

# Diffraction Radiation Generated by a Density-Modulated Electron Beam Flying over the Periodic Boundary of the Medium Section.

## I. Analytical Basis

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**Abstract**—The paper is focused on reliable modeling of the effects associated with the resonant transformation of the field of a plane, density modulated electron beam, flying over the periodically uneven boundary of a natural or artificial medium, in the field of volume outgoing waves. Here, the general information (analytical basis) is presented on the peculiarities and principal characteristics of electromagnetic fields arising in the situations under consideration, on the procedures for regularization of model boundary value problems describing these situations, and on possible eigen modes of periodic structures. Without relying on this information, it is impossible to advance considerably effectively in solving numerous urgent physical problems (establishing the conditions providing anomalously high levels of Vavilov-Cherenkov and/or Smith-Purcell radiation; diagnostics of beams of charged particles, artificial materials and media) and in practical implementation of new knowledge about the effects of diffraction radiation and their wave analogues in new devices and instruments of optoelectronics, high-power electronics, antenna, and accelerator technology.

## 1. INTRODUCTION

Vavilov-Cherenkov radiation (VChR) [1] and Smith-Purcell radiation (SPR) [2] are among the most significant physical phenomena discovered in the 20th century. Classical works [3–6] are devoted to the theory of these phenomena and their practical use. The surge of interest to them in modern science (see, for example, [7–18]) is mainly due to: (a) a significant progress of computational physics, which allows to formulate and solve rather complex problems that are adequate to the situations under study; (b) the invention and fabrication of new artificial materials and media with unique properties not previously considered; (c) a growing list of relevant physical and applied problems, whose solution is facilitated or could be facilitated by new knowledge about the processes of diffraction radiation (VChR or SPR).

We address this topic because, in a number of computational experiments, based on reliable mathematical models and corresponding computational schemes, rather interesting results had been obtained (see, for example, [18]). These results proved the existence of anomalously high levels of coherent diffraction radiation when a density modulated electron beam flies over the periodic boundary separating the ordinary and artificial medium, characterized by a specific frequency dispersion of permittivity and permeability. The recorded radiation levels were much higher than those we noted as anomalously high in [15], where the classical metal-dielectric diffraction gratings conventionally used in millimeter and submillimeter-wave techniques were considered as a periodic scatterer. The high levels of diffraction radiation achieved in [15] got a fairly convincing explanation, based on the previously

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obtained results of the electrodynamic theory of gratings, describing the space-frequency and space-time transformations of electromagnetic waves by open periodic resonators [19–21]. In the case of a periodic interface between media, the analysis of this phenomenon is more complicated. It requires a partial revision of previously used models, an extensional work related to the study of possible ‘eigen’ modes of the corresponding periodic structures, and their influence on the formation of response of these structures to any external excitation. This case also requires revision of formulation and accurate physical reading of the results of many problem-oriented computational experiments.

We plan to present the results of the extensive study of the problem in several papers. The present one (and the first of this series) is devoted to the description of general electromagnetic model, providing the investigation (in the approximation of a given current) of all the physical features and regularities in the processes of diffraction radiation generated by a density-modulated electron beam moving over a periodically uneven interface between media with different constitutive parameters.

The time dependence  $t$  for harmonic processes considered in the work is determined by the factor  $\exp(-i\omega t)$  omitted everywhere, and  $\omega$  is a circular frequency. The dimensions of the SI system of all mentioned physical quantities are also omitted.

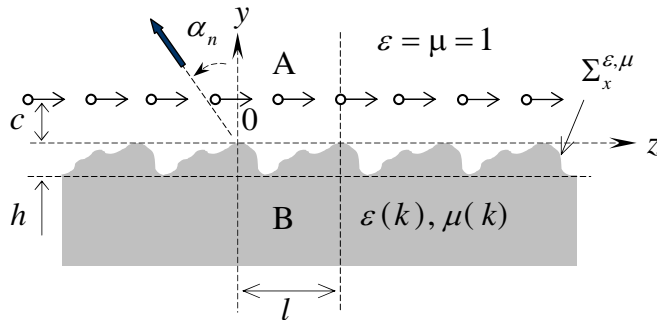
## 2. BASIC ELECTROMAGNETIC MODEL

Consider the boundary  $\Sigma^{\varepsilon,\mu} = \Sigma_x^{\varepsilon,\mu} \times (-\infty < x < \infty)$  separating ordinary ( $\varepsilon = \mu = 1$ ) and dispersive ( $\varepsilon = \varepsilon(k)$ ,  $\mu = \mu(k)$ ) media (see Fig. 1). It is periodic along coordinate  $z$  and homogeneous along coordinate  $x$ . When being excited by  $H$ -polarized ( $E_x^i = H_y^i = H_z^i = 0$ ) wave  $V_0(g, k) = H_x^i(g, k) = \exp(-i\Gamma_0^+ y)\varphi_0(z)$ , this boundary forms the total field  $\{\mathbf{E}(g, k), \mathbf{H}(g, k)\}$ ,  $g = \{y, z\}$ ,  $\partial_x = 0$ , which is defined by the solution  $U(g, k) = H_x(g, k)$  to the following boundary value problem [19–21] within the Floquet channel  $\mathbf{R} = \{g = \{y, z\} \in \mathbf{R}^2 : 0 < z < l\}$ :

$$\left\{ \begin{array}{l} [\partial_y^2 + \partial_z^2 + \varepsilon(g, k) \mu(g, k) k^2] U(g, k) = 0; \quad g = \{y, z\} \in \Omega_{int} \\ \mathbf{E}_{tg}(q, k), \quad \mathbf{H}_{tg}(q, k) \text{ are continuous when crossing } \Sigma^{\varepsilon,\mu} \\ \text{and virtual boundaries } y = 0, y = -h; q = \{x, y, z\} \\ U\{\partial_z U\}(y, l, k) = \exp(2\pi i \zeta) U\{\partial_z U\}(y, 0, k) \text{ for } -h \leq y \leq 0 \end{array} \right., \quad (1a)$$

$$\begin{aligned} U(g, k) &= V_0(g, k) + U^+(g, k) = V_0(g, k) + \sum_{n=-\infty}^{\infty} U_n^+(g, k) \\ &= \exp(-i\Gamma_0^+ y) \varphi_0(z) + \sum_{n=-\infty}^{\infty} R_n(k) \exp(i\Gamma_n^+ y) \varphi_n(z); \quad g \in \bar{\mathbf{A}}, \end{aligned} \quad (1b)$$

$$U(g, k) = U^-(g, k) = \sum_{n=-\infty}^{\infty} U_n^-(g, k) = \sum_{n=-\infty}^{\infty} T_n(k) \exp(-i\Gamma_n^- (y+h)) \varphi_n(z); \quad g \in \bar{\mathbf{B}}. \quad (1c)$$



**Figure 1.** Periodic boundary between two media: upper half-space filled with vacuum and lower half-space filled with dispersive material.

Here,  $\bar{G}$  is the closure of the domain  $G$ ;  $\Omega_{int} = \{g \in \mathbb{R} : -h < y < 0\}$ ,  $A = \{g \in \mathbb{R} : y > 0\}$  and  $B = \{g \in \mathbb{R} : y < -h\}$ ;  $\mathbf{E}(g, k) = \{E_x, E_y, E_z\}$  and  $\mathbf{H}(g, k) = \{H_x, H_y, H_z\}$  are the electric and magnetic field vectors;  $\mathbf{E}_{tg}$  and  $\mathbf{H}_{tg}$  are their components tangential (tangent) to the corresponding surfaces;  $U(g, k) = H_x(g, k)$  is the only nonzero component of  $\mathbf{H}(g, k)$ . The nonzero components of  $\mathbf{E}(g, k)$  are determined by the relations [20, 21]

$$E_y(g, k) = -\frac{\eta_0}{ik\varepsilon(g, k)}\partial_z H_x(g, k), \quad E_z(g, k) = \frac{\eta_0}{ik\varepsilon(g, k)}\partial_y H_x(g, k). \quad (2)$$

Real-value functions  $\varepsilon(g, k)$  and  $\mu(g, k)$  set relative permittivity and permeability of the wave propagation medium ( $\varepsilon(g, k) = \mu(g, k) \equiv 1.0$  for  $y > f(z)$  and  $\varepsilon(g, k) = \varepsilon(k)$ ,  $\mu(g, k) = \mu(k)$  for  $y < f(z)$ );  $l$  and  $h$  are period and height of the boundary  $\Sigma_x^{\varepsilon, \mu} = \{g : y = f(z), -h \leq f(z) \leq 0\}$ , the surface  $\Sigma^{\varepsilon, \mu} = \Sigma_x^{\varepsilon, \mu} \times (-\infty < x < \infty)$  of discontinuities of material parameters  $\varepsilon(g, k)$  and  $\mu(g, k)$  is assumed to be sufficiently smooth.  $\Gamma_n^+ = \sqrt{k^2 - \Phi_n^2}$ ,  $\text{Re}\Gamma_n^+ \geq 0$ ,  $\text{Im}\Gamma_n^+ \geq 0$  and  $\Phi_n = (n + \zeta)2\pi/l$  are vertical and horizontal wave numbers for spatial harmonics (plane waves)  $U_n^+(g, k) = R_n(k) \exp(i\Gamma_n^+ y) \varphi_n(z)$  propagating in the domain  $A$  with attenuation (when  $\text{Im}\Gamma_n^+ > 0$ ) or without it (when  $\text{Im}\Gamma_n^+ = 0$ ). The transverse functions  $\varphi_n(z) = l^{-1/2} \exp(i\Phi_n z)$ ,  $n = 0, \pm 1, \pm 2, \dots$  form a complete (in space  $L_2(0, l)$ ) orthonormal system in the cross section of the Floquet channel  $R$ .  $k = 2\pi/\lambda$  is a frequency parameter or just frequency,  $\lambda$  is the wavelength of electromagnetic waves in free space;  $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$  is an impedance of free space;  $\varepsilon_0$  and  $\mu_0$  are electric and magnetic vacuum constants;  $\zeta$ ,  $\text{Im}\zeta = 0$  is a numeric parameter.

We determine the signs of real and imaginary parts of the square root  $\Gamma_n^+$  using the radiation condition at infinity [19–21], which ensures the unique solvability of the model problem (1) for almost all values of real parameters  $k$  and  $\zeta$ . Exceptions are possible for no more than a countable number of values  $k = \bar{k}$  and  $\zeta = \bar{\zeta}$  belonging to the sets  $\Theta_k$  and  $\Theta_\zeta$  (see Section 3), which are spectral sets corresponding to possible eigen field oscillations and possible eigen waves in the open electrodynamic structure [20, 21]. In the reflection zone of the periodic structure (in the half-space  $y > 0$ ), due to the radiation condition, the field  $U^+(g, k)$  consists only of the waves  $U_n^+(g, k)$  outgoing from the boundary  $\Sigma_x^{\varepsilon, \mu}$ . The same condition determines the branch of the square root  $\Gamma_n^- = \sqrt{k^2 \varepsilon(k) \mu(k) - \Phi_n^2}$ : all partial components  $U_n^-(g, k) = T_n(k) \exp[-i\Gamma_n^-(y + h)] \varphi_n(z)$  of the field  $U^-(g, k)$  in the transmission zone of the periodic structure (in the half-space  $y < -h$ ) should be plane homogeneous waves (in the case when  $\text{Im}\Gamma_n^- = 0$ ) moving away from the boundary  $\Sigma_x^{\varepsilon, \mu}$ , which transfer energy in the general direction  $y = -\infty$ , or inhomogeneous waves (in the case when  $\text{Re}\Gamma_n^- = 0$ ), exponentially decaying when moving in the same direction. This general statement allows us to define the sign of  $\text{Im}\Gamma_n^-$  ( $\text{Im}\Gamma_n^- \geq 0$ ). The sign of  $\text{Re}\Gamma_n^-$  we shall define below.

In the electrodynamic theory of gratings, the plane waves  $U_n^+(g, k)$  and  $U_n^-(g, k)$  are called spatial harmonics of periodic structure [19–21]. The ones with numbers  $n$  corresponding to real propagation constants  $\Gamma_n^+$  and  $\Gamma_n^-$  are able to propagate infinitely far from the boundary  $\Sigma_x^{\varepsilon, \mu}$ . In the reflection zone  $y > 0$ , they leave the boundary at the angles  $\alpha_n(k) = -\arcsin[\Phi_n(k)/k]$  that are counted anti-clockwise from the axis  $y$ . Obviously, for any fixed finite values of real parameters  $k$  and  $\zeta$ , the number of such waves is finite.

The Poynting's complex power theorem [22] for the field  $\{\mathbf{E}(g, k), \mathbf{H}(g, k)\}$  in the volume  $[0 \leq x \leq l] \times [0 \leq y \leq -h] \times [0 \leq z \leq l]$  implies the fundamental relation [21]

$$\sum_{n=-\infty}^{\infty} \left[ |R_n|^2 \text{Re}\Gamma_n^+ + \varepsilon^{-1}(k) |T_n|^2 \text{Re}\Gamma_n^- \right] = \text{Re}\Gamma_0^+ + 2 \text{Im} R_0 \text{Im}\Gamma_0^+. \quad (3)$$

According to Eq. (3), the values

$$W_n^+(k) = |R_n|^2 \frac{\text{Re}\Gamma_n^+}{|\Gamma_0^+|}, \quad W_n^-(k) = \varepsilon^{-1}(k) |T_n|^2 \frac{\text{Re}\Gamma_n^-}{|\Gamma_0^+|} \quad (4)$$

determine the relative part of energy directed by the boundary  $\Sigma_x^{\varepsilon, \mu}$  into the relevant spatial harmonic,  $U_n^+(g, k)$  or  $U_n^-(g, k)$ . If  $\text{Im}\Gamma_0^+ = 0$  (the boundary is excited by a homogeneous plane wave  $V_0(g, k)$ , coming onto the grating at an angle  $\alpha_0^+ = \arcsin[\Phi_0(k)/k]$ ), we have from Eqs. (3) and (4)

$$\sum_n [W_n^+(k) + W_n^-(k)] = 1. \quad (5)$$

If the boundary is excited by an inhomogeneous plane wave  $V_0(g, k)$  ( $\text{Re}\Gamma_0^+ = 0$ ,  $\text{Im}\Gamma_0^+ > 0$ ), the near-field to far-field conversion efficiency is determined by the value of  $2 \text{Im} R_0$  (see Eq. (3)), which is nonnegative in this case and

$$2 \text{Im} R_0 = \sum_n [W_n^+(k) + W_n^-(k)]. \quad (6)$$

The equality in Eq. (3) defines the sign of  $\text{Re}\Gamma_n^-$  ( $\varepsilon^{-1}(k) \text{Re}\Gamma_n^- \geq 0$ ) and together with the relation

$$\mathbf{P} = \frac{1}{l} \int_0^l (\mathbf{E} \times \mathbf{H}^*) dl = \frac{1}{l} \int_0^l (E_z H_x^* \mathbf{y} - E_y H_x^* \mathbf{z}) dz = \frac{1}{l} \frac{\eta_0}{k \varepsilon(k)} \left[ \sum_n \left( -|T_n|^2 \Gamma_n^- \mathbf{y} + |T_n|^2 \Phi_n \mathbf{z} \right) \right], \quad (7)$$

which is the complex Poynting vector  $\mathbf{P}(k)$  of the field  $\{\mathbf{E}(g, k), \mathbf{H}(g, k)\}$  in the plane  $y = -h$  averaged over the period  $l$  of the boundary  $\Sigma_x^{\varepsilon, \mu}$ , allows strict, unambiguous, and complete determination of the direction of phase velocity of the harmonics  $U_n^-(g, k)$ ,  $\text{Im}\Gamma_n^- = 0$  propagating in the domain  $y < -h$  and the direction of energy transfer performed by these harmonics. For a conventional (right hand) medium, these directions coincide and are set by the vector  $-\Gamma_n^- \mathbf{y} + \Phi_n \mathbf{z}$ ,  $\Gamma_n^- > 0$  ( $\mathbf{y}$  and  $\mathbf{z}$  are the unit vectors for the axes  $0y$  and  $0z$ ). For a bi-negative medium,  $\Gamma_n^- < 0$ , the phase velocity is oriented along the vector  $-\Gamma_n^- \mathbf{y} + \Phi_n \mathbf{z}$ , and the direction of energy transfer is oriented along the vector  $\Gamma_n^- \mathbf{y} - \Phi_n \mathbf{z}$ . In a medium with only one negative constitutive parameter, the harmonics  $U_n^-(g, k)$  transferring energy in the direction  $z = -\infty$  are not excited.

To solve the problem (1) numerically, we use the approach from [18, 23, 24], and it is a version of the analytical regularization method [19, 21, 25], which can be described briefly as follows. The system of orthonormal functions  $\varphi_n(z)$ ,  $n = 0, \pm 1, \pm 2, \dots$  is complete in the space  $L_2(0, l)$  of functions with the integrable (on the interval  $0 \leq z \leq l$ ) squared module. This allows us to write down the conditions in (1), related to the continuity of tangential field components on the boundaries  $\Sigma_x^{\varepsilon, \mu}$ ,  $y = 0$ , and  $y = -h$  in the form of an ill-conditioned infinite system of linear algebraic equations. The right and left side analytical regularization of this system [25, 26] allows one to construct an operator equation of the second kind equivalent to it (the Fredholm operator equation); its numerical solution can be obtained by a truncation method converging in a norm in one of the Hilbert spaces of infinite sequences  $\{R_n\}_{n=-\infty}^{\infty}$ ,  $\{T_n\}_{n=-\infty}^{\infty}$  [27, 28].

### 3. EIGEN-MODES OF A PERIODIC INTERFACE

Main peculiarities of the field  $U(g, k)$  transformation are associated with the so-called normal (or eigen) modes [20, 29–31]. When modes of this kind are excited in a periodic structure, it operates as an open periodic resonator or an open periodic waveguide. One can simulate such regimes by extending analytically homogeneous (spectral) frequency-domain problems (see, for example, problem (1) for  $V_0(g, k) \equiv 0$ ) into the domain of complex values of one of the spectral parameters: the frequency  $k$  or the longitudinal propagation number  $\zeta$  [20, 21, 29, 32]. The domain of analytical extension coincides with the infinite-sheeted Riemann surfaces  $K$  (real-valued  $\zeta$  is fixed, and  $k \in K$  is a complex-valued spectral parameter) or  $F$  ( $k > 0$  is fixed, and  $\zeta \in F$  is a spectral parameter) with algebraic branch points  $k = k_n^\pm$  :  $\Gamma_n^\pm(k_n^\pm) = 0$  or  $\zeta = \zeta_n^\pm$  :  $\Gamma_n^\pm(\zeta_n^\pm) = 0$ ,  $n = 0, \pm 1, \pm 2, \dots$ . The choice of branches of the square roots in  $\Gamma_n^\pm$  on the real axes of the first (physical) sheets of the Riemann surfaces  $K$  and  $F$  has been done earlier. Such a choice has to ensure the physically understandable requirement that the fields  $U^+(g, k)$  and  $U^-(g, k)$  do not contain waves coming onto the boundary  $\Sigma_x^{\varepsilon, \mu}$  from infinity.

The set  $\Theta_k$  of eigenfrequencies  $\bar{k}$  is the frequency spectral set or the frequency spectra if for the complex-valued frequencies  $k = \bar{k} \in K$ , the spectral problem (1) has nontrivial solutions  $U(g, \bar{k})$ . Every solution of this kind corresponds to a free field oscillation at the eigenfrequency  $\bar{k}$  in a periodic structure. Likewise, one can define the set  $\Theta_\zeta$  of propagation constants  $\bar{\zeta}$  for surface, leaky, and other types [20, 29] of eigen waves  $U(g, \bar{\zeta})$  of a periodic media interface. If any eigenvalue  $\bar{\zeta}$  belongs to the axis  $\text{Re}\zeta$  of the first (physical) sheet of the surface  $\Theta_\zeta$  and  $\text{Im}\Gamma_n^\pm(\bar{\zeta}) > 0$  for all  $n = 0, \pm 1, \pm 2, \dots$ , then we have an ordinary surface real eigen wave (or true eigen wave) propagating near and along a media boundary without attenuation. For periodic structures, the sets  $\Theta_k$  and  $\Theta_\zeta$  are countable nonempty ones. For a number of canonical (elementary) periodic structures, the existence of eigen-modes can be proved by

constructing explicit solutions of the corresponding spectral problems [20, 29]. It is possible to obtain a similar result for much more complex structures, relying on the existence of eigen-modes for simple structures and the results from [20, 29, 33], proving that for ‘smooth’ variations of any parameter of the spectral problem, the existing eigen frequencies of the free field oscillation  $U(g, \bar{k}(\tau))$  and propagation constants  $\bar{\zeta}(\tau)$  of the eigen waves  $U(g, \bar{\zeta}(\tau))$ , moving along the sheets of surfaces K and F, cannot disappear anywhere in their finite part.

A detailed discussion of spectral sets, localization, and dynamics of their elements on the surfaces K and F, as well as the relation between anomalous or resonant scattering of monochromatic and pulsed waves by periodic structures, and generation in these structures of high-Q free oscillations and weak decaying eigen waves can be found in [20, 21, 29–35]. Here, we would like to note only one result, which must be taken into account in a profound analysis of features and regularities in the behavior of principal electrodynamic characteristics of the considered periodic structure. The Green’s function  $G(g, g_0, k, \zeta)$  of the boundary  $\Sigma_x^{\varepsilon, \mu}$  in the field of point quasiperiodic sources  $\psi(g) = \sum_{n=-\infty}^{\infty} \delta(z - z_0 - nl) \delta(y - y_0) \exp(2\pi i \zeta n)$  (here  $g_0 = \{y_0, z_0\} \in \Omega_{int}$  and  $\delta(\dots)$  is the Dirac delta-function) for any fixed value  $\zeta$  is a meromorphic function of the parameter  $k$  in local variables on the surface K [36]; its poles coincide with points  $\bar{k} \in \Theta_k$  and, in the neighborhood of these points, the function  $G(g, g_0, k, \zeta)$  can be represented by the Laurent series

$$G(g, g_0, k, \zeta) = \sum_{m=-M}^{\infty} a_m(g, g_0, \zeta) \left\{ \begin{array}{l} (k - \bar{k})^m \\ (k - \bar{k})^{m/2} \end{array} \right\}; \quad \left\{ \begin{array}{l} \bar{k} \notin \{k_n^{\pm}\}_n \\ \bar{k} \in \{k_n^{\pm}\}_n \end{array} \right\}; \quad (8)$$

$M$  is a pole order at a point  $k = \bar{k}$ . Similarly, in points  $\bar{\zeta} \in \Theta_{\zeta}$  for any fixed value  $k$ , we have

$$G(g, g_0, k, \zeta) = \sum_{m=-M}^{\infty} b_m(g, g_0, k) \left\{ \begin{array}{l} (\zeta - \bar{\zeta})^m \\ (\zeta - \bar{\zeta})^{m/2} \end{array} \right\}; \quad \left\{ \begin{array}{l} \bar{\zeta} \notin \{\zeta_n^{\pm}\}_n \\ \bar{\zeta} \in \{\zeta_n^{\pm}\}_n \end{array} \right\}. \quad (9)$$

It follows from Eqs. (8) and (9) that sharp (resonant) changes in electrodynamic characteristics of the boundary  $\Sigma_x^{\varepsilon, \mu}$  are possible in those regions of the parameter values for which points  $\bar{k} \in \Theta_k$  (points  $\bar{\zeta} \in \Theta_{\zeta}$ ) are found near the corresponding intervals of the axis  $\text{Re}k$  (axis  $\text{Re}\zeta$ ) of the first physical sheet of the surface K (surface F). The use of local representations following from Eq. (8) for the first time allowed to give a sufficiently convincing explanation of the Wood’s anomalies, which are non-differentiable singularities in the behavior of diffraction characteristics of periodic structures in the vicinity of threshold points [20, 37].

The real parameters  $k$  and  $\zeta$  of the problem (1) rarely fall into the near vicinity of spectral points from the higher sheets of the surfaces K and F [20, 29, 36]; therefore the corresponding normal modes cannot significantly affect diffraction characteristics of the periodic structure. However, we should find an accurate way to calculate points  $\bar{k}$  and  $\bar{\zeta}$ . For example, determining the elements  $\bar{k} : \text{Im}\bar{k} = 0$  of the spectral set  $\Theta_k$  on the higher sheets of the surface K, we, in essence, synthesize a periodic structure able to implement the effects of complete transformation of plane waves and packets of plane waves, and it has great applied interest [38, 39]. Including the effects associated with Vavilov-Cherenkov and Smith-Purcell radiation, their theoretical study may be based on the numerical solution of the boundary value problem (1).

#### 4. TRANSITION TO A MODEL OF DIFFRACTION RADIATION PROCESSES

Now let us find the relation between the problems of analysis of effects of diffraction radiation induced by the density modulated electron beam moving over a periodic boundary and the model problems of the electrodynamic theory of gratings (1).

Suppose that the density-modulated electron beam, whose instantaneous charge density has the form  $\rho \delta(y - c) \exp[i(k/\beta)z]$ ,  $c \geq 0$ , is moving over the boundary  $\Sigma_x^{\varepsilon, \mu}$  (see Fig. 1). Here,  $\rho$  and  $k$  are the modulation amplitude and modulation frequency of the beam, and  $\beta < 1$  is its relative velocity. Own electromagnetic field of such an electron beam is  $H$ -polarized field ( $E_x^{\text{beam}} = H_y^{\text{beam}} = H_z^{\text{beam}} = 0$ ,  $\partial_x = 0$ ) with [6, 40, 41]

$$H_x^{\text{beam}}(g, k) = 2\pi\rho\beta \exp \left\{ i \left[ \sqrt{k^2 - (k/\beta)^2} |y - c| + (k/\beta)z \right] \right\} [|y - c|/(y - c)]; \quad y \neq c. \quad (10)$$

From Eq. (10) it follows that in the presence of a plane (in case  $h = 0$ ) or periodic ( $h > 0$ ) boundary  $\Sigma^{\varepsilon,\mu}$  between vacuum and dispersive medium, the density-modulated electron beam with  $-2\pi\rho\beta\sqrt{l}\exp[-kc\sqrt{(1/\beta)^2 - 1}] = 1$  and  $k/\beta = \Phi_0 = \zeta 2\pi/l$  (with this value  $\Phi_0$ ,  $\Gamma_0^+$  is an imaginary value) generates in domains  $y > 0$  and  $y < -h$   $H$ -polarized field, whose  $H_x$ -components  $U^+(g, k)$  and  $U^-(g, k)$  are determined by the solution  $U(g, k)$  of the boundary value of problem (1). Indeed, under the conditions specified above,  $H_x^{\text{beam}}(g, k) = V_0(g, k) = H_x^i(g, k)$  for  $y < c$ , an electromagnetic field of electron beam and a field of an inhomogeneous plane wave exciting the boundary  $\Sigma_x^{\varepsilon,\mu}$  coincide. Consequently, secondary fields arising as a result of such excitation also coincide.

## 5. CONCLUSION

This work is of principal importance for the study of diffraction radiation generated by a density-modulated electron beam flying over a periodically uneven interface between media, in particular, over the boundary separating vacuum and artificial dispersive medium.

The essence of the presented study is the quality of numerical implementation and adequate physical treatment of its results. The proposed quality is of particular importance for problem-oriented mathematical models aimed at working in rather harsh conditions of possible resonant wave scattering, which is a key feature of the problem.

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## REFERENCES

1. Cherenkov, P., "Visible radiation produced by electrons moving in a medium with velocities exceeding that of light," *Physical Review*, Vol. 52, 378–379, 1937.
2. Smith, S. and E. Purcell, "Visible light from localized surface charges moving across a grating," *Physical Review*, Vol. 92, No. 4, 1069–1073, 1953.
3. Jelley, J., *Cherenkov Radiation and Its Applications*, Pergamon Press, London, 1958.
4. Bolotovskiy, B. and G. Voskresenskiy, "Radiation of charged particles in periodic structures," *Uspehi Fizicheskikh Nauk*, Vol. 94, No. 3, 378–416, 1968 (in Russian).
5. Frank, I., *Vavilov-Cherenkov Radiation*, Nauka, Moscow, 1988 (in Russian).
6. Shestopalov, V., *The Smith-Purcell Effect*, Nova Science Publishes, New York, 1998.
7. Kesar, A., M. Hess, S. Korbly, and R. Temkin, "Time- and frequency-domain models for Smith-Purcell radiation from a two-dimensional charge moving above a finite length grating," *Physical Review E*, Vol. 71, 016501-1–016501-9, 2005.
8. Duan, Z., B.-I. Wu, S. Xi, H. Chen, and M. Chen, "Research progress in reversed Cherenkov radiation in double-negative metamaterials," *Progress In Electromagnetics Research*, Vol. 90, 75–87, 2009.
9. Kesar, A., "Smith-Purcell radiation from a charge moving above a grating of finite length and width," *Physical Review Special Topics — Accelerators and Beams*, Vol. 13, 022804-1–022804-8, 2010.
10. Bakunov, M., R. Mikhaylovskiy, S. Bodrov, and B. Luk'yanchuk, "Reversed Cherenkov emission of terahertz waves from an ultrashort laser pulse in a sandwich structure with nonlinear core and left-handed cladding," *Optics Express*, Vol. 18, No. 2, 1684–1694, 2010.
11. Burlak, G., "Spectrum of Cherenkov radiation in dispersive metamaterials with negative refraction index," *Progress In Electromagnetics Research*, Vol. 132, 149–158, 2013.
12. Sautbekov, S., K. Sirenko, Y. Sirenko, and A. Yevdokimov, "Diffraction radiation phenomena: Physical analysis and applications," *IEEE Antennas and Propagation Magazine*, Vol. 57, No. 5, 73–93, 2015.

13. Granet, G., P. Melezhhik, A. Poyedinchuk, S. Sautbekov, Y. Sirenko, and N. Yashina, "Resonances in reverse Vavilov-Cherenkov radiation produced by electron beam passage over periodic interface," *International Journal of Antennas and Propagation*, Vol. 2015, 10, 2015.
14. Sirenko, Y. and L. Velychko (eds.), *Electromagnetic Waves in Complex Systems: Selected Theoretical and Applied Problems*, Springer, New York, 2016.
15. Sautbekov, S., K. Sirenko, Y. Sirenko, A. Poyedinchuk, N. Yashina, and A. Yevdokymov, "Smith-Purcell effect. Anomalously high level of outgoing wave excitation," *Telecommunications and Radio Engineering*, Vol. 77, No. 6, 469–487, 2018.
16. Melezhhik, P., M. Ney, S. Sautbekov, K. Sirenko, Y. Sirenko, A. Vertiy, and N. Yashina, "Cherenkov radiation based antenna with the funnel-shaped directional pattern," *Electromagnetics*, Vol. 38, No. 1, 34–44, 2018.
17. Sautbekov, S., K. Sirenko, Y. Sirenko, and A. Yevdokymov, "Diffraction antennas. Synthesis of radiating elements," *Telecommunications and Radio Engineering*, Vol. 77, No. 11, 925–943, 2018.
18. Sirenko, Y., P. Melezhhik, A. Poyedinchuk, S. Sautbekov, A. Shmat'ko, K. Sirenko, A. Vertiy, and N. Yashina, "Radiation of electromagnetic waves induced by electron beam passage over artificial material periodic interfaces," *An Essential Guide to Electrodynamics*, N. Brewer (ed.), Chap. 5, 169–206, Nova Science Publishers, New York, 2019.
19. Shestopalov, V., A. Kirilenko, S. Masalov, and Y. Sirenko, *Resonance Wave Scattering. Vol. 1. Diffraction Gratings*, Naukova Dumka, Kiev, 1986 (in Russian).
20. Shestopalov, V. and Y. Sirenko, *Dynamic Theory of Gratings*, Naukova Dumka, Kiev, 1989 (in Russian).
21. Sirenko, Y. and S. Strom (eds.), *Modern Theory of Gratings. Resonant Scattering: Analysis Techniques and Phenomena*, Springer, New York, 2010.
22. Jackson, J., *Classical Electrodynamics*, Wiley, New York, 1975.
23. Melezhhik, P. N., A. Y. Poyedinchuk, N. P. Yashina, G. Granet, and M. M. Ney, "Radiation from surface with periodic boundary of metamaterials excited by a current," *Progress In Electromagnetic Research*, Vol. 65, 1–14, 2006.
24. Melezhhik, P., A. Poyedinchuk, N. Yashina, and G. Granet, "Periodic boundary of metamaterial: Eigen regimes and resonant radiation," *Journal of Optics A: Pure and Applied Optics*, Vol. 9, S403–S409, 2007.
25. Shestopalov, V., Y. Tuchkin, A., Poyedinchuk, and Y. Sirenko, *New Solution Methods for Direct and Inverse Problems of the Diffraction Theory. Analytical Regularization of the Boundary Value Problems in Electromagnetic Theory*, Osnova, Kharkov, 1997 (in Russian).
26. Manzhyrov, A. and A. Polyanin, *Solution Methods for Integral Equations*, Faktorial, Moscow, 1999 (in Russian).
27. Shestopalov, V., A. Kirilenko, and S. Masalov, *Matrix Convolution-Type Equations in the Diffraction Theory*, Naukova Dumka, Kiev, 1984 (in Russian).
28. Kantorovich, L. and G. Akilov, *Functional Analysis in Normed Spaces*, Pergamon Press, New York, 1982.
29. Shestopalov, V. and Y. Shestopalov, *Spectral Theory and Excitation of Open Structures*, The Institution of Electrical Engineers, London, 1996.
30. Perov, A. O., Y. K. Sirenko, and N. P. Yashina, "Periodic open resonators: Peculiarities of pulse scattering and spectral features," *Progress In Electromagnetics Research*, Vol. 46, 33–75, 2004.
31. Sirenko, Y., S. Strom, and N. Yashina, *Modeling and Analysis of Transient Processes in Open Resonant Structures. New Methods and Techniques*, Springer, New York, 2007.
32. Sirenko, Y., "A grating in the field of a compact monochromatic source," *Electromagnetics*, Vol. 13, No. 3, 255–272, 1993.
33. Sanchez-Palencia, E., *Nonhomogeneous Media and Vibration Theory*, Springer-Verlag, New York, 1980.
34. Velychko, L. G., Y. K. Sirenko, and O. S. Velychko, "Time-domain analysis of open resonators. Analytical grounds," *Progress In Electromagnetics Research*, Vol. 61, 1–26, 2006.

35. Kravchenko, V., Y. Sirenko, and K. Sirenko, *Electromagnetic Wave Transformation and Radiation by the Open Resonant Structures. Modelling and Analysis of Transient and Steady-State Processes*, Fizmatlit, Moscow, 2011 (in Russian).
36. Von Hurwitz, A. and R. von Courant, *Allgemeine Funktionentheorie und Elliptische Funktionen, Geometrische Funktionentheorie*, Springer-Verlag, Berlin, 1964 (in German).
37. Sirenko, Y., “Analytical extension of diffraction problems and threshold effects in electromagnetics,” *Doklady Akademii Nauk Ukrainiskoy SSR, Seriya A*, No. 8, 65–68, 1986 (in Russian).
38. Rud’, L., Y. Sirenko, V. Yatsik, and N. Yashina, “Spectral method to analyze the effects of total transformation of waves by open periodic and waveguide resonators,” *Izvestiya Vuzov. Radiofizika*, Vol. 31, No. 10, 1246–1252, 1988 (in Russian).
39. Sirenko, Y., N. Yashina, and K. Schuenemann, “Synthesis of mode converters in waveguides and gratings based on spectral theory,” *Journal of Electromagnetic Waves and Applications*, Vol. 16, No. 5, 611–628, 2002.
40. Tretyakov, O., S. Tretyakova, and V. Shestopalov, “Electromagnetic wave radiation by electron beam moving over diffraction grating,” *Radiotekhnika I Elektronika*, Vol. 10, No. 7, 1233–1243, 1965 (in Russian).
41. Shestopalov, V., *Diffraction Electronics*, Vysshaya Shkola, Kharkov, 1976 (in Russian).