Spatial Magnetic Field Calculations for Coreless Circular Coils with Rectangular Cross-Section of Arbitrary Turn Numbers

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Abstract—In a wireless magnetic induction communication system, the magnetic field distribution of the current-carrying coil affects the communication effect between the communication transceiver and receiver. In the study of magnetic field distribution, it was found that magnetic induction intensity and magnetic flux were important parameters to measure the effectiveness of communication. Aiming at the circular coils with rectangular cross-section of any turn numbers, this paper proposed an improved algorithm to calculate the magnetic induction intensity at any spatial position based on Biot-Savart law. At the same time, the calculation formula of the magnetic flux at the receiving point was also given. The coils were modeled and simulated with COMSOL software. The correctness of the improved algorithm was verified and compared with the traditional formula and simulation results, especially in the near field, which provided an important theoretical support for the further study of mutual inductance in the wireless magnetic induction communication system.

1. INTRODUCTION

In wireless magnetic induction communication, it is necessary to study the magnetic field distribution of a current-carrying coil, and Biot-Savart law is the most basic formula to analyze and calculate the magnetic field excited by current-carrying wire in space. The magnetic field distribution of current-carrying circular coil in space has been studied by many scholars [1–6]. In [1], formulas were derived giving the vector potential and magnetic field components of a general coil of a rectangular cross section and constant winding density. The solution was given in a cylindrical coordinate system in terms of trigonometric integrals. Ref. [2] presented analytical calculations of magnetic parameters in cylindrical magnets and coils, and used elliptic integration to calculate the analytical expressions of radial and axial magnetic field components produced by either a thin coil or a ring permanent magnet whose polarization is axial. In [3], a numerical calculation method of magnetic induction intensity at any point in the space around the circular current was derived by using the principle of "circle cutting method", which can conveniently calculate the magnetic induction intensity at any point in the space with high accuracy. In [4], the integral expression of the magnetic induction intensity distribution of a single current-carrying circular coil at any point in space was analyzed and obtained analytical calculation results. Based on the results of a single current-carrying circular coil, the magnetic induction distribution of a long straight solenoid at any point in space was analyzed, and the "uniform region" of its magnetic field distribution was discussed by numerical calculation. In [5], the magnetic field distribution of a single current-carrying circular coil in space was extended to a Helmholtz coil, and the analytical expression of magnetic induction intensity of a Helmholtz coil at any point in space was obtained. According to the axial magnetic field formula of circular coil and spiral coil in [6], the axial magnetic field of a circular coil and spiral coil was simulated respectively with the help of finite element simulation software. It

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analyzed the influence of axial distance, coil radius, and coil turns on magnetic field, and obtained the optimal coil radius for maximum magnetic field strength under any axial distance, which provided some ideas for the design of a coil coupler.

Although scholars have made a full research and analysis on the magnetic field excited by the current-carrying coil in space, there is little research on the coreless circular coil with rectangular cross-section, and there is a lack of accurate modeling and analysis of the coreless circular coil with rectangular cross-section. In this paper, the model of a coreless circular coil with rectangular cross-section in an arbitrary spatial position was established. According to Biot-Savart law, a numerical calculation method of magnetic induction intensity distribution of coreless circular coil with rectangular cross-section at any point in space was derived, so as to accurately calculate the coil magnetic flux at the wireless magnetic induction communication receiver, and provided an important theoretical support for further analysis of the effectiveness of wireless magnetic induction communication.

2. BASIC MODEL OF MAGNETIC INDUCTION COMMUNICATION

Magnetic induction communication is mainly based on Faraday's law of electromagnetic induction, using the coupling between magnetic dipoles to transfer information. This section mainly introduces and analyzes the transmission principle and basic model of wireless magnetic induction.

2.1. Principle of Magnetic Induction Signal Transmission

The basic principle of wireless magnetic induction communication technology is Faraday's law of electromagnetic induction. When current changes with time in a closed conductor, a time-varying magnetic field will be generated around the closed conductor. At this time, a time-varying electromotive force will be induced on another closed conductor in this space. Wireless magnetic induction communication is to load the modulated baseband signal onto the transmitting coil, which makes the transmitting coil generate an alternating magnetic field in space. At this time, the magnetic flux in the closed area surrounded by the receiving coil in the changing magnetic field also changes. Therefore, the corresponding induced electromotive force is generated on the receiving coil, and the baseband signal is completely restored after signal demodulation to complete the transmission of information.

As shown in Fig. 1, two coreless circular coils C_1 and C_2 are used as the transmitting and receiving antennas for magnetic induction communication. Their areas are S_1 and S_2; the turns are N_1 and N_2; and the distance between the geometric centers of the two coils is r. When the alternating current I_1 is loaded into C_1, a variable magnetic field is generated around it. Φ_1 is the magnetic flux generated by current I_1 in the transmitting coil C_1, and the partial flux Φ_{21} cross linked to the receiving coil C_2 is:

$$\Phi_{21} = \int_S \mathbf{B} \cdot \mathbf{n} ds \tag{1}$$

where $\mathbf{n}$ is the normal vector of the receiving coil, and $\mathbf{B}$ is the magnetic induction intensity at the receiving coil position.
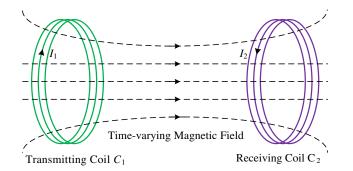

Figure 1. Basic model of wireless magnetic induction communication.

The magnetic linkage generated in the closed area surrounded by C_2 is:

$$\Psi = N_2\Phi_{21} \tag{2}$$

According to Faraday's law of electromagnetic induction, the induced electromotive force on the receiving coil C_2 is [7]:

$$\varepsilon = -\frac{d\Psi}{dt} = -N_2\frac{d\Phi_{21}}{dt} = -M\frac{dI_1}{dt} \tag{3}$$

where M is the mutual inductance between the transceiver coils.

2.2. Basic Model of Magnetic Dipole

Figure 2 shows the radiation produced by current-carrying coreless circular coil at point P in space. In the spherical coordinate system, the radius of the transmitting coil is a. The geometric center and central axis of the coil coincide with the origin O and z-axis of the coordinate system O-xyz, respectively. r is the distance from point P in space to the origin O, θ the angle between OP and z-axis, φ the angle between the line of the projection point of point P and the origin in xoy plane and x-axis, R the distance from point P to a current element on the transmitting coil, and ϕ the angle between the line of the current element dl and the origin and x-axis. The alternating current loaded in the coil $I = I_m\sin(\omega t)$, where I_m is the current amplitude, $\omega = 2\pi f$ the angular frequency of the current, and f the frequency of the current.
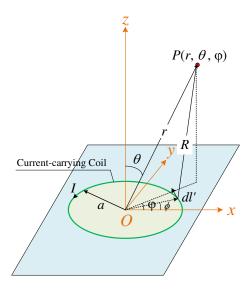

Figure 2. Radiation produced by the current-carrying coil at point P in space.

The coil can be regarded as a magnetic dipole, which generates a time-varying electromagnetic field around it. In order to calculate the magnetic field of the magnetic dipole at any point $P(r, \theta, \varphi)$ in space, the vector magnetic potential $\mathbf{A}$ generated by the current element in the coil needs to be analyzed, and the magnetic induction intensity $\mathbf{B}$ can be obtained by calculating the curl of $\mathbf{A}$. According to the relationship between the magnetic induction intensity and the magnetic field intensity $\mathbf{B} = \mu\mathbf{H}$, the magnetic field intensity $\mathbf{H}$ is finally obtained. Assuming $r \gg a$, the magnetic induction intensity produced by the circular coil at any point P is obtained [8]:

$$\mathbf{B} = \frac{\mu NIS}{4\pi r^3}(2\cos\theta\mathbf{e}_r + \sin\theta\mathbf{e}_\theta) \tag{4}$$

where $\mu = \mu_0\mu_r$ is the magnetic permeability, $\mu_0 = 4\pi\times10^{-7}$ H/m the magnetic permeability of vacuum, and μ_r the relative magnetic permeability of the medium. Since the research environment of this paper is air, set $\mu_r = 1$. N is the turn of the coil, I the loaded alternating current, $S = \pi a^2$ the area of the coil,

r the length of line OP, θ the angle between OP and z-axis, and $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors along the increasing direction of distance r and angle θ, respectively in a spherical coordinate system. The traditional model can be equivalently used to calculate the magnetic induction intensity of the circular coil with rectangular cross-section at a certain point in space. More details are provided in Appendix A. However, the accuracy of the model is poor, especially at close range.

3. MAGNETIC INDUCTION INTENSITY FOR CORELESS CIRCULAR COIL WITH RECTANGULAR CROSS-SECTION

Since Eq. (4) is approximately calculated under the condition that the distance r between point O and point P is far larger than the coil radius a, it is not suitable for solving the magnetic field distribution of point P in close distance, and multi-turns, large-radius coil. In this section, according to Eq. (4), the algorithm for magnetic induction intensity generated by the circular coil with rectangular cross-section at any point P in space is improved based on Biot-Savart law to make it more suitable for the calculation of coils with multi-turns and large radius in the near field. In order to simplify the analysis, the placement angle is limited to rotating only around its x-axis.

3.1. Coreless Circular Coil with Rectangular Cross-Section in Arbitrary Spatial Position

Figure 3 shows the point P and the circular coil with rectangular cross-section in arbitrary spatial position. The geometric center and central axis of the coil coincide with the origin O' and z'-axis of the coordinate system O'-$x'y'z'$, respectively. In the coordinate system O-xyz, the coordinate of point P is (x, y, z), and the origin O' of the coordinate system O'-$x'y'z'$ is (x', y', z'). Among them, α is the angle between z'-axis and the normal vector of the coil when it rotates around the x'-axis, $\alpha \in [0, 2\pi]$. N and n are the axial and radial turns of the coil, respectively, and d_0 is the diameter of the coil wire. Take a coil element with radius r on the coil, the point O'' is the center of the coil element, and $\vec{dl}$ is the infinitesimal element on it.
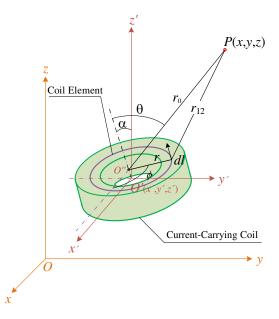

Figure 3. Radiation produced by the circular coil with rectangular cross-section in arbitrary spatial position at point P.

3.1.1. Axial Turn N is an Even Number

As shown in Fig. 3 and Fig. 4, in the coordinate system O'-$x'y'z'$, the coordinate of point O'' is $(0, 0, d_0(m - m/2|m|))$, and the coordinate of $\vec{dl}$ is $(r\cos\phi, r\sin\phi, d_0(m - m/2|m|))$, in which $\phi \in [0, 2\pi]$,
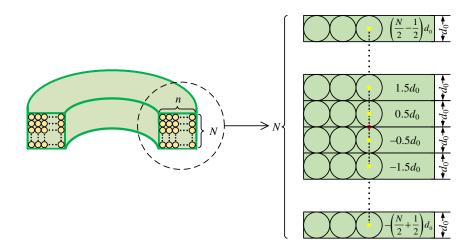
Figure 4. Rectangular cross-section with an even number of axial turn N.

$m \in [-N/2, N/2]$, and $m \neq 0$. When the rotating angle of the coil around its x'-axis is α, according to the coordinate transformation formula [9], the coordinates of the current element $\vec{dl}$ in the coordinate system $O\text{-}xyz$ of any turn of coil elements with the point O'' as the center in the axial direction of the coil can be obtained, and then the geometric distance r_{12} between point P and the infinitesimal element can also be obtained. After a series of calculation, the following parameters can be obtained:

Normal vector of the coil after rotating $\mathbf{n}$: $\{0, -\sin\alpha, \cos\alpha\}$;

The coordinates of the center O'' of the coil element in the coordinate system $O\text{-}xyz$ is: $(x', y' - d_0(m - m/2\,|m|)\sin\alpha, z' + d_0(m - m/2\,|m|)\cos\alpha)$;

From the vector multiplication operation, we know that $\mathbf{n} \cdot \overrightarrow{O''P} = |\mathbf{n}| \left|\overrightarrow{O''P}\right| \cos\theta$. Then the angle θ between $O''P$ and the central axis of the coil is:

$$\theta = \arccos \frac{-\left(y - y' + d_0\left(m - \dfrac{m}{2\,|m|}\right)\sin\alpha\right)\sin\alpha + \left(z - z' - d_0\left(m - \dfrac{m}{2\,|m|}\right)\cos\alpha\right)\cos\alpha}{\sqrt{(x - x')^2 + \left(y - y' + d_0\left(m - \dfrac{m}{2\,|m|}\right)\sin\alpha\right)^2 + \left(z - z' - d_0\left(m - \dfrac{m}{2\,|m|}\right)\cos\alpha\right)^2}};$$

The coordinates of $\vec{dl}$ in the coordinate system $O\text{-}xyz$ are: $(x' + r\cos\phi, y' + r\cos\alpha\sin\phi - d_0(m - m/2|m|)\sin\alpha, z' + r\sin\alpha\sin\phi + d_0(m - m/2|m|)\cos\alpha)$.

The geometric distance r_{12} between point P and the infinitesimal on the coil element is:

$$r_{12}^2 = (x' + r\cos\phi - x)^2 + \left(y' + r\cos\alpha\sin\phi - d_0\left(m - \frac{m}{2\,|m|}\right)\sin\alpha - y\right)^2$$
$$+ \left(z' + r\sin\alpha\sin\phi + d_0\left(m - \frac{m}{2\,|m|}\right)\cos\alpha - z\right)^2.$$

Therefore, according to Eq. (4), the magnetic induction intensity produced by any single-turn circular coil element with radius r in the axial direction at any point P in the space is:

$$\mathbf{B} = \int_0^{2\pi} \frac{\mu_0 I \pi r^2}{8\pi^2 r_{12}^3} (2\cos\theta \mathbf{e}_r + \sin\theta \mathbf{e}_\theta) d\phi \tag{5}$$

where θ is the angle between $O''P$ and the central axis of the coil, and $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors of the center O'' of the coil element along the increasing direction of distance r_0 and angle θ from point P in spherical coordinate system, respectively.
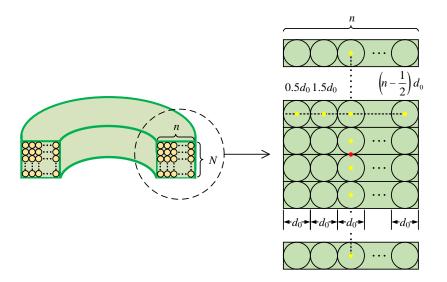
Figure 5. Rectangular cross-section with the radial turn n.

Through the superposition calculations of single-turn circular coil radially and axially, the calculation model of circular coils with rectangular cross-section in arbitrary spatial positions can be further obtained. As shown in Fig. 5, the radial radius of the circular coil element with rectangular cross-section is:

$$r_i = r_{\text{pi}} + \frac{r_{\text{po}} - r_{\text{pi}}}{n}\left(i - \frac{1}{2}\right) \tag{6}$$

where r_{pi} and r_{po} are the inner and outer radii of the coil, and n is the radial turn of the coil, $i \in [1, n]$. For the coil with tight wiring harness in the ideal state, the relationship $(r_{\text{po}} - r_{\text{pi}})/n = d_0$ is satisfied.

Substituting Eq. (6) into Eq. (5) yields the magnetic induction intensity produced by any coil element at point P in space:

$$\mathbf{B}_{im} = \int_0^{2\pi} \frac{\mu_0 I \pi r_i^2}{8\pi^2 r_{12_{im}}^3}(2\cos\theta \mathbf{e}_r + \sin\theta \mathbf{e}_\theta)d\phi \tag{7}$$

where $r_{12_{im}}$ is the geometric distance between point P and the infinitesimal on any turn of coil elements,

$$r_{12_{im}}^2 = \left(x' + r_i\cos\phi - x\right)^2 + \left(y' + r_i\cos\alpha\sin\phi - d_0\left(m - \frac{m}{2|m|}\right)\sin\alpha - y\right)^2$$
$$+ \left(z' + r_i\sin\alpha\sin\phi + d_0\left(m - \frac{m}{2|m|}\right)\cos\alpha - z\right)^2.$$

Based on Biot-Savart law [10], through the radial and axial superimpositions of any coil element, the magnetic induction intensity of a rectangular cross-section circular coil with an even number of axial turn at arbitrary point P in space is finally obtained:

$$\mathbf{B}_{even} = \sum_{m=-N/2}^{N/2}\sum_{i=1}^{n}\mathbf{B}_{im} = \sum_{m=-N/2}^{N/2}\sum_{i=1}^{n}\int_0^{2\pi} \frac{\mu_0 I \pi r_i^2}{8\pi^2 r_{12_{im}}^3}(2\cos\theta \mathbf{e}_r + \sin\theta \mathbf{e}_\theta)d\phi \tag{8}$$

It can be seen from Eq. (1) that the single turn's cross-section magnetic flux generated by the circular coil with rectangular cross-section at the receiving coil with point P as the geometric center in space is:

$$\Phi_{even} = \sum_{m=-N/2}^{N/2}\sum_{i=1}^{n}\int_0^{2\pi} \frac{\mu_0 I \pi r_i^2 S_r}{8\pi^2 r_{12_{im}}^3}(2\cos\theta \mathbf{e}_r \cdot \mathbf{e}_n + \sin\theta \mathbf{e}_\theta \cdot \mathbf{e}_n)d\phi \tag{9}$$

where $S_{\rm r}$ is the area of the receiving coil, and $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors of the center O'' of the transmitting coil element along the increasing direction of distance r_0 and angle θ from point P in spherical coordinate system, respectively. $\mathbf{e}_n$ is the normal vector of the receiving coil. The specific calculation steps are shown in Appendix A.

3.1.2. Axial Turn N Is an Odd Number

As shown in Fig. 3 and Fig. 6, in the coordinate system O'-$x'y'z'$, the coordinate of point O'' is $(0, 0, d_0 m)$, and the coordinate of $d\vec{l}$ is $(r\cos\phi, r\sin\phi, d_0 m)$, in which $\phi \in [0, 2\pi]$, $m \in [-(N-1)/2, (N-1)/2]$. After a series of calculation, the magnetic induction intensity of a rectangular cross-section circular coil with an odd number of axial turn at arbitrary point P in space is finally obtained:

$$\mathbf{B}_{odd} = \sum_{m=-(N-1)/2}^{(N-1)/2} \sum_{i=1}^{n} \mathbf{B}_{im} = \sum_{m=-(N-1)/2}^{(N-1)/2} \sum_{i=1}^{n} \int_{0}^{2\pi} \frac{\mu_0 I \pi r_i^2}{8\pi^2 r_{12_{im}}^3} (2\cos\theta \mathbf{e}_r + \sin\theta \mathbf{e}_\theta)d\phi \qquad (10)$$

where $r_{12_{im}}$ is the geometric distance between point P and the infinitesimal on any turn of coil elements,

$$r_{12_{im}}^2 = \left(x' + r_i\cos\phi - x\right)^2 + \left(y' + r_i\cos\alpha\sin\phi - d_0 m\sin\alpha - y\right)^2$$
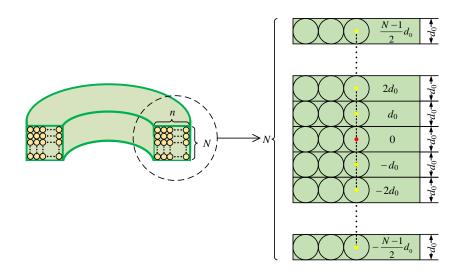$$+ \left(z' + r_i\sin\alpha\sin\phi + d_0 m\cos\alpha - z\right)^2.$$

Figure 6. Rectangular cross-section with an odd number of axial turn N.

It can be seen from Eq. (1) that the single turn's cross-section magnetic flux generated by the circular coil with rectangular cross-section at the receiving coil with point P as the geometric center in space is:

$$\Phi_{odd} = \sum_{m=-(N-1)/2}^{(N-1)/2} \sum_{i=1}^{n} \int_{0}^{2\pi} \frac{\mu_0 I \pi r_i^2 S_{\rm r}}{8\pi^2 r_{12_{im}}^3} (2\cos\theta \mathbf{e}_r \cdot \mathbf{e}_n + \sin\theta \mathbf{e}_\theta \cdot \mathbf{e}_n)d\phi \qquad (11)$$

3.2. Thin Solenoid Coil in Simplified Form

In order to preferably illustrate the applicability of the above formulas in solving the magnetic induction intensity at any point in space, a thin solenoid coil with even number of axial turns N and radius r is selected for a further study. Let point P and geometric center of the coil move only in yoz plane (i.e., $x = 0$), and when the point P is fixed, the magnetic fields excited by coils at different positions on y-axis are analyzed, respectively.
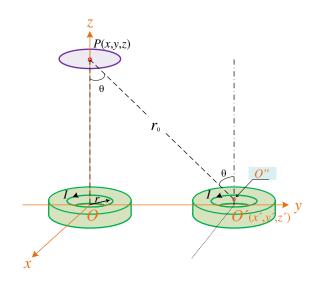
Figure 7. Radiation produced by the circular coils with rectangular cross-section at point P in space.

3.2.1. Coincide with the Coordinate Origin

As shown in Fig. 7, the coordinate of point P in the space is $(0, 0, z)$, and the geometric center of the coil is at the origin O, whose coordinates are $(0, 0, 0)$, and the rotating angle of the coil $\alpha = 0$. Then the coordinates of the current element on any turn of coil elements in the axial direction of the coil are $(r \cos \phi, r \sin \phi, d_0(m - m/2|m|))$, in which $\phi \in [0, 2\pi]$, $m \in [-N/2, N/2]$, and $m \neq 0$. Therefore, the geometric distance r_{12} between point P and the infinitesimal on the coil element can be obtained by calculating:

$$r_{12} = \sqrt{(r \cos \phi)^2 + (r \sin \phi)^2 + (z - d_0 (m - m/2 |m|))^2}$$
$$= \sqrt{r^2 + (z - d_0 m (1 - 1/2 |m|))^2}.$$

It can be seen from Eq. (8) that the magnetic induction intensity of the circular coil with rectangular cross-section at any point P in space is:

$$\mathbf{B}_{sd} = \sum_{m=-N/2}^{N/2} \int_0^{2\pi} \frac{\mu_0 I \pi r^2}{8\pi^2 r_{12}^3} (2\cos\theta \mathbf{e}_r + \sin\theta \mathbf{e}_\theta) d\phi \tag{12}$$

where $\theta = 0$. Therefore, the above formula can be further simplified as:

$$\mathbf{B}_{sd} = \sum_{m=-N/2}^{N/2} \frac{\mu_0 I \pi r^2}{2\pi r_{12}^3} \mathbf{e}_r \tag{13}$$

It can be seen from Eq. (9) that the single turn's cross-section magnetic flux generated by the circular coil with rectangular cross-section at the receiving coil with point P as the geometric center is:

$$\Phi_{sd} = \sum_{m=-N/2}^{N/2} \frac{\mu_0 I \pi r^2 S_r}{2\pi r_{12}^3} \mathbf{e}_r \cdot \mathbf{e}_n \tag{14}$$

In the spherical coordinate system of Fig. 7, when the normal vector of a single turn's cross section is the same as the transmitting coil, there is $\mathbf{e}_r \cdot \mathbf{e}_n = 1$, so the above formula can be further simplified as:

$$\Phi_{sd} = \sum_{m=-N/2}^{N/2} \frac{\mu_0 I \pi r^2 S_r}{2\pi r_{12}^3} \tag{15}$$

3.2.2. The Offset Distance from the Coordinate Origin Is d and No Rotating Angle

As shown in Fig. 7, the coordinate of point P in the space is $(0, 0, z)$; the coordinate of the geometric center of the coil is $(0, d, 0)$; and the rotating angle of the coil $\alpha = 0$. Then the coordinates of the current element $\vec{dl}$ on any turn of coil elements in the axial direction of the coil are $(r \cos \phi, d + r \sin \phi, d_0(m - m/2|m|))$, in which $\phi \in [0, 2\pi]$, $m \in [-N/2, N/2]$, and $m \neq 0$. Therefore, the geometric distance r_{12} between point P and the infinitesimal on the coil element can be obtained by calculating:

$$
\begin{aligned}
r_{12} &= \sqrt{(r \cos \phi)^2 + (d + r \sin \phi)^2 + (z - d_0 m (1 - 1/2 \, |m|))^2} \\
&= \sqrt{r^2 + d^2 + 2rd \sin \phi + + (z - d_0 m (1 - 1/2 \, |m|))^2}.
\end{aligned}
$$

It can be seen from Eq. (8) that the magnetic induction intensity of the circular coil with rectangular cross-section at any point P in space is:

$$
\mathbf{B}_{\mathrm{sd}} = \sum_{m=-N/2}^{N/2} \int_0^{2\pi} \frac{\mu_0 I \pi r^2}{8\pi^2 r_{12}^3} (2 \cos \theta \mathbf{e}_r + \sin \theta \mathbf{e}_\theta) d\phi \tag{16}
$$

where θ is the angle between $O''P$ and the central axis of the coil, and there is a relationship $\theta = \arctan [d/ (z - d_0 (m - m/2 \, |m|))]$.

It can be seen from Eq. (9) that the single turn's cross-section magnetic flux generated by the circular coil with rectangular cross-section at the receiving coil with point P as the geometric center is:

$$
\Phi_{\mathrm{sd}} = \sum_{m=-N/2}^{N/2} \int_0^{2\pi} \frac{\mu_0 I \pi r^2 S_{\mathrm{r}}}{8\pi^2 r_{12}^3} (2 \cos \theta \mathbf{e}_r \cdot \mathbf{e}_n + \sin \theta \mathbf{e}_\theta \cdot \mathbf{e}_n) d\phi \tag{17}
$$

In the spherical coordinate system of Fig. 7, when the normal vector of a single turn's cross section is the same as the transmitting coil, there are $\mathbf{e}_r \cdot \mathbf{e}_n = \cos \theta$ and $\mathbf{e}_\theta \cdot \mathbf{e}_n = -\sin \theta$, so the above formula can be further simplified as:

$$
\Phi_{\mathrm{sd}} = \sum_{m=-N/2}^{N/2} \int_0^{2\pi} \frac{\mu_0 I \pi r^2 S_{\mathrm{r}}}{8\pi^2 r_{12}^3} (2 \cos^2 \theta - \sin^2 \theta) d\phi \tag{18}
$$

3.2.3. The Offset Distance from the Coordinate Origin Is d and the Rotating Angle

As shown in Fig. 8, the coordinate of point P in the space is $(0, 0, z)$; the coordinate of the geometric center of the coil is $(0, d, 0)$; and the rotating angle of the coil is α. Then the coordinates of the current element $\vec{dl}$ of any turn of coil elements in the axial direction of the coil is $(r \cos \phi, d + r \cos \alpha \sin \phi - d_0 m (1 - 1/2 \, |m|) \sin \alpha, d_0 m (1 - 1/2 \, |m|) \cos \alpha + r \sin \alpha \sin \phi)$, in which $\alpha, \phi \in [0, 2\pi]$, $m \in [-N/2, N/2]$, and $m \neq 0$. Therefore, the geometric distance r_{12} between point P and the infinitesimal on the coil element can be obtained by calculating:

$$
\begin{aligned}
r_{12}^2 &= (r \cos \phi)^2 + (d + r \cos \alpha \sin \phi - d_0 m (1 - 1/2 \, |m|) \sin \alpha)^2 \\
&\quad + (z - d_0 m (1 - 1/2 \, |m|) \cos \alpha - r \sin \alpha \sin \phi)^2 = r^2 + d^2 + d_0^2/4 + d_0^2 m^2 - d_0^2 \, |m| + z^2 \\
&\quad + \cos \alpha (-2 d_0 m z + d_0 \, |m| \, z/m + 2rd \sin \phi) + \sin \alpha (-2 d d_0 m + d d_0 \, |m| \, /m - 2rz \sin \phi).
\end{aligned}
$$

The angle between $O''P$ and the central axis of the coil is $\theta = \gamma - \alpha$. Then, it can be seen from Eq. (8) that the magnetic induction intensity of the circular coil with rectangular cross-section at any point P in space is:

$$
\mathbf{B}_{\mathrm{sd}} = \sum_{m=-N/2}^{N/2} \int_0^{2\pi} \frac{\mu_0 I \pi r^2}{8\pi^2 r_{12}^3} (2 \cos (\gamma - \alpha) \mathbf{e}_r + \sin (\gamma - \alpha) \mathbf{e}_\theta) d\phi \tag{19}
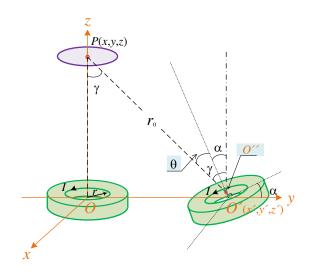$$

Figure 8. Radiation produced by the rotating circular coils with rectangular cross-section at point P in space.

where γ is the angle between $O''P$ and z-axis; α is the rotating angle of the coil around its x'-axis; and there is a relationship $\gamma = \arctan[(d - d_0(m - \frac{m}{2|m|})\sin\alpha)/(z - d_0(m - \frac{m}{2|m|})\cos\alpha)]$.

It can be seen from Eq. (9) that the single turn's cross-section magnetic flux generated by the circular coil with rectangular cross-section at the receiving coil with point P as the geometric center is:

$$\Phi_{\mathrm{sd}} = \sum_{m=-N/2}^{N/2} \int_0^{2\pi} \frac{\mu_0 I \pi r^2 S_{\mathrm{r}}}{8\pi^2 r_{12}^3} \left(2\cos\left(\gamma - \alpha\right)\mathbf{e}_r \cdot \mathbf{e}_n + \sin\left(\gamma - \alpha\right)\mathbf{e}_\theta \cdot \mathbf{e}_n\right) d\phi \tag{20}$$

In the spherical coordinate system of Fig. 8, when the normal vector of the single turn's cross section is the same as z-axis, there are $\mathbf{e}_r \cdot \mathbf{e}_n = \cos\gamma$ and $\mathbf{e}_\theta \cdot \mathbf{e}_n = -\sin\gamma$, so the above formula can be further simplified as:

$$\Phi_{\mathrm{sd}} = \sum_{m=-N/2}^{N/2} \int_0^{2\pi} \frac{\mu_0 I \pi r^2 S_{\mathrm{r}}}{8\pi^2 r_{12}^3} G d\phi \tag{21}$$

where $G = 2\cos\left(\gamma - \alpha\right)\cos\gamma - \sin\left(\gamma - \alpha\right)\sin\gamma$.

In addition, the first derivative of G is taken as zero in order to maximize Φ_{sd}, and the rotating angle α of the coil rotating around its x'-axis should satisfy the following condition:

$$\alpha = \arctan\left[3\sin\left(2\gamma_0\right)/\left(3\cos\left(2\gamma_0\right) + 1\right)\right] \tag{22}$$

where, $\gamma_0 = \arctan\left(d/z\right)$.

4. MODELING AND SIMULATION

In order to verify the accuracy of the formula in this paper, COMSOL Multiphysics software is used to model and analyze the magnetic induction intensity generated by coreless circular coil with rectangular cross-section in space. The geometric parameters used for coil modeling are described in Table 1.
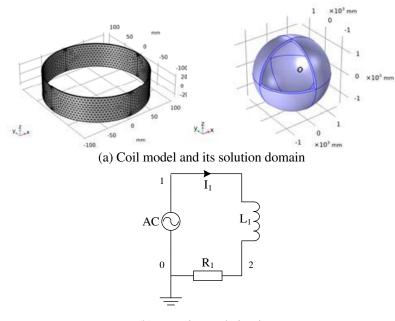
Table 1. Coil parameters.

Coil type	Wire diameter	Axial turns	Radial turns	Inner radius
Solenoid	1.0 mm	50	1	100.0 mm

4.1. COMSOL Modeling and Simulation

COMSOL Multiphysics software is a whole process simulation platform, which can realize the modeling, analysis, and result drawing of single or multiple physical field problems on the same interface. Among them, the AC/DC module applied in the field of electromagnetics has a variety of physical field interfaces including circuit, electric field, and magnetic field. The circuit interface is used to model the current and voltage in the circuit, including the voltage source, current source, resistance, capacitor, inductor, etc., which can solve the Kirchhoff voltage, Kirchhoff current, and the principle of charge conservation related to the electric circuit. The magnetic field interface is used to calculate distributions of magnetic field and induced current inside and around the coil, which can solve the Maxwell equations based on vector magnetic potential.

In COMSOL, the two-dimensional (2D) axisymmetric space dimension can be selected for simulation when the coil has no rotating angle, which has the advantages of short modeling time and fast simulation speed. While the three-dimensional (3D) space dimension can be selected for simulation when the coil has a certain rotating angle, which is more intuitive than the 2D model. Fig. 9 shows the coil model, its solution domain, and experimental circuit established when COMSOL software is used to solve the magnetic induction intensity of coils in space, and the two are coupled together.

(a) Coil model and its solution domain

(b) Experimental circuit

Figure 9. Model and experimental circuit of circular coils with rectangular cross-section.

4.2. Analysis of Numerical Results

Figures 10–12 show the comparison between the simulation value and the formula calculation value of the magnetic induction intensity norm of the coil in different spatial positions, where the x-axis represents the size of z in the observation point $(0, 0, z)$. Through calculations, the relative errors among traditional formula, improved formula, and simulation were obtained.

It can be seen from Figs. 10–12 that in the axial magnetic field direction, the maximum relative error between the improved formula calculation value and the simulated value is no more than 0.5%, i.e., the simulated value has a high consistency with the calculated value. In the other two cases with offset distance and rotating angle, the relative error between the calculated value of the improved formula and the simulated value at the same observation points of z-axis is also within 5%. However, the effect of traditional formula is very poor when it is applied to short distance, especially to calculate the axial magnetic induction intensity of the coil.
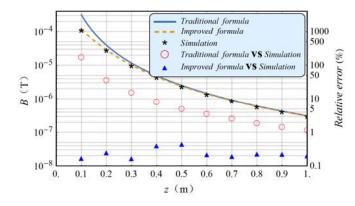
Figure 10. Variation of magnetic induction intensity norm of the coil at the coordinate origin with observation points.
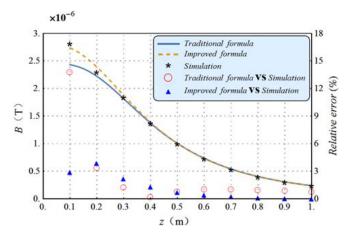

Figure 11. Variation of magnetic induction intensity norm of the coil at the position of the offset distance $d = 0.4\,\text{m}$ with observation points.
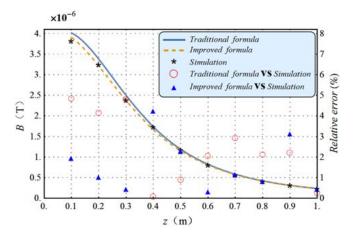

Figure 12. Variation of magnetic induction intensity norm of the coil at the position of the offset distance $d = 0.4\,\text{m}$ and rotating angle $\alpha = 45°$ with observation points.

Therefore, compared with the traditional formula, the simulation results are in good agreement with the improved formula calculation results. Especially, the error will be greatly reduced when the magnetic induction intensity of a certain point at close range of the coil is calculated, which verifies the correctness of the improved formula in this paper.

5. CONCLUSIONS

This paper analyzed the magnetic field distribution of a single current-carrying circular coil in space based on Biot-Savart law and then extends it to the coreless circular coils with rectangular cross-section of any turn numbers. COMSOL Multiphysics software was used for modeling and simulation to eventually obtain the analytical results of magnetic induction intensity at any one point in space. The correctness of the improved theoretical formula is verified by comparing the numerical calculation and simulation results, and it is more suitable for magnetic-field calculations at close range of the current-carrying coil. It also provides a theoretical support for further research on mutual inductance of coreless circular coils with rectangular cross-section in a wireless magnetic induction communication system.

APPENDIX A.

Next, more details about the specific calculation steps for the magnetic induction intensity produced by the circular coil with rectangular cross-section at arbitrary point P in space are provided in Figs. A1–A2.
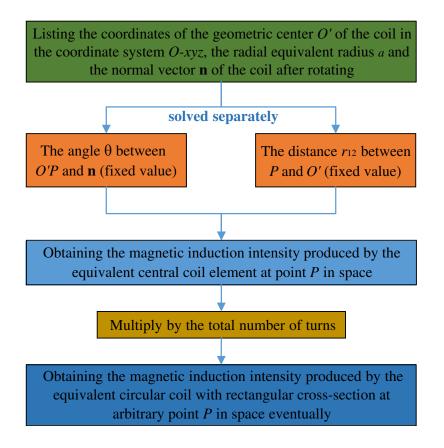
Listing the coordinates of the geometric center O' of the coil in the coordinate system O-xyz, the radial equivalent radius a and the normal vector **n** of the coil after rotating

solved separately

The angle θ between $O'P$ and **n** (fixed value)

The distance r_{12} between P and O' (fixed value)

Obtaining the magnetic induction intensity produced by the equivalent central coil element at point P in space

Multiply by the total number of turns

Obtaining the magnetic induction intensity produced by the equivalent circular coil with rectangular cross-section at arbitrary point P in space eventually

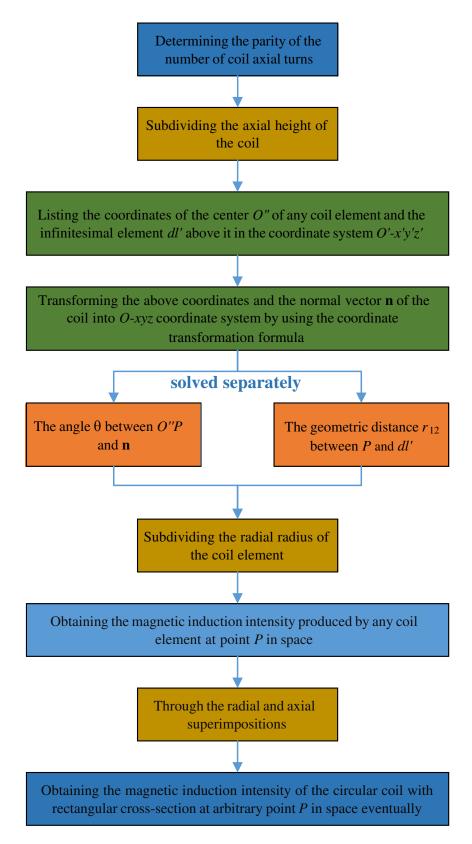**Figure A1.** Traditional calculation model.

Figure A2. Improved calculation model.

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