

A Prior Parameter Extraction Method for the Solution of Wide-Angle Electromagnetic Scattering Problems Based on Compressed Sensing

Daoping Wang^{1, 2}, Mingsheng Chen^{3, *}, Xinyuan Cao²,
Qi Qi², Xiangxiang Liu², and Chundong Hu¹

Abstract—A fast solution for electromagnetic scattering problems over a wide incident angle based on compressed sensing (CS) was proposed in recent years. Since current expansion coefficients are not known in advance, the parameters of this solution (e.g., the times of measurements, the selection of sparse transforms) for different scattering objects are difficult to determine. In order to solve this problem, this paper presents a prior parameter extraction method based on the principle of on-surface discretized boundary equation (OS-DBE), in which an approximate distribution of current expansion coefficients at any given point of the scatterer is first obtained with low-coverage and low-complexity, and then the prior parameters can be determined by CS tests for the approximate result. The implementation method is elaborated, and its effectiveness is verified by numerical results.

1. INTRODUCTION

As a classic numerical method in computational electromagnetics (CEM), method of moments (MoM) [1] is widely used in the analysis of electromagnetic (EM) scattering. With the development of improved schemes for MoM, many fast algorithms have been proposed, e.g., fast multipole method (FMM) [2], conjugate gradient fast Fourier transform (CG-FFT) [3], adaptive integral method (AIM) [4], and adaptive cross approximation (ACA) [5].

Recently, aiming at the EM scattering problems over a wide incident angle, a fast solution, in which compressed sensing (CS) [6] theory is introduced into MoM, is proposed [7, 8]. This CS-MoM solution constructs a new kind of incident sources by randomly adding the incident waves from different angles together, and unknown current expansion coefficients can be reconstructed precisely with only a few times of traditional MoM computation.

However, since current expansion coefficients are not known in advance, while solving practical EM problems, the key parameters of the CS-MoM solution, e.g., the times of measurements (i.e., the times of MoM computation), the selection of sparse transforms, are difficult to predetermine. In order to obtain the prior knowledge, a prior parameter extraction method based on the principle of on-surface discretized boundary equation (OS-DBE) [9] is proposed for the CS-MoM solution in this paper. The formulas are described in detail, and its effectiveness will be verified by numerical results.

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* Corresponding author: Mingsheng Chen (chenms@ustc.edu.cn).

¹ Institute of Plasma Physics, Chinese Academy of Science, Hefei, Anhui, China. ² Anhui Province Key Laboratory of Simulation and Design for Electronic Information System, Hefei Normal University, Hefei, Anhui, China. ³ School of Electronic Engineering, Huainan Normal University, Huainan, Anhui, China.

2. FORMULATIONS

2.1. CS-MoM Solution

As electromagnetic integral equations are solved by MoM, a matrix equation could be established as

$$\mathbf{Z}_{N \times N} \mathbf{I}_{N \times 1} = \mathbf{V}_{N \times 1} \quad (1)$$

in which \mathbf{Z} , \mathbf{I} , \mathbf{V} represent the impedance matrix, current expansion coefficient vector, and excitation vector respectively, and N is the number of basis functions.

While the scatterer is illuminated by incident wave with a wide angle, Eq. (1) could be transformed to the form of multiple right-hand sides as

$$\mathbf{Z}_{N \times N} [\mathbf{I}_1 \ \mathbf{I}_2 \ \dots \ \mathbf{I}_n]_{N \times n} = [\mathbf{V}_1 \ \mathbf{V}_2 \ \dots \ \mathbf{V}_n]_{N \times n} \quad (2)$$

in which n represents the discrete number of the wide angle.

Generally, in order to solve Eq. (2), n times of MoM computation need to be performed. But in the CS-MoM solution, with the establishment of a new kind of incident sources, this number can be reduced effectively. The new excitations could be represented as

$$\mathbf{V}_i^{\text{CS}} = \alpha_{i1} \mathbf{V}_1 + \alpha_{i2} \mathbf{V}_2 + \dots + \alpha_{in} \mathbf{V}_n \quad (i = 1, 2, \dots, m, m \ll n) \quad (3)$$

Since the impedance matrix does not change with the incident angle, the matrix equations excited by the new sources can be established as

$$\mathbf{Z}_{N \times N} [\mathbf{I}_1^{\text{CS}} \ \mathbf{I}_2^{\text{CS}} \ \dots \ \mathbf{I}_m^{\text{CS}}]_{N \times m} = [\mathbf{V}_1^{\text{CS}} \ \mathbf{V}_2^{\text{CS}} \ \dots \ \mathbf{V}_m^{\text{CS}}]_{N \times m} \quad (4)$$

in which

$$\mathbf{I}_i^{\text{CS}} = \alpha_{i1} \mathbf{I}_1 + \alpha_{i2} \mathbf{I}_2 + \dots + \alpha_{in} \mathbf{I}_n \quad (i = 1, 2, \dots, m) \quad (5)$$

and $m \ll n$. With m times of calculation for solving Eq. (4), \mathbf{I}_i^{CS} could be obtained.

In the view of CS theory, \mathbf{I}_1^{CS} , \mathbf{I}_2^{CS} , ..., \mathbf{I}_m^{CS} can be exactly regarded as m measurements of $[\mathbf{I}_1, \mathbf{I}_2, \dots, \mathbf{I}_n]^T$, which could be described as

$$\mathbf{\Phi}_{m \times n} [\mathbf{I}_1 \ \mathbf{I}_2 \ \dots \ \mathbf{I}_n]_{n \times N}^T = [\mathbf{I}_1^{\text{CS}} \ \mathbf{I}_2^{\text{CS}} \ \dots \ \mathbf{I}_m^{\text{CS}}]_{m \times N}^T \quad (6)$$

in which $\mathbf{\Phi}$ is the measurement matrix, and its i th row is $[\alpha_{i1} \ \alpha_{i2} \ \dots \ \alpha_{in}]$. As the sparse transform matrix (denoted by $\mathbf{\Psi}$) is introduced, Eq. (6) could be rewritten as

$$\mathbf{\Phi}_{m \times n} \mathbf{\Psi}_{n \times k} \mathbf{\Gamma}_{k \times N} = [\mathbf{I}_1^{\text{CS}} \ \mathbf{I}_2^{\text{CS}} \ \dots \ \mathbf{I}_m^{\text{CS}}]_{m \times N}^T \quad (7)$$

in which the j th column of $\mathbf{\Gamma}$ is the K -sparse representation for the j th column of $[\mathbf{I}_1 \ \mathbf{I}_2 \ \dots \ \mathbf{I}_n]^T$ ($j = 1, 2, \dots, N$). Finally, with the application of recovery algorithms [10], $\mathbf{\Gamma}$ could be obtained, and the unknown current expansion coefficients can be calculated via

$$[\mathbf{I}_1 \ \mathbf{I}_2 \ \dots \ \mathbf{I}_n]_{n \times N}^T = \mathbf{\Psi}_{n \times k} \mathbf{\Gamma}_{k \times N} \quad (8)$$

In summary, the number of times of MoM computation is reduced from n to m in the CS-MoM solution, thus accomplishing fast calculation of EM scattering problems over a wide incident angle. The total complexity of CS-MoM method can be represented as $O(pmN^2 + KmnN)$, and compared with the traditional n times of MoM computation with the complexity of $O(pnN^2)$, the effectiveness improved by this method is about $m/n + Km/pN$, in which p is the iteration counter for solving each matrix equation in MOM by iteration methods, and generally $m \ll n$, $m \ll N$ and $K \ll p$ [7]. Moreover, unlike interpolation and approximation schemes, e.g., Lagrange/spectrum (FFT with zero padding) interpolation method, each new excitation (shown as Eq. (3)) established in the CS-MoM solution essentially includes all the information from n incident angles, which implies that the K non-zero elements in each column of $\mathbf{\Gamma}$ contain the major information and properties of the induced current signal. Therefore, compared with the interpolation schemes, the CS-MoM method theoretically has the potential to take less time of MoM computation, since the value of K could be possibly further reduced with more suitable sparse transforms. However, for the reason that $[\mathbf{I}_1 \ \mathbf{I}_2 \ \dots \ \mathbf{I}_n]^T$ is not known in advance, it is difficult to predetermine the value of m (i.e., the times of measurements) and judge the performances of different sparse transform matrices for different scatterers. To solve this difficulty, a prior parameter extraction method based on the principle of OS-DBE is proposed in this paper.

2.2. Basic Principle of OS-DBE

OS-DBE is a numerical method focusing on the surface current computation at any given point of the scatterer independently, which mainly includes two steps:

Step 1: A small-scale matrix equation is established based on “partial coverage” centering at the given point (node “0”), which is often written as

$$\mathbf{B}_{M \times M'} \mathbf{C}_{M' \times 1} = \mathbf{E}_{M \times 1}^i \quad (9)$$

in which \mathbf{B} is the small-scale impedance matrix which is usually generated from the electric field integral equation (EFIE) discretization; \mathbf{C} and \mathbf{E}^i represent the current expansion coefficients and incident electric field, respectively; $M \ll N$, $M' \ll N$ (N represents the total number of basis functions); meanwhile, $M > M'$, $M = M'$, $M < M'$ are all suitable. By solving Eq. (9), an approximate current expansion coefficient vector (i.e., \mathbf{C}) could be obtained.

Step 2: The scattering magnetic field on node “0” is calculated based on the approximate current expansion coefficients as

$$H_0^s = [\mathbf{d}_0]_{1 \times M'}^T \mathbf{C}_{M' \times 1} \quad (10)$$

in which H_0^s represents the tangential projection of the scattering magnetic field on node “0”, and \mathbf{d}_0 is generated from the discretization of the magnetic field expression (MFE). Eventually, the current expansion coefficient at the given point can be determined by

$$I_0 = -(H_0^i + H_0^s) \quad (11)$$

in which H_0^i represents the tangential projection of the incident magnetic field on node “0”.

2.3. Prior Parameter Extraction Method

Based on OS-DBE, the prior parameter extraction method for the CS-MoM solution is implemented as follows:

First, denoting the coupling between the scattered and incident fields on the node “0” as \mathbf{f}_0 [11], the following matrix equation is established and solved with a low coverage rate:

$$[\mathbf{B}]_{M' \times M}^T [\mathbf{f}_0]_{M \times 1} = [\mathbf{d}_0]_{M' \times 1} \quad (M \ll N, M' \ll N). \quad (12)$$

Second, the tangential components of the scattering magnetic field on the node “0” excited by incident waves with n discrete angles are calculated by

$$[H_0^s \ 1 \ H_0^s \ 2 \ \dots \ H_0^s \ n]_{1 \times n} = [\mathbf{f}_0]_{1 \times M}^T [\mathbf{E}_1^i \ \mathbf{E}_2^i \ \dots \ \mathbf{E}_n^i]_{M \times n}, \quad (13)$$

and then according to Eq. (11), the current expansion coefficient vector on the node “0” over the wide incident angles (denoted by $\mathbf{I}_0^{\text{OS-DBE}}$) is obtained as

$$[\mathbf{I}_0^{\text{OS-DBE}}]_{n \times 1} = [-(H_0^i \ 1 + H_0^s \ 1) - (H_0^i \ 2 + H_0^s \ 2) \dots - (H_0^i \ n + H_0^s \ n)]^T. \quad (14)$$

Third, taking $\mathbf{I}_0^{\text{OS-DBE}}$ as the known signal, CS algorithm could be tested via

$$\Phi_{m \times n} [\mathbf{I}_0^{\text{OS-DBE}}]_{n \times 1} = \mathbf{S}_{m \times 1} \quad (15)$$

$$(\Phi_{m \times n} \Psi_{n \times k}) \Gamma'_{k \times 1} = \mathbf{S}_{m \times 1} \quad (16)$$

$$\text{argmin} \|\Gamma'_{k \times 1}\|_1 \quad \text{s.t.} \quad \Phi_{m \times n} \Psi_{n \times k} \Gamma'_{k \times 1} = \mathbf{S}_{m \times 1} \quad (17)$$

in which Φ and Ψ represent the measurement matrix and sparse transform matrix; \mathbf{S} is the measurement result; and Γ' is the sparse representation of $\mathbf{I}_0^{\text{OS-DBE}}$. Thus, the CS test results, e.g., the times of measurements needed for accurate reconstruction of $\mathbf{I}_0^{\text{OS-DBE}}$ with different measurement matrices, sparse transform matrices and recovery algorithms, could be collected.

Finally, taking the above test results as the prior knowledge, the key parameters of the CS-MoM solution, e.g., the times of measurements (i.e., the number of the new incident sources, as shown in Eq. (3)), the selection of sparse transforms, can be pre-extracted.

2.4. Discussion

In the above prior parameter extraction method, the current expansion coefficient vector at any given point of the scatterer over a wide incident angle obtained based on OS-DBE is taken as the prior signal for the CS-MoM solution, which originates from the fact that there exist strong correlations among current expansion coefficient vectors at each point (i.e., each column in $[\mathbf{I}_1 \ \mathbf{I}_2 \ \dots \ \mathbf{I}_n]^T$, as shown in Eq. (6)). Hence, the CS test results of any current expansion coefficient vector over wide incident angles can provide prior knowledge for CS calculations of other vectors.

Besides, differing from the exact OS-DBE method which has a certain requirement on the coverage rate (CR) for obtaining an accurate result (especially for 3-D objects, CR is usually demanded to be high), the requirement of CR is not so strict in the proposed method for the reason that an approximation of real current expansion coefficients is sufficient to provide prior knowledge for the calculation of the CS-MoM solution.

Furthermore, with the introduction of the coupling between the scattered and incident fields (i.e., \mathbf{f}_0 in Eq. (12)), $\mathbf{I}_0^{\text{OS-DBE}}$ could be obtained by only one-time calculation of a small-scale matrix equation (shown as Eq. (12)) and one-time matrix-vector multiplication (shown as Eq. (13)), so the computational complexity is low.

3. NUMERICAL RESULTS

In order to verify the effectiveness of the proposed prior parameter extraction method clearly, three numerical experiments of 3-D EM scattering problems are analyzed.

3.1. Sphere

A perfect electric conductor (PEC) sphere with a radius of 1 m is illuminated by the wide-angle incident wave with a frequency of 300 MHz in the xy plane, and the range of the incident angle is $0^\circ \sim 360^\circ$ which is discretized by the interval of 1° . EFIE is solved by MoM with RWG basis functions ($N = 960$). Gaussian random matrix, Fourier basis, and orthogonal matching pursuit (OMP) are employed as the measurement matrix, sparse transform matrix, and recovery algorithm, respectively, in the CS-MoM solution. An arbitrary RWG panel is selected randomly, and the partial coverage is set to revolve around the given basis function (CR = 0.1) in the proposed prior parameter extraction method.

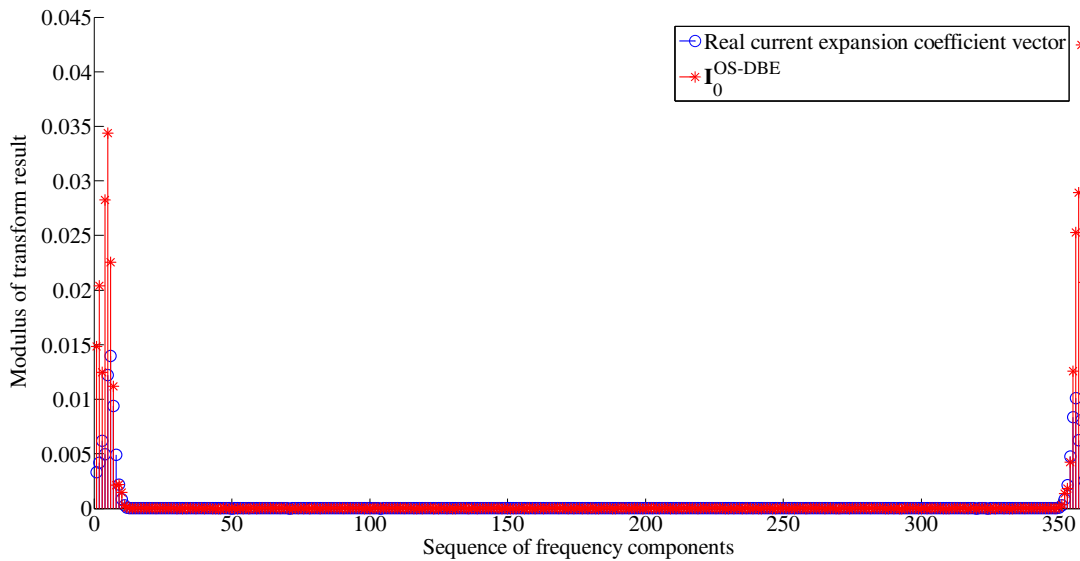


Figure 1. Sparse distributions in frequency domain transformed from the real current expansion coefficient vector and $\mathbf{I}_0^{\text{OS-DBE}}$ at the same RWG panel of the sphere.

Firstly, the Fourier transform result of the real current expansion coefficient vector at the given RWG panel is compared with the one calculated from the prior parameter extraction method (i.e., $\mathbf{I}_0^{\text{OS-DBE}}$, as shown in Eq. (14)) in Figure 1, which implies that the sparsity of the signal transformed from $\mathbf{I}_0^{\text{OS-DBE}}$ with a low CR is consistent with the one transformed from the real current expansion coefficient vector by the same Fourier basis.

Then, the recovery errors changing with the times of measurements in practical calculation of the CS-MoM solution and the corresponding CS test results of $\mathbf{I}_0^{\text{OS-DBE}}$ are compared in Figure 2. One can see that the recovery error curve in the practical CS-MoM solution and the CS test result of $\mathbf{I}_0^{\text{OS-DBE}}$ approach to 0 from the 68th measurement (the specific error values are respectively 3.219×10^{-9} and 3.880×10^{-9}), which means that the prior knowledge of the time of measurements provided by the proposed method is correct.

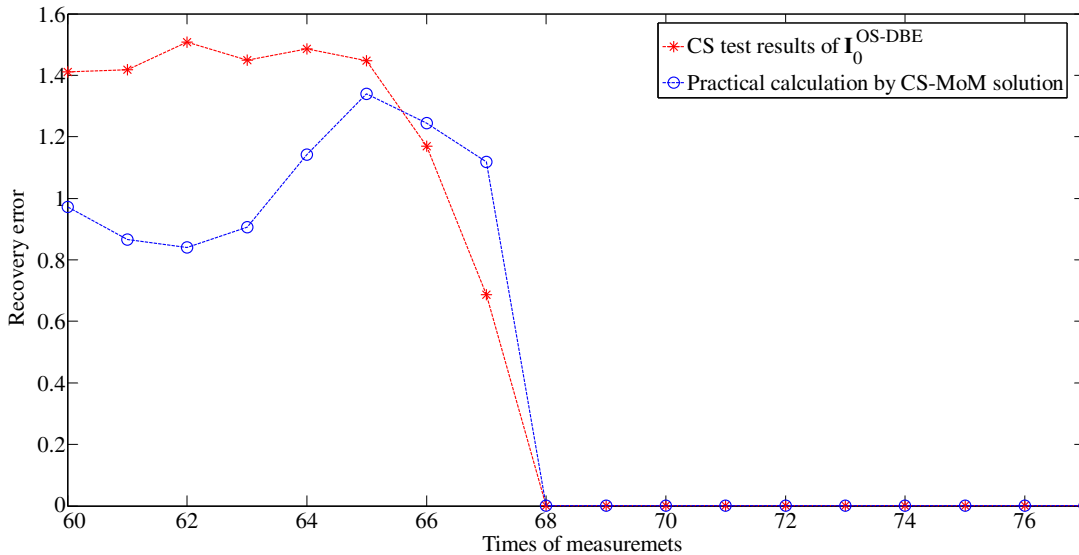


Figure 2. Comparison of recovery errors changing with times of measurements for the sphere.

3.2. Cube

A PEC cube with an edge length of 1 m is chosen as the scatterer, and 900 RWG basis functions are established on its surface. Considering that the current on vertices usually changes dramatically, the center of partial coverage is selected at one vertex of the cube. In order to further verify the correctness of the prior knowledge provided by the prior parameter extraction method for selection of suitable sparse transforms in the CS-MoM solution, the excitation matrix is added as another sparse transform matrix [12]. Other experimental parameters keep the same as the former numerical example.

The recovery errors in the practical CS-MoM solution with the two sparse transform matrices relative to the CS test results of $\mathbf{I}_0^{\text{OS-DBE}}$ are described in Figure 3. From Figure 3, one can see that the prior minimum numbers of measurements for accurate recovery with the sparse transform of Fourier basis and the one with the sparse transform of excitation matrix are both correct, and the number of measurements (i.e., the times of MoM computation in the CS-MoM solution) is much fewer while using the excitation matrix as the sparse transform (the specific error values of the practical CS-MoM solution and the CS test result of $\mathbf{I}_0^{\text{OS-DBE}}$ at 74th measurement in (a) are respectively 4.476×10^{-5} and 3.428×10^{-5} , and the ones at 25th measurement in (b) are respectively 5.974×10^{-5} and 1.016×10^{-5}). Therefore, compared with Fourier basis, the excitation matrix is a more suitable sparse transform matrix, which is in accord with the conclusion of Ref. [12].

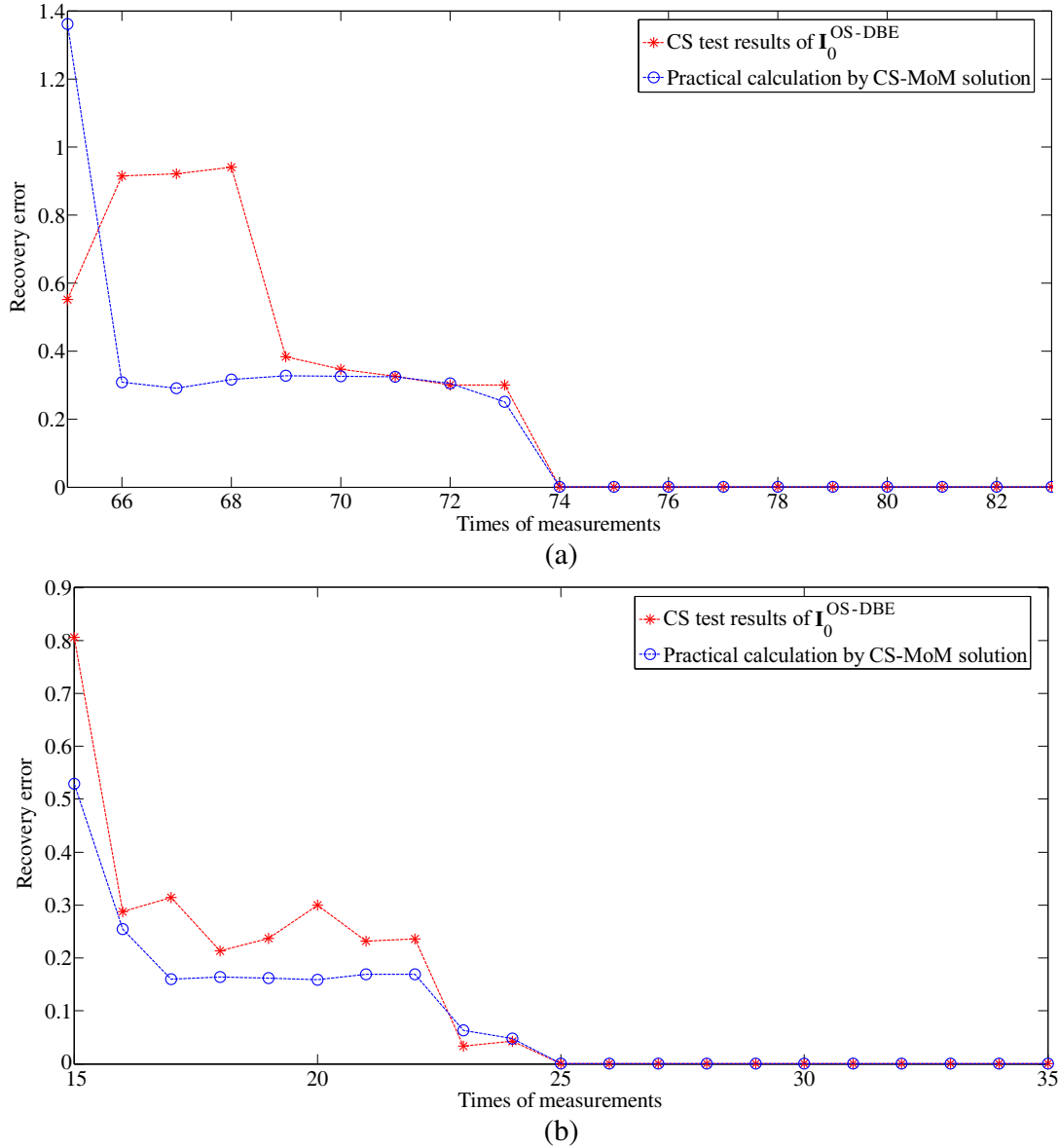


Figure 3. Comparison of recovery errors changing with times of measurements for the cube with the sparse transform of (a) Fourier basis and (b) excitation matrix.

3.3. Missile Model

In the third numerical experiment, the scatterer is changed to a missile model (as shown in Figure 4), and other experimental parameters are set to be the same as the second numerical example. The comparisons of relationships between recovery errors and the times of measurements while using Fourier basis and the excitation matrix as the sparse transform are shown in Figure 5(a) and Figure 5(b), respectively. The results indicate that the prior knowledge from the proposed method is still correct. To obtain an accurate recovery result, the prior minimum number of measurements matched with Fourier basis is 88, and the one matched with the excitation matrix is 38 (the corresponding specific error values are 2.357×10^{-5} and 9.909×10^{-5} , respectively, while the errors of the practical CS-MoM solution with the same numbers of measurements are 1.818×10^{-5} and 9.121×10^{-5}), which implies that the excitation matrix is a more suitable selection for sparse transform in the practical calculation of the CS-MoM solution.

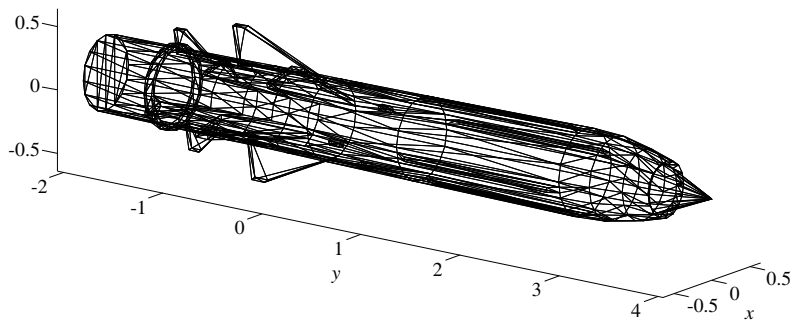


Figure 4. The missile model.

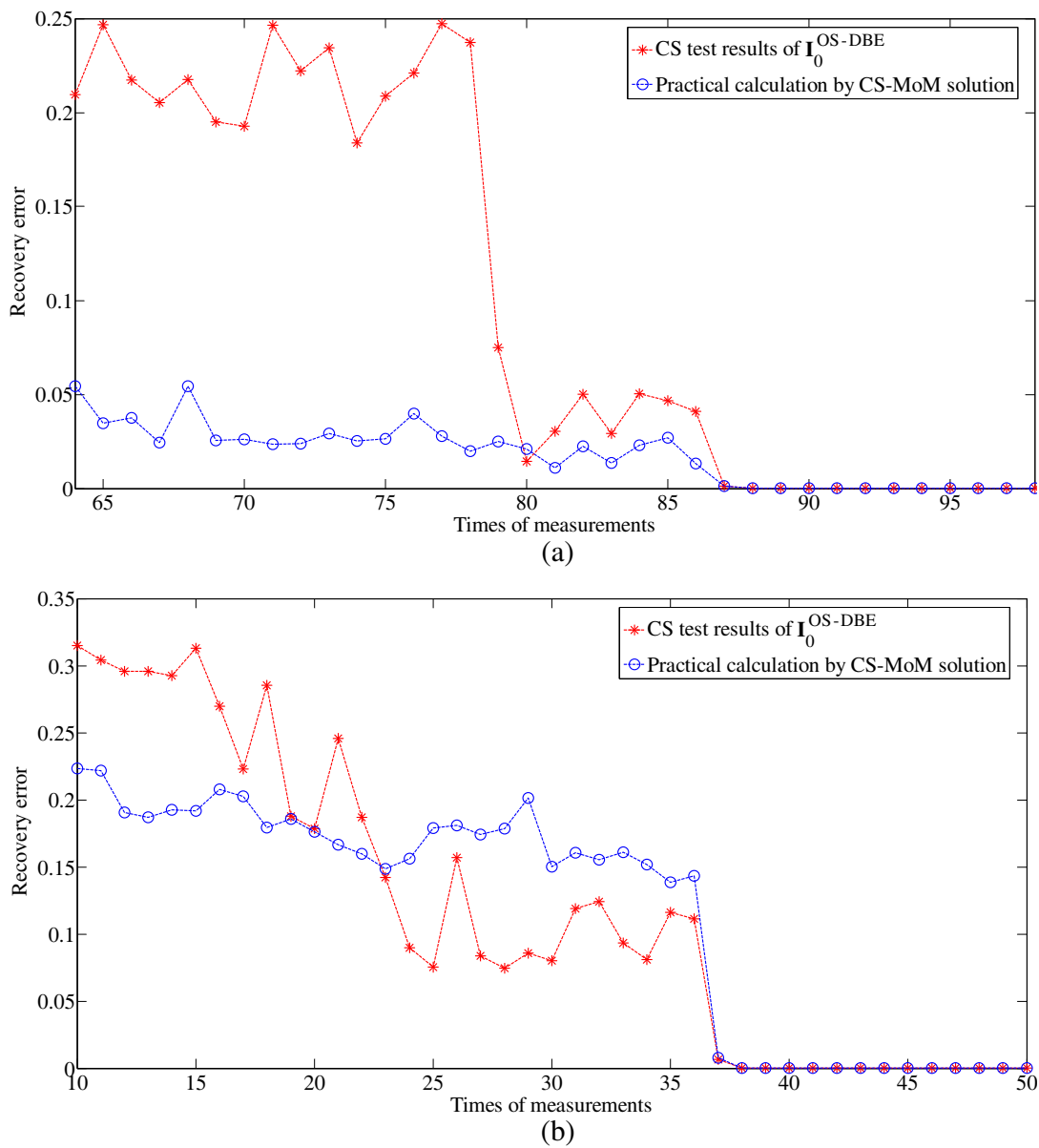


Figure 5. Comparison of relationships between recovery errors and times of measurements for the missile model while the sparse transform is set to (a) Fourier basis and (b) excitation matrix.

4. CONCLUSION

With the introduction of CS theory into MoM, a fast solution of EM scattering problems over a wide incident angle has been established. For the purpose of predetermination of parameters of the CS-MoM solution in practical application, this paper puts forward a prior parameter extraction method. The method adopts partial coverage to obtain an approximate result of current expansion coefficient vector at any given point of the scatterer based on the principle of OS-DBE with low computational cost, and then taking the approximate result as a prior signal, performs CS test to acquire the prior information for setting the key parameters in the practical calculation of the CS-MoM solution.

Numerical results show that the prior knowledge provided by the proposed method is correct and effective, since for the prior signal obtained by OS-DBE and the real current expansion coefficients, as the recovery error approaches to zero, the relationship curves between the recovery errors and the numbers of measurements for the prior signal obtained by OS-DBE and the one for the real current expansion coefficients agree well with each other. Especially, while using different sparse transforms (e.g., Fourier basis, the excitation matrix), the above phenomenon still exists.

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