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Robust Adaptive Beamforming Based on Interference-Plus-Noise Covariance Matrix Reconstruction Method

Yang Bi[1, 2, *], Xi'an Feng[1], and Tuo Guo[3]

Abstract—Aiming at the problem of look direction error in the desired signal, a robust adaptive beamforming method based on covariance matrix reconstruction is proposed. The sparse Bayesian learning (SBL) is performed to acquire the true signal direction and the spatial spectrum simultaneously. Furthermore, the SBL spatial spectrum is used to reconstruct the interference-plus-noise covariance matrix. Compared with other reconstruction algorithms, this approach can realize the position estimation without any optimization procedures. Theoretical analysis, simulation results, and water pool experiments demonstrate the effectiveness and robustness of the proposed algorithm.

1. INTRODUCTION

When the array element covariance matrix is accurately known and the steering vector (SV) is accurate, the adaptive beamformer is an optimal spatial filter based on the maximization of the output signal-to-interference-noise ratio (SINR) criterion. However, in the actual underwater acoustic signal processing environment, the number of sampling snapshots is limited, and the signal-of-interest (SOI) is always present in the training data. Furthermore, there may also be a look direction mismatch problem. These problems will cause the performance of the beamformer to be seriously degraded, thereby greatly reducing the robustness of the beamformer. In response to these problems, many robust adaptive beamforming algorithms were proposed based on interference-plus-noise covariance matrix (INCM) reconstruction [1–4]. In [1], the proposed algorithm eliminates the effects of expected signal components by reconstructing the INCM and adopts solving quadratic constraint quadratic programming to modify the steering vector. The integration in [1] can be regarded as linear, and the method in [2] modifies the linear integration area into annular volume with high computation complexity. In [3], a sparse INCM reconstruction algorithm has been proposed which makes explicit use of sparsity of source distribution, but it still cannot cope with array calibration errors. In [4], a spatial power sampling method is proposed to reconstruct the INCM. For these reconstruction-based algorithms, the desired signal direction-of-arrival (DOA) must be found firstly by a low-resolution DOA estimation algorithm. Then, the INCM can be reconstructed by adopting Capon spatial spectrum. Hence, two DOA estimation algorithms are employed during this process. Besides, the accuracy of Capon spatial spectrum degrades severely when coherent interference exists.

In order to solve the above problems, some modified algorithms have been proposed in [5–7]. In [5], a new estimator for the INCM based on interference SV and power estimation is presented, and a quadratic convex optimization problem with new inequality constraint is established to estimate the steering vector of the desired signal. In [6], subspace methods are developed for RAB. The proposed methods utilize the orthogonality of interference subspace to reconstruct the INCM. In [7], the desired

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* Corresponding author: Yang Bi (200707002@xaau.edu.cn).
[1] School of Marine Science and Technology, Northwestern Polytechnical University, Xi'an, Shanxi 710072, China. [2] School of Electronic Engineering, Xi'an Aeronautical University, Xi'an, Shaanxi 710077, China. [3] School of Electronic Information and Artificial Intelligence, Shaanxi University of Science & Technology, Xi'an, Shaanxi 710021, China.

signal steering vector is estimated by minimizing the sine value of the angle between the presumed desired signal steering vector and the eigenvectors of sample covariance matrix. The sample covariance matrix is reconstructed by mitigating the effects of noise eigenvalues and desired signal on the performance of adaptive beamformer.

However, these modified algorithms should jointly deal with the covariance matrix uncertainty and SV mismatch, which has high computational complexity and could not effectively reconstruct the INCM. Therefore, in this paper a new robust adaptive beamforming method based on INCM reconstruction is proposed. This method introduces sparse Bayesian learning (SBL) algorithm, with high accuracy of DOA and power estimation, in the INCM reconstruction process. Due to the superior DOA estimation accuracy of the SBL algorithm, the SV estimation procedure adopted in [5–7] is no longer necessary in this method. The algorithm uses the SBL algorithm to determine the true signal direction and spatial spectrum, then uses the SBL algorithm to determine the desired signal orientation from the received signal and reconstructs the interference noise covariance matrix to obtain the optimal weight required for beamforming. Simulation results and water pool experiments prove that the proposed algorithm could effectively solve the problem when the incident signal contains expected signal components.

The rest of this paper is organized as follows. The data model of array output and some necessary background about adaptive beamformer are described in Section 2. In Section 3, a novel INCM reconstruction method is proposed. The SBL algorithm is introduced in the INCM reconstruction process. Simulation results are presented in Section 4. Water pool experiments are reported in Section 5. Finally, some conclusions are drawn in Section 6.

2. ARRAY DATA MODEL AND PROBLEM FORMULATION

Assume an array of M isotropic sensors spaced half-wavelength d apart. There are Q underwater acoustic target signals, of which a desired signal is incident from direction θ_0 and P interference signals with direction θ_j, $j = 1, 2, \cdots, P$. M and Q satisfy $M > Q$. The array observation vector $x(k)$ can be modeled as,

$$x(k) = x_s(k) + x_j(k) + x_n(k) \tag{1}$$

where $x_s(k)$, $x_j(k)$, and $x_n(k)$ are the desired signal, interference, and Gaussian white noise components, respectively.

The covariance matrix of the signal received by the array is,

$$\begin{aligned} R &= E\left[x(k)\,x(k)^H\right] = E\left\{\left[x_s(k) + x_j(k) + x_n(k)\right]\left[x_s(k) + x_j(k) + x_n(k)\right]^H\right\} \\ &= E\left[x_s(k)\,x_s^H(k)\right] + E\left[x_j(k)\,x_j^H(k)\right] + E\left[x_n(k)\,x_n^H(k)\right] = R_s + R_j + R_n = R_s + R_{j+n} \end{aligned} \tag{2}$$

where $(\cdot)^H$ denotes the Hermitian transpose, and R_s, R_j, R_n, R_{j+n} are the covariance matrix of the desired signal, interference, noise, and interference-plus-noise, respectively.

$$R_s = \sigma_0^2 a(\theta_0)\, a^H(\theta_0) \tag{3}$$

$$R_{j+n} = \sum_{j=1}^{P} \sigma_j^2 a(\theta_j)\, a^H(\theta_j) + \sigma_n^2 I \tag{4}$$

where σ_0^2 is the power of the desired signal, $a(\theta_0)$ the steering vector of the desired signal, θ_j the direction of the jth interference signal, $a(\theta_j)$ the corresponding steering vector, σ_j^2 the corresponding power of the interference signal, and σ_n^2 the noise power.

According to the maximum output signal to interference and noise ratio (MSINR) criterion:

$$\max_{\omega} \mathrm{SINR} = \max_{\omega} \frac{\omega^H R_s \omega}{\omega^H R_{j+n} \omega} \tag{5}$$

The optimal weight ω can be obtained by the above formula under the criterion of the MSINR. In an ideal situation, that is, under the constraint that the desired signal passes without distortion, Equation (5) turns to,

$$\begin{cases} \min & \omega^H R_{j+n} \omega \\ \text{s.t.} & \omega^H a(\theta_0) = 1 \end{cases} \tag{6}$$

This is the Capon beamformer. The optimal beamformer weight vector can be obtained as,

$$\omega_{opt} = \frac{R_{j+n}^{-1} a(\theta_0)}{a^H(\theta_0) R_{j+n}^{-1} a(\theta_0)} \tag{7}$$

According to the determined optimal weight, the output power of the Capon beamformer can be obtained as,

$$P = \omega_{opt}^H R_{j+n} \omega_{opt} = \frac{1}{a^H(\theta_0) R_{j+n}^{-1} a(\theta_0)} \tag{8}$$

In an ideal situation, the optimal weights obtained by R and R_{j+n} are equivalent [8]. The output power of Capon beamformer can be obtained as,

$$P = \omega_{opt}^H R \omega_{opt} = \frac{1}{a^H(\theta_0) R^{-1} a(\theta_0)} \tag{9}$$

In practical applications, R_{j+n} is difficult to obtain, so the sample covariance matrix $\hat{R}$ is usually used to obtain the optimal value,

$$\hat{R} = \frac{1}{N} \sum_{i=1}^{N} x(i) x^H(i) \tag{10}$$

where N is the number of snapshots.

Then the optimal beamformer weight vector can be obtained as,

$$\omega_{\text{capon}} = \frac{\hat{R}^{-1} a(\theta_0)}{a^H(\theta_0) \hat{R}^{-1} a(\theta_0)} \tag{11}$$

3. PROPOSED ALGORITHM

In this paper, a new INCM reconstruction approach based on SBL is proposed. The basic idea of the proposed adaptive beamformer is to make full use of the SBL spatial spectrum to revise SOI DOA and reconstruct R_{j+n}.

Firstly, the SBL is performed to acquire the true signal direction and the spatial spectrum simultaneously. Secondly, the SBL algorithm is used to reconstruct the interference-plus-noise covariance matrix, then the optimal weight can be obtained.

3.1. SBL Algorithm

We establish a DOA estimation model for underwater acoustic targets in the SBL framework and then determine the values of hyper parameters to determine the maximum posterior probability of underwater acoustic target signal sources, that is, to achieve DOA estimation of the target signal sources. Simultaneously, the spatial spectrum required for the reconstruction of the covariance matrix can be determined, and the orientation of the desired signal can be determined from the received signal using the SBL algorithm.

Under the same array element condition, Equation (1) can also be expressed by steering vector [9, 10],

$$X = A(\theta) S + N \tag{12}$$

where $X = [x_1, \cdots, x_K] \in \mathbb{C}^{M \times K}$ is the observation matrix of measurement vector; $S = [s_1, \cdots, s_K] \in \mathbb{C}^{Q \times K}$ is the unknown source matrix; and $N = [n_1, \cdots, n_K] \in \mathbb{C}^{M \times K}$ is the unknown noise matrix. Among them, $A(\theta) = [a(\theta_1), a(\theta_2), \cdots, a(\theta_Q)] \in \mathbb{C}^{M \times Q}$, $a(\theta_q) = \left[1, \cdots, e^{-j(m-1)\frac{\omega d}{c}\sin\theta_q}\right]$, where $a(\theta_q)$ is the steering vector of the incident signal from the direction of θ_q, c the sound speed, d the element spacing, and ω the signal frequency [11, 12].

Using Bayesian inference to solve the linear problem in Eq. (12) involves determining the posterior distribution of the complex source amplitudes S from the likelihood and a prior model. Equation (12) is sparse decomposed as [13, 14],

$$X = A_D\tilde{S} + N \tag{13}$$

$$A_D = [a(\theta_1), a(\theta_2), \cdots, a(\theta_Q)] \in \mathbb{C}^{M\times Q} \tag{14}$$

where A_D is the overcomplete dictionary, and $\tilde{S}$ is the row sparse vector. The objective function of $\tilde{S}$ is determined by,

$$L\left(\tilde{S}|X\right) = \left\|X - A_D\tilde{S}\right\|_2^2 + \lambda\left\|\tilde{S}\right\|_{p,2} \tag{15}$$

The intermediate parameter $\Upsilon = [\gamma_1, \cdots, \gamma_Q]^T$ is introduced to represent the source power of the incident signal in the direction set θ_Q, and assume that,

$$p\left(\tilde{S}; \Upsilon\right) \sim CN(0, \Gamma) \tag{16}$$

where $N(0, \Gamma)$ is a zero-mean Gaussian distribution with variance Γ, $\Gamma = \text{diag}(\Upsilon)$. Assuming that the observation noise is also Gaussian distribution, the posterior probability of $\tilde{S}$ about X obtained by Bayesian probability theory is as follows [15],

$$p\left(\tilde{S}|X; \Upsilon, \sigma^2\right) \sim CN(\mu_S, \Sigma_s) \tag{17}$$

Among them,

$$\mu_S = \Gamma A_D^H \Sigma_x^{-1} X \tag{18}$$

$$\Sigma_s = \left(\frac{1}{\sigma^2}A_D^H A_D + \Gamma^{-1}\right)^{-1} \tag{19}$$

$$\Sigma_x = \sigma^2 I + A_D\Gamma A_D^H \tag{20}$$

$$\Sigma_x^{-1} = \sigma^{-2}I - \sigma^{-2}A_D\left(\frac{1}{\sigma^2}A_D^H A_D + \Gamma^{-1}\right)^{-1} A_D^H\sigma^{-2} \tag{21}$$

Therefore, when the posterior probability of $\tilde{S}$ with respect to X reaches the maximum value conditioned on Υ and σ^2, the spatial spectrum estimate of the target signal can be obtained. The likelihood function of X with respect to Υ and σ^2 is,

$$p\left(X|\Upsilon; \sigma^2\right) = |\pi\Sigma_x|^{-N}\exp\left\{-\text{tr}\left(X^H\Sigma_x^{-1}X\right)\right\} \tag{22}$$

The objective function obtained from Equation (22) is,

$$L\left(\Upsilon, \sigma^2\right) = \ln|\Sigma_x| + \text{tr}\left(\Sigma_x^{-1}S_x\right) \tag{23}$$

where $S_x = \frac{1}{L}XX^H$.

The above formula can be optimized by using [16] as,

$$\gamma_q^{(\text{new})} = \frac{\gamma_q^{\text{old}}}{\sqrt{L}}\left\|X^H\Sigma_x^{-1}a(\theta_q)\right\|_2 \Big/ \sqrt{a(\theta_q)^H S_x a(\theta_q)} \tag{24}$$

$$M = \{q \in N \,|\, \text{The largest } M \text{ peaks in } \gamma\} = \{q_1, \cdots, q_M\}$$

$$A_M = \left(a_{q_1}, \cdots, a_{q_M}\right) \tag{25}$$

$$\left(\sigma^2\right)^{\text{new}} = \frac{1}{N-M}\text{tr}\left(\left(I_N - A_M A_M^+\right)S_x\right) \tag{26}$$

3.2. INCM Reconstruction

When the number of snapshots is limited and the look direction mismatches exist in the expected signal steering vectors, the interference plus noise covariance matrix is replaced by $\hat{R}$ of Equation (10), and the performance of adaptive beamforming degrades seriously. Therefore, it is necessary to reconstruct the interference plus noise covariance variance matrix.

Using the SBL spatial spectrum obtained by Eq. (24), the interference-plus-noise covariance matrix can be reconstructed as [17],

$$\hat{R}_{j+n} = \int_{\bar{\Theta}} \hat{P}(\theta) a(\theta) a^H(\theta)\, d\theta \tag{27}$$

where $a(\theta)$ is the steering vector associated with the direction θ based on the known array structure. $\bar{\Theta}$ is the complement sector of Θ. That is to say, $\Theta \cup \bar{\Theta}$ covers the whole spatial domain, and $\Theta \cap \bar{\Theta}$ is empty. The main requirement is that the signal's direction is in Θ while the interferers are not. Hence, $\hat{R}_{j+n}$ collects all information on interference and noise in the out-of-sector $\bar{\Theta}$. Consequently, the effect of the desired signal is removed from the reconstructed covariance matrix, as long as the desired signal's direction is located inside Θ. Θ can be obtained by SBL algorithm, then the interference-plus-noise covariance matrix can be reconstructed as,

$$\hat{R}_{j+n} = \sum_{\theta_k \in \bar{\Theta}} \hat{P}_k a(\theta_k) a^H(\theta_k) \tag{28}$$

where $\hat{P}_k$ is acquired through the SBL algorithm. Besides, due to the superior DOA estimation accuracy of the SBL algorithm, the SV estimation procedure adopted in other literature is no longer necessary in this method when SOI DOA mismatch exists.

3.3. Algorithm Flow

Based on the above analysis, the algorithm can be summarized as,

Step 1: Determine the true signal direction Θ and spatial frequency spectrum $\hat{P}_k$ by using the SBL algorithm.

Step 2: Reconstruct the interference-plus-noise covariance matrix $\hat{R}_{j+n}$ in Eq. (28) based on Θ and $\hat{P}_k$.

Step 3: Calculate the adaptive beamformer weight in Eq. (11) based on $\hat{R}_{j+n}$ and $a(\theta_0)$, $\omega_{\text{Rec}} = \frac{\hat{R}_{j+n}^{-1} a(\theta_0)}{a^H(\theta_0)\hat{R}_{j+n}^{-1} a(\theta_0)}$.

4. SIMULATION RESULTS

Five sets of simulation experiments were carried out to verify the effectiveness of the proposed algorithm in this paper. In our simulation, we assume a uniform linear array of 20 isotropic sensors and half-wavelength space. The desired signal and four interferences impinge on the array from directions 10°, −65°, −35°, 50°, and 60°, respectively. The presumed direction of the desired signal is 10°. In all the simulations, the interference-to-noise ratio is fixed at 0 dB. Additive noise is modeled as independent complex Gaussian noise with zero mean and unit variance. The number of snapshots is 40000, and for each scenario 100 Monte-Carlo runs are performed.

The output SINR of the beamformer can be expressed as [18],

$$\text{SINR} = \frac{\sigma_0 \left|\mathbf{W}^H \mathbf{a}_0(\theta_0)\right|}{\mathbf{W}^H \mathbf{R}_{j+n} \mathbf{W}} = \frac{\sigma_0 \left|\mathbf{W}^H \mathbf{a}_0(\theta_0)\right|}{\mathbf{W}^H (\mathbf{R}_j + \mathbf{R}_n) \mathbf{W}} = \frac{\sigma_0 \left|\mathbf{W}^H \mathbf{a}_0(\theta_0)\right|}{\mathbf{W}^H \left(\sum_{j=1}^{P} \sigma_j \mathbf{a}(\theta_j) \mathbf{a}^H(\theta_j) + \sigma_n \mathbf{I}\right) \mathbf{W}} \tag{29}$$

where σ_0, σ_j, σ_n are the power of the desired signal, interference, and noise, respectively; I is the identity matrix; P is the number of interference signals. The design goal of the optimal beamformer is to maximize the SINR, so the SINR becomes an important indicator for evaluating the beamformer.

4.1. Performance of the SBL Algorithm for DOA Estimation

In the first simulation, the SBL algorithm for DOA estimation was examined. When there are 5 signal sources in space, the comparison performance of the SBL algorithm for DOA estimation with MVDR algorithm is shown in Figure 1.

As can be seen from Figure 1, the incident angles of the signal source estimated by SBL are $-65°$, $-35°$, $10°$, $50°$, $60°$, which are the same as the assumed incident signal. Therefore, the SBL method can achieve high-resolution and reliable DOA estimation. Compared with the MVDR algorithm, although two sources are closely spaced, the SBL algorithm still has a better resolution.

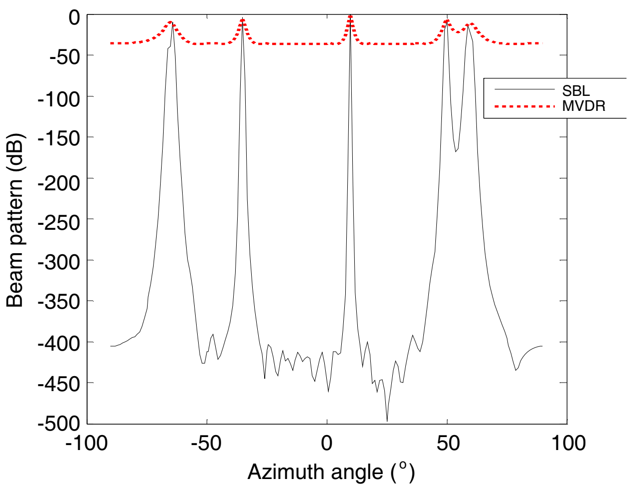

Figure 1. DOA estimation of SBL algorithm.

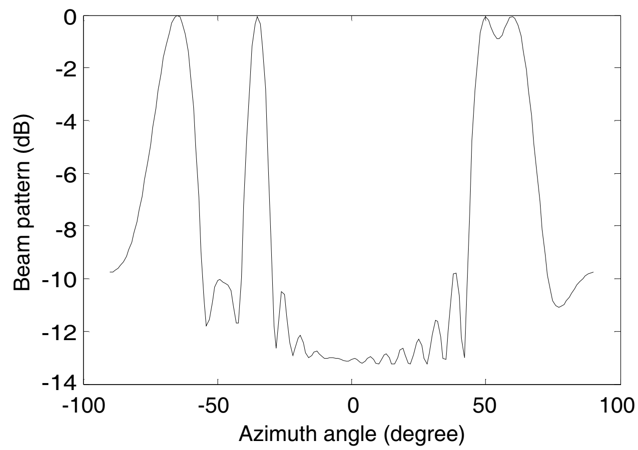

Figure 2. Beam pattern after INCM reconstruction.

4.2. Performance of the SBL Algorithm for INCM

This paper proposes an INCM reconstruction method based on the SBL algorithm. The true signal direction and spatial spectrum are acquired by the SBL algorithm. They are Θ and $\hat{P}_k$ required by INCM algorithm. $\hat{R}_{j+n}$ is calculated using Equation (29). Figure 2 is the beam pattern after INCM reconstruction.

It can be seen from Figure 2 that the useful signal in the $10°$ azimuth is completely removed in the reconstructed beam pattern, leaving only the interference signal in four azimuths. Therefore, the INCM reconstruction method based on the SBL algorithm proposed in this paper can better realize the separation of useful signals, ensure that the incident signal does not contain useful signal components, and eliminate the influence of expected signal components.

4.3. Analysis of Output SINR under Different Input SNR

When the input SNR changes from -10 dB to 30 dB, the output SINR of the proposed method, compared with the reconstruction-estimation beamformer (REB) and the theoretical optimal one are shown in Figure 3. It can be seen from Figure 3 that that the performance of the proposed beamformer is close to the optimal output SINR in a large range from -10 to 30 dB.

4.4. Analysis of Output SINR under Different Snapshot Number

When the input signal SNR is fixed at 0 dB, and the number of snapshots changes from 1000 to 40000, the comparison of the output SINR of three different methods with the change of the number of snapshots is shown in Figure 4. It can be seen that the algorithm in this paper can achieve convergence when the number of snapshots is about 12000, and its output SINR is closer to the optimal value, and the performance is better.

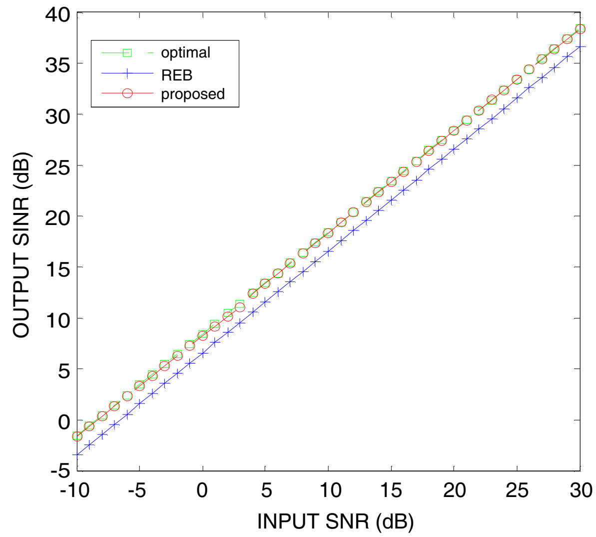

Figure 3. Output SINR versus input SNR.

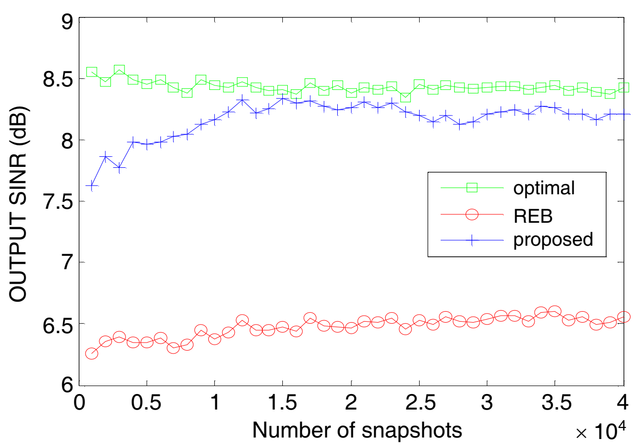

Figure 4. Output SINR versus snapshot number.

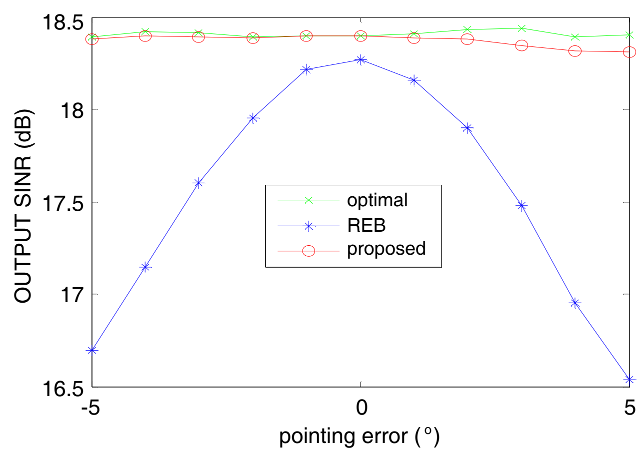

Figure 5. Output SINR versus mismatch angle.

4.5. Analysis of Output SINR with Angle of Mismatch

When the input signal SNR is fixed at 0 dB, the number of snapshots is 40000, and the mismatch angle of the desired signal changes from -5° to 5°. The output SINR change curves of the three methods are compared as shown in Figure 5. It can be seen that when the angle deviation of the desired signal steering vector is large, the algorithm in this paper is still close to the optimal output SINR. Thus, it has been demonstrated that the performance of the proposed beamformer against DOA errors is obviously better than that of REB.

5. WATER POOL EXPERIMENTS

The algorithm in this paper was verified by pool experiments in an anechoic pool. The Layout of the water pool experiment is shown in Figure 6. The water pool was 20 m long, 8 m wide, and 7 m deep. The water depth was approximately 6.7 m. The pool was a straight-wall reinforced concrete structure, and the tapered rubber wedge was used in the silencing unit. Units were arranged on six surfaces of the pool at equal intervals to eliminate reflected sound and simulate the marine environment. The 10-element

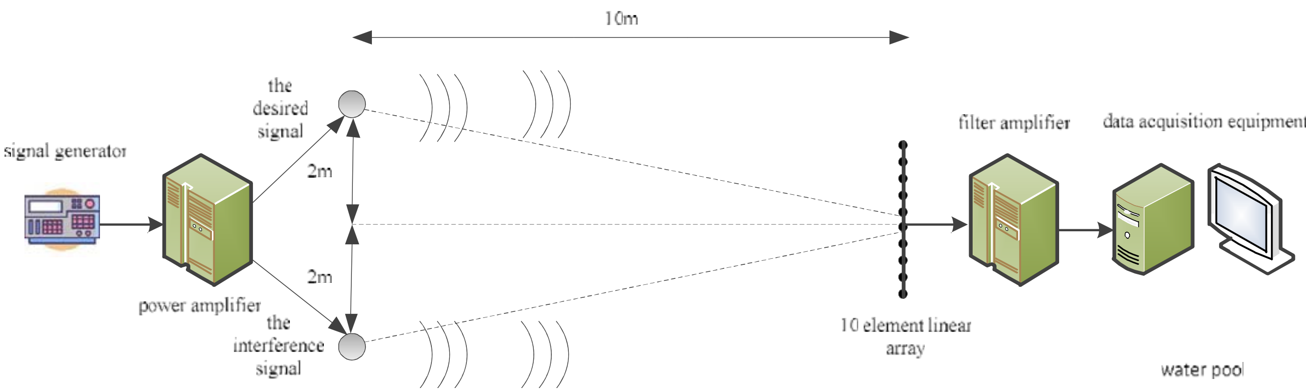

Figure 6. Layout of the water pool experiment.

linear array was placed at a depth of 2.5 m. The array element spacing is 0.12 m. The incident signal is two far-field narrowband and uncorrelated CW signals. The desired signal and interference signal are from directions 11.3° and −11.3°, respectively. They are at the same depth as the receiving array.

At the transmitting end, firstly, the signal generator generates two independent CW signals. Then the signals are driven amplified by the power amplifier. Furthermore, the two transmitting transducers receive the amplified electrical signals and convert them into acoustic signals for transmission. At the receiving end, the linear array receives weak acoustic signals, then the signals are processed by the filter amplifier and data acquisition equipment, respectively.

In the experiment, the sampling frequency is 40 kHz, and the sound velocity of the pool is set to 1490 m/s. Figure 7 is the scene graph of the pool experimental, and Figure 8 is the 10-element linear array.

Figure 7. Scene graph of the water pool experiment.

Figure 8. The 10-element linear array.

The results of the DOA estimation of SBL algorithm and the beam pattern after INCM reconstruction are shown in Figure 9 and Figure 10, respectively. As can be seen from Figure 9, the incident angles of the signal source are −11° and 11°. The incident angles are almost the same as the actual incident signal. Therefore, the SBL method can achieve high-resolution and reliable DOA estimation. Furthermore, the processing results of SBL can be used in the INCM reconstruction.

It can be seen from Figure 10 that the useful signal in the 11° azimuth is completely removed in the reconstructed beam pattern, leaving only the interference signal in the −11° azimuth. The proposed algorithm in this paper can better realize the separation of useful signals, ensure that the incident signal does not contain useful signal components, and eliminate the influence of expected signal components.

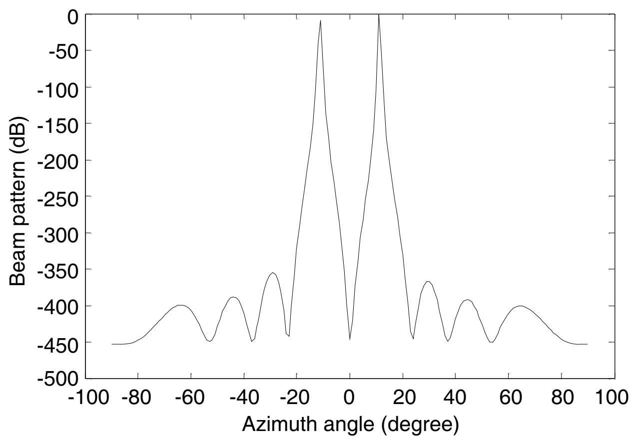

Figure 9. The DOA estimation of SBL algorithm.

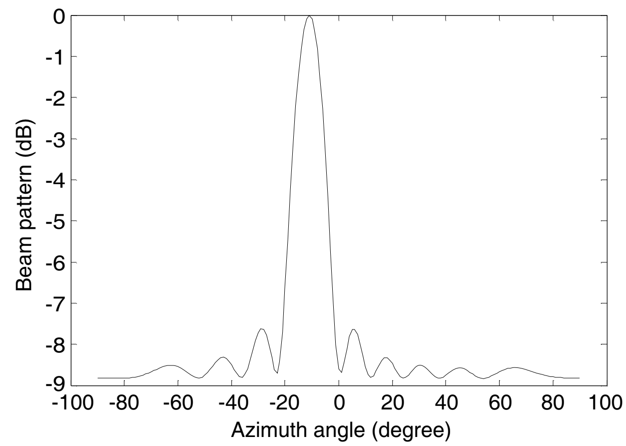

Figure 10. Beam pattern after INCM reconstruction.

6. CONCLUSIONS

This paper combines the idea of SBL algorithm with interference noise covariance matrix reconstruction and proposes a robust adaptive beamforming method based on covariance matrix reconstruction. The algorithm uses the SBL algorithm to determine the true signal direction and spatial spectrum, then uses the SBL algorithm to determine the desired signal orientation from the received signal and reconstructs the interference noise covariance matrix to obtain the optimal weight required for beamforming. The algorithm in this paper does not require any additional optimization process. Theoretical analysis, simulation results, and water pool experiments demonstrate that the proposed algorithm could achieve high-precision and robust position estimation when the incident signal contains expected signal components. Broadband signals which contain more information than narrowband signals are widely used in underwater signal processing. Therefore, I will apply the proposed algorithm to the broadband signals in future.

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