

# Average BER Analysis of Free-Space Optical Communications with Adaptive Threshold Technique over Exponentiated Weibull Distribution

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**Abstract**—Average bit error rate (BER) performance of on-off keying (OOK) modulation in a free space optical (FSO) system, which is based on adaptive threshold technique under atmospheric turbulence described by exponentiated Weibull (EW) distribution, is studied and compared with that of using fixed threshold technique. In order to solve the adaptive threshold, the equation is simplified by using the generalized Gauss-Laguerre polynomial function, which significantly improves the operational efficiency. The simulation results show that the adaptive threshold varies with the average transmitted power under different noise variances, receiving aperture sizes and turbulence conditions. Compared with the fixed threshold technique, the adaptive threshold technique can greatly improve the BER performance of FSO communication system.

## 1. INTRODUCTION

Free space optical (FSO) communication has attracted wide attention due to its advantages of broadband, large capacity, high rate, strong anti-interference, and free spectrum [1, 2]. In FSO communication channel, atmospheric turbulence is one of the most important factors, which affect the performance of the system. The irradiance scintillation caused by atmospheric turbulence will seriously affect the performance of FSO communication system.

In recent years, many researchers have studied the related content in the field of FSO communication. In [3], the atmospheric turbulence and its effect on laser beam have been studied by the OptiSystem software. In [4], the misalignment loss of FSO communication link has been investigated. In [5], the bit error rate (BER) performance of an OOK modulation system, which is based on Gamma-Gamma model under the combined effects of atmospheric turbulence and pointing error, has been discussed. In [6, 7], the BER performance of an OOK modulation system under the condition of atmospheric turbulence, which is based on EW model with and without pointing error, has been researched. In [8, 9], the BER and outage probability for various binary modulation schemes of the FSO communication system, which is based on EW model with and without the misalignment errors, have been studied. In [10], the advantages and disadvantages of FSO communication as well as various turbulence channel models have been discussed. Reference [11] has analyzed the average channel capacity of airborne optical links by using EW model. Reference [12] has investigated the performance of pulse position modulation (PPM) under atmospheric turbulence. Reference [13] has evaluated the performance of an FSO system using wavelength diversity and time diversity. Reference [14] has analyzed the BER performance of satellite-underground link. Reference [15] has examined the performance of an FSO system under atmospheric turbulence with partial coherent beam as laser source. However,

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the above researches are all based on fixed threshold technique, and the performance analysis based on adaptive threshold technique is still relatively little. The relationship between adaptive threshold and turbulence level under different noise variances based on the log-Normal distribution model has been studied in [16]. However, there is no method to solve the adaptive threshold. The performance of a modulating retro-reflector (MRR) system under Log-Normal (LN) distribution model with adaptive threshold technique has been analyzed in [17]. Nevertheless, it only studies the case of weak turbulence and does not consider the aperture averaging effect. The performance of an MMR system with adaptive threshold over correlated Gamma-Gamma model has been investigated in [18]. The adaptive threshold was constructed by using the training data received from the detector, not by solving the equation.

This paper analyzes the performance of an OOK modulated FSO system by using adaptive threshold technique and compares with the fixed threshold technique under weak and moderate turbulence based on exponentiated Weibull (EW) distribution model. The performance of the system with different turbulence levels and a range of receiving aperture sizes is studied. The results show that the adaptive threshold technique is better than the fixed threshold technique in improving the BER performance of FSO communication system.

## 2. CHANNEL MODEL

In [19] and [20], the EW model proposed by Mudholkar and Srivastava [21] was adopted to describe the distribution characteristics of light intensity attenuation under atmospheric turbulence. The results show that the EW distribution model is more accurate than LN model and Gamma-Gamma model when considering the aperture averaging based on experiments. We model  $h$  by the EW distribution, and the probability density function (PDF) is expressed as:

$$f_h(h) = \frac{\alpha\beta}{\eta} \left(\frac{h}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{h}{\eta}\right)^\beta\right] \left\{1 - \exp\left[-\left(\frac{h}{\eta}\right)^\beta\right]\right\}^{\alpha-1}, \quad h \geq 0, \quad (1)$$

and the corresponding cumulative distribution function (CDF) is:

$$F_h(h) = \left\{1 - \exp\left[-\left(\frac{h}{\eta}\right)^\beta\right]\right\}^\alpha, \quad h \geq 0, \quad (2)$$

where  $\alpha > 0$ ,  $\beta > 0$  are shape parameters, and  $\eta > 0$  is a scale parameter [19]. The empirical formulas of these three parameters have been given in [20]. Similar to [6, 8], this paper adopts the three parameter values obtained by experiments in [19] to analyze the performance of OOK modulation, because these values are more accurate than those obtained from the empirical formulas.

## 3. PERFORMANCE ANALYSIS

The photocurrent generated by the photodetector can be expressed as:

$$i_r(t) = \begin{cases} 0 + n(t), & \text{'0'} \\ 2P_t R h + n(t), & \text{'1'} \end{cases}, \quad (3)$$

where  $P_t$  is the average transmitted power,  $R$  the photodiode responsivity, and  $n(t) \sim N(0, \sigma^2)$  the additive noise. When the signal is '0', no pulse is transmitted, so it is not affected by turbulence, and the marginal probability is:

$$P(i_r/0) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-i_r^2}{2\sigma^2}\right). \quad (4)$$

When the signal is '1', it is affected by turbulence:

$$\begin{aligned} P(i_r/1) &= \int_0^\infty P(i_r/1, h) P_h(h) dh \\ &= \int_0^\infty \exp\left[\frac{-(i_r - 2P_t R h)^2}{2\sigma^2}\right] \frac{\alpha\beta}{\eta} \left(\frac{h}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{h}{\eta}\right)^\beta\right] \left\{1 - \exp\left[-\left(\frac{h}{\eta}\right)^\beta\right]\right\}^{\alpha-1} dh. \end{aligned} \quad (5)$$

The adaptive threshold denoted by  $i_{th}$  is obtained at  $P(i_r/0) = P(i_r/1)$ . Namely, the adaptive threshold  $i_{th}$  is obtained from Eq. (6) with  $\Lambda = 1$ .

$$\Lambda = \int_0^\infty \exp\left\{-\frac{[(i_r - 2P_tRh)^2 - i_r^2]}{2\sigma^2}\right\} \frac{\alpha\beta}{\eta} \left(\frac{h}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{h}{\eta}\right)^\beta\right] \left\{1 - \exp\left[-\left(\frac{h}{\eta}\right)^\beta\right]\right\}^{\alpha-1} dh. \quad (6)$$

The BER  $P_e$  of adaptive threshold determination can be expressed as:

$$P_e = P(0) \int_{i_{th}}^\infty P(i_r/0) di_r + P(1) \int_0^{i_{th}} P(i_r/1) di_r. \quad (7)$$

It takes time to solve the adaptive threshold  $i_{th}$  directly. In order to improve the operation efficiency, Eq. (6) is deformed as follows:

$$\begin{aligned} \Lambda &= \int_0^\infty \exp\left\{-\frac{[(i_r - 2P_tRh)^2 - i_r^2]}{2\sigma^2}\right\} f_h(h) dh = \int_0^\infty \exp\left\{-\frac{[(i_r - 2P_tRh)^2 - i_r^2]}{2\sigma^2}\right\} dF_h(h) \\ &= -\int_0^\infty \left\{ \exp\left\{-\frac{[(i_r - 2P_tRh)^2 - i_r^2]}{2\sigma^2}\right\} \right\}' F_h(h) dh \\ &= \int_0^\infty \left[ \frac{(4P_t^2R^2h - 2P_tRi_r)}{\sigma^2} \exp\left(\frac{-2P_t^2R^2h^2 + 2P_tRhi_r}{\sigma^2}\right) \right] F_h(h) dh \\ &= \int_0^\infty \left[ \frac{4P_t^2R^2h}{\sigma^2} \exp\left(\frac{-2P_t^2R^2h^2}{\sigma^2}\right) \right] \left[ \exp\left(\frac{2P_tRhi_r}{\sigma^2}\right) F_h(h) \right] dh \\ &\quad - \int_0^\infty \left[ \frac{2P_tRi_r}{\sigma^2} \exp\left(\frac{-2P_t^2R^2h^2}{\sigma^2}\right) \right] \left[ \exp\left(\frac{2P_tRhi_r}{\sigma^2}\right) F_h(h) \right] dh, \end{aligned} \quad (8)$$

we make the variable substitution  $Z = 2P_t^2R^2h^2/\sigma^2$ , submitted into Eq. (8):

$$\begin{aligned} \Lambda &= \int_0^\infty \exp(-Z) \exp\left(\sqrt{2i}Z^{1/2}/\sigma\right) \left\{1 - \exp\left[-\left(\sqrt{2}\sigma Z^{1/2}/2P_tR\eta\right)^\beta\right]\right\}^\alpha dZ \\ &\quad - \int_0^\infty \left(\sqrt{2i}/2\sigma\right) Z^{-1/2} \exp(-Z) \exp\left(\sqrt{2i}Z^{1/2}/\sigma\right) \left\{1 - \exp\left[-\left(\sqrt{2}\sigma Z^{1/2}/2P_tR\eta\right)^\beta\right]\right\}^\alpha dZ \\ &= \Lambda_1 - \Lambda_2, \end{aligned} \quad (9)$$

both  $\Lambda_1$  and  $\Lambda_2$  have the form of  $\int_0^\infty x^\alpha e^{-x} f(x) dx$ , and  $\Lambda$  can be expressed as Eq. (10) by using the generalized Gauss-Laguerre polynomials [22]:

$$\begin{aligned} \Lambda &= \sum_{k=1}^n W_{1k} \exp\left(\sqrt{2i}Z_{1k}^{1/2}/\sigma\right) \left\{1 - \exp\left[-\left(\sqrt{2}\sigma Z_{1k}^{1/2}/2P_tR\eta\right)^\beta\right]\right\}^\alpha \\ &\quad - \left(\sqrt{2i}/2\sigma\right) \sum_{k=1}^n W_{2k} \exp\left(\sqrt{2i}Z_{2k}^{1/2}/\sigma\right) \left\{1 - \exp\left[-\left(\sqrt{2}\sigma Z_{2k}^{1/2}/2P_tR\eta\right)^\beta\right]\right\}^\alpha, \end{aligned} \quad (10)$$

where  $Z_{1k}$  is the  $k$ -th root of  $L_n^0(x) = 0$ ,  $W_{1k}$  the weight function,  $Z_{2k}$  the  $k$ -th root of  $L_n^{-1/2}(x) = 0$ , and  $W_{2k}$  the weight function,

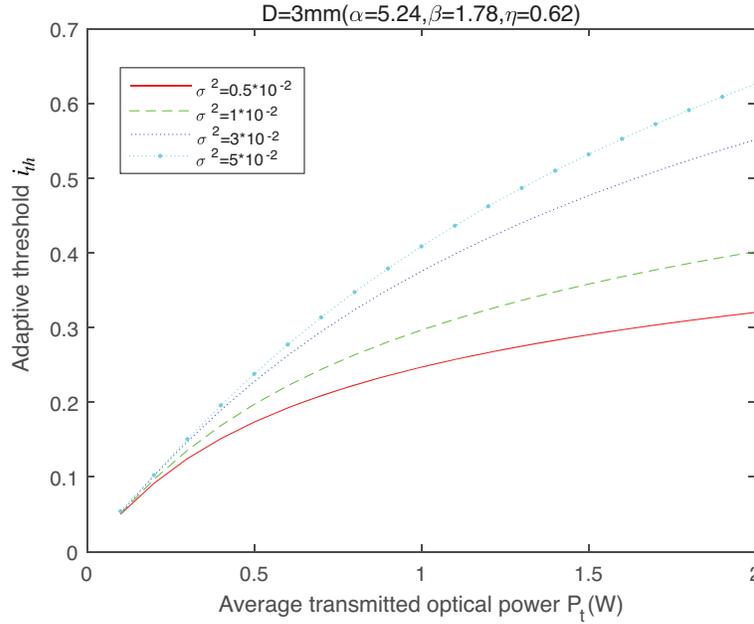
$$W_{1k} = \frac{\Gamma(n+1)Z_{1k}}{n!(n+1)^2[L_{n+1}^0(Z_{1k})]^2}, \quad (11)$$

$$W_{2k} = \frac{\Gamma(n+1/2)Z_{2k}}{n!(n+1)^2[L_{n+1}^{-1/2}(Z_{2k})]^2}, \quad (12)$$

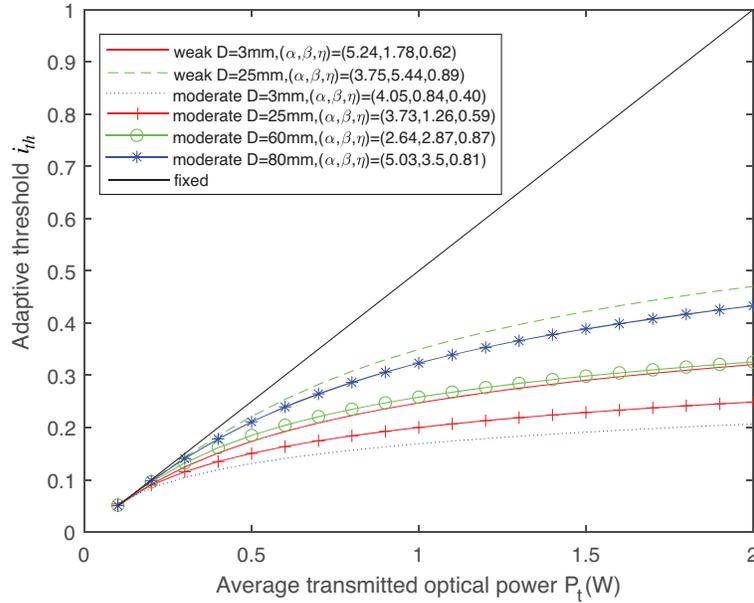
where  $n$  is chosen to be 40 in the following simulations.

### 4. RESULTS AND DISCUSSIONS

Figure 1 shows the relationship between the adaptive threshold and the average transmitted power with different noise variances. The receiving aperture size represented by  $D$  is 3 mm. As the average transmitted power increases, the adaptive threshold increases with the same noise variance. Under the same average transmitted power, the adaptive threshold increases with the increase of the noise variance.



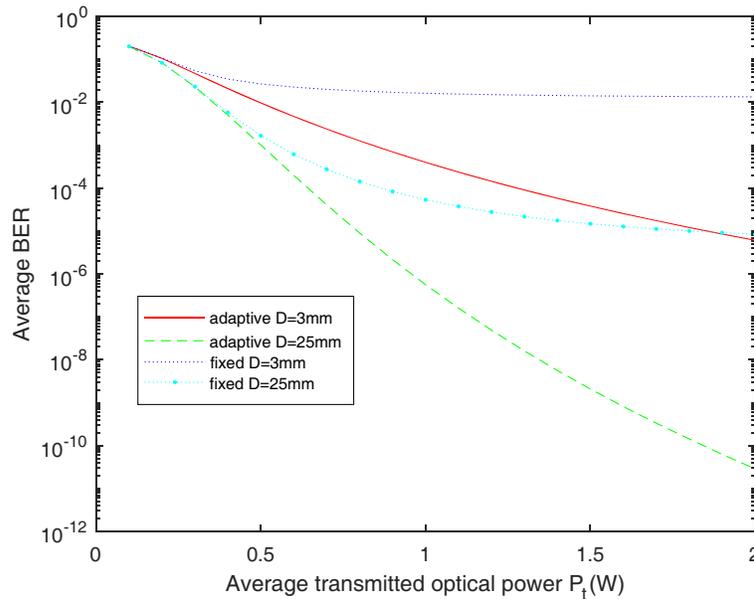
**Figure 1.** Adaptive threshold  $i_{th}$  against the average transmitted power with different noise variance.



**Figure 2.** Adaptive threshold  $i_{th}$  against the average transmitted power under weak and moderate turbulence with different receiving aperture size.

Figure 2 clearly shows the relationship between the adaptive threshold and the average transmitted power under weak and moderate turbulence. The adaptive threshold is smaller than the fixed threshold level in the presence of turbulence. The adaptive threshold increases with the increase of the receiving aperture size and decreases with the increase of the turbulence intensity.

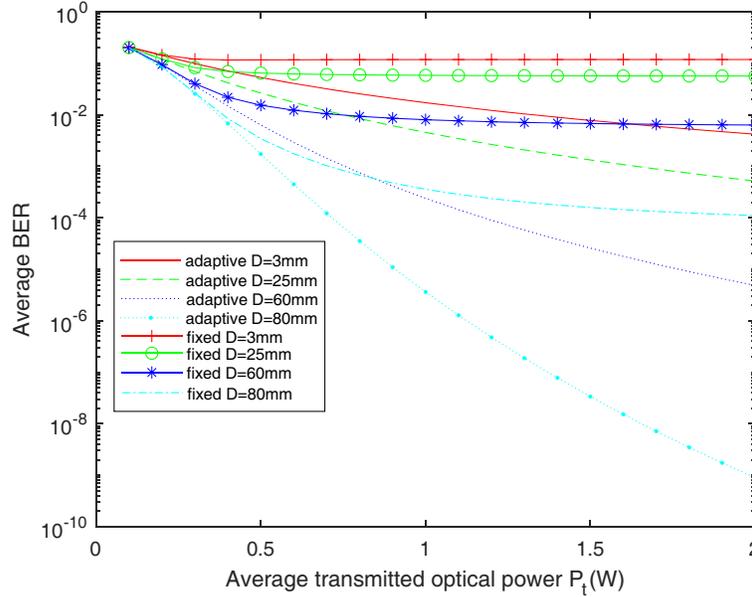
Figure 3 shows the relationship between the BER performance of the system with different receiving aperture sizes by using adaptive threshold technique and fixed threshold technique under weak turbulence. The results show that the BER performance of the system with adaptive threshold technique is better than that of the system with fixed threshold technique under weak turbulence. Taking  $P_t = 2w$  as an example, the BER performance of the system with adaptive threshold technique is  $6.10 \times 10^{-6}$  while fixed threshold technique is  $1.34 \times 10^{-2}$  at  $D = 3$  mm. When  $D$  increases to 25 mm, the BER performance of the system with adaptive threshold technique is  $2.87 \times 10^{-11}$ , whereas fixed threshold technique is  $8.42 \times 10^{-6}$ .



**Figure 3.** BER performance of the system with adaptive threshold technique and fixed threshold technique against the average transmitted power with different receiving aperture size under weak turbulence.

Figure 4 shows the relationship between the BER performance of the system with different receiving aperture sizes by using adaptive threshold technique and fixed threshold technique under moderate turbulence. The results are similar to those in weak turbulence. Taking  $P_t = 2w$  as an example again, the BER performance of the adaptive threshold decision is  $4.20 \times 10^{-3}$  while fixed threshold technique is  $1.18 \times 10^{-1}$  at  $D = 3$  mm. The BER performance of the system with adaptive threshold technique is  $5.17 \times 10^{-4}$ , whereas fixed threshold technique is  $5.66 \times 10^{-2}$  at  $D = 25$  mm. The BER performance of the system with adaptive threshold technique is  $4.85 \times 10^{-6}$ , but fixed threshold technique is  $6.30 \times 10^{-3}$  at  $D = 60$  mm. While  $D = 80$  mm, the BER performance of the system with adaptive threshold technique is  $8.97 \times 10^{-10}$  compared with  $1.11 \times 10^{-4}$  of the fixed threshold technique.

Through the above comparative analysis, it can be seen that the performance of BER can be greatly improved by adopting adaptive threshold technique. In Figures 3 and 4, the BER is less than  $10^{-9}$  only when the power reaches the order of watts, which is not suitable for application between buildings or streets near the ground, as it will cause harm to human eyes. It can be used for long-range FSO communication, such as unmanned aerial vehicle (UAV) optical communication system. The drone can transmit power to the level of watt, because the power reaching the ground after flying several kilometers is too small to harm the human eyes. Just as the transmission power of satellite communication is at the level of tens of watts or even hundreds of watts, the signal strength after reaching the ground is relatively small, and it will not be harmful to human eyes.



**Figure 4.** BER performance of the system with adaptive threshold technique and fixed threshold technique against the average transmitted power with different receiving aperture size under moderate turbulence.

## 5. CONCLUSION

In summary, we have studied the average BER of OOK modulation in FSO system based on adaptive threshold technique under atmospheric turbulence described by EW distribution. In order to improve the operational efficiency, the generalized Gauss-Laguerre polynomial function is employed to deform the equation when solving the adaptive threshold. The simulation results show that the adaptive threshold technique can deeply improve the BER performance of the system compared to the fixed threshold technique. The performance is improved by 4 and 5 orders of magnitude, respectively, when  $D = 3$  mm and  $D = 25$  mm with  $P_t = 2w$  under weak turbulence. The performance is improved by 2, 2, 3, and 6 orders of magnitude, respectively, when  $D = 3$  mm,  $D = 25$  mm,  $D = 60$  mm, and  $D = 80$  mm under moderate turbulence.

There is still a lot of work to be done in the field of FSO communication. This paper studies the adaptive threshold technique by numerical simulation and provides a new idea for improving the performance of FSO communication system. In the future, we will carry out experiments to further promote the application of this theory.

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