from both the radar as well as from the HPRF DEW perspective. Towards this objective, it is useful to note that the RCS of a conducting plate with a physical area  $\mathcal{A}$  observed at the normal direction is approximately  $\sigma = \frac{4\pi\mathcal{A}^2}{\lambda^2}$ , where  $\lambda$  is the wavelength of the waveform [16]. Hence, given that the radar and HPRF DEW are illuminating the same object, this expression can be used to relate the two required average RCS and the different waveforms. Since the radar is operating with  $\lambda_1 = 0.03$ , while the HPRF DEW has  $\lambda_2 = 0.1$ , it follows that  $\sigma_1 \approx 3.333\sigma_2$ , where  $\sigma_1$  is the RCS for the radar, and  $\sigma_2$  is that for the HPRF DEW. For the example to follow  $\sigma_2 = 0.6$  has been selected, and hence  $\sigma_1 = 1.998$ .

Figure 3 plots the probability of individual as well as team survivability, as a function of the HPRF DEW disruption threshold of the missile. As expected, the target of the missile ( $B_1$ ) has the smallest individual survivability, while the other team members have increasing survivability. The latter is due to their distance from  $B_1$  and the values assigned to  $d_j$ . Due to the definition of team survivability, the probability of the latter is thus smaller than individual team member survivability as shown. At smaller HPRF DEW disruption thresholds of the missile, individual and team survivability is maximised, as expected, since it is more likely that the missile is disrupted in this case.

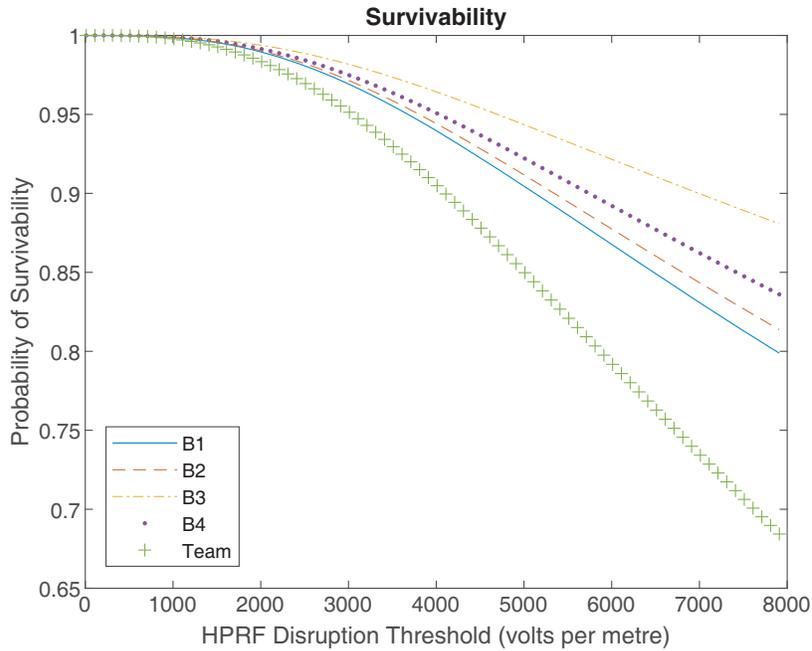
The results of team survivability for the second configuration of technology is shown in Figure 4. Comparing this with the results in Figure 3 it is clear that Blue Force survivability has increased considerably. Observe that the probability of team survivability reaches a minimum of about 0.67 at an HPRF DEW disruption threshold of 8000 volts per metre. By contrast, for the first configuration as shown in Figure 3, this probability is reduced to about 0.29.

Some comments on variations to the power level of the HPRF DEW are merited. When the power level is reduced to  $10^2$  Watts a significant decline in team survivability is observed, especially if the field intensity is to be maintained at a level to guarantee disruptions. Under configuration 1 examined above, the probability that the missile is defeated is almost reduced to zero, and individual team member survivability is almost equal to the respective  $d_j$ . The probability of team survivability is less than 0.05 in this case. This is also explained from the fact that the effective range of an HPRF effector is  $\epsilon = 9.1529$  in this situation. In the second configuration, with the same reduction in HPRF DEW power, the same significant reduction in survivability is also observed.

If the radar output power is also reduced, then this does not appear to alter this outcome considerably. Reducing the output power of the HPRF DEW is necessary to maintain portability;



**Figure 3.** Plot of Blue Force team survivability, as a function of the HPRF DEW disruption threshold. In this case the HPRF DEWs, operated by  $B_2$  and  $B_4$ , admit a field intensity of 1500 volts per metre. Team members  $B_1$  and  $B_3$  operate radars.



**Figure 4.** Probability of team and individual survivability in the second configuration. Here radars are operated by  $B_2$  and  $B_3$ , while HPRF DEWs are applied in vehicles  $B_1$  and  $B_4$ .

this is also why a higher operating frequency has been selected, since the antenna size will be governed by the wavelength and desired power on the target. If, however, the HPRF DEW's output is increased to  $10^6$  Watts, preserving the initial radar operating characteristics, both individual and team survivability is highly likely. As an example, under configuration 1, it can be shown that at an HPRF disruption

threshold of 8000 volts per metre, the operating protocol yields a combat team survivability of roughly 0.9965. When being applied to configuration 2, this probability is increased to 0.999.

## 6. CONCLUSIONS

This paper introduced a mathematical framework in which a collaborative active protection system's performance could be quantified in terms of individual and team survivability in the presence of a missile threat. Examples were considered in the case where an X-band radar is used for detection, and an HPRF DEW is used for defense. The two operating configurations demonstrated that team survivability is enhanced in the case where the target of the missile has an active defense mechanism. Further work will examine whether an optimal selection of sensors and effectors can be determined. In addition to this, it is important to investigate whether the assumptions adopted throughout the paper can be relaxed, such as independent detections and non-cumulative HPRF DEW effects on targets.

## REFERENCES

1. NATO Panel SCI-294, <https://www.sto.nato.int/Lists/test1/activitydetails.aspx?ID=16085&ContentTypeId=0x0100880490B4FA57FB4E87FCC3786387978C>.
2. Braidwood, S. and K. Hong, "Stopping car engines using high power electromagnetic pulses," *International Conference on Electromagnetics in Advanced Applications*, 2010.
3. Bayram, Y., J. L. Volakis, S. K. Myoung, S. J. Doo, and P. Robin, "High power EMI on RF amplifier and digital modulation schemes," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 50, 849–860, 2008.
4. Esteves, J., E. Cottais, and C. Kasmi, "Analysis of HPEM perturbations induced on the navigation system of a UAV," *AMEREM*, 2018.
5. Graswald, M., R. Gutser, F. Grabner, B. Meyer, C. Winter, and A. Oelerich, "Defeating UAVs through novel HPEM effectors," *31st International Symposium on Ballistics*, India, 2019.
6. Radasky, W. A., "The threat of Intentional Electromagnetic Interference (IEMI) to wired and wireless systems," *17th International Zurich Symposium on Electromagnetic Compatibility*, 2006.
7. Clarke, T., A. Taylor, N. Estep, S. Yakura, D. Brumit, D. Dietz, S. Hemmady, and J. Duffet, "Predictive modeling of high-power electromagnetic effects on electronics," *International Conference on Electromagnetics in Advanced Applications*, 2011.
8. Radasky, W. A., C. E. Baum, and M. W. Wik, "Introduction to the special issue on High Power Electromagnetics (HPEM) and Intentional Electromagnetic Interference (IEMI)," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 46, 314–321, 2004.
9. Giri, D. V. and F. M. Tesche, "Classification of Intentional Electromagnetic Environments (IEME)," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 46, 322–328, 2004.
10. Barnes, F. S. and B. Greenebaum, (Editors), *Handbook of Biological Effects of Electromagnetic Fields: Biological and Medical Aspects of Electromagnetic Fields*, 3rd Edition, CRC Press, 2006.
11. Simon, M. D., "Solid-state high power radio frequency directed energy systems in support of USMC force protection operations," Masters Thesis, Naval Postgraduate School, Monterey, 2015.
12. Barton, D. K., "Land clutter models for radar design and analysis," *Proceedings of the IEEE*, 73, 1985.
13. Skolnik, M., *Radar Handbook*, McGraw-Hill, New York, 2008.
14. Levanon, N., *Radar Principles*, John Wiley and Sons, New York, 1988.
15. Shnidman, D. A., "The calculation of the probability of detection and the generalized Marcum Q-function," *IEEE Transactions on Information Theory*, Vol. 35, 389–400, 1989.
16. Currie, N. C., *Radar Reflectivity Measurement, Techniques and Applications*, Artech House, 1989.