Progress In Electromagnetics Research M, Vol. 100, 81–91, 2021

Inductive Multi-Frequency Diversity Using Split Resonant Frequency

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Abstract—Although wireless power transfer systems suffer from splitting frequency conditions under strong coupling, this could create an opportunity for initiating other frequencies for power and data transfer. This paper introduces a model of an inductive transmitter containing a transmitter and many internal resonators to diversify the magnetic link to the receiver. Using the proposed architecture and solution, the efficiency and received power can be increased, and it also supports multiple frequency diversity.

1. INTRODUCTION

The electromagnetic resonance coupling experiment between two coils in a middle range undertaken by the MIT research group in 2006 has enabled considerable international research on wireless power transfer [1]. Since then, it has emerged that the power at the received coil at resonance frequency however repeatedly dropped at close distances due to strong couplings [2]. Historically, little attention was paid to this issue of frequency splitting. In the recent past, researchers have extensively investigated and analysed frequency splitting. Excellent analyses of frequency splitting phenomenon are presented in [3–7]. When transmitter and receiver are closer to the condition for coupling greater than the critical coupling, the single resonant peak at the receiver load quickly changes to double peaks, and thus power is shared between the two frequencies. The variation of the gap between the transmitter and receiver is the main cause for splitting frequencies. Therefore, this paper has taken advantage of this phenomenon to extend the magnetic link to receiver.

The pioneering research at MIT [8] provides a physical demonstration from a single source to multiple small receivers. Multiple investigators have gone further and inserted between the transmitter and receiver three coils [9, 10], four coils [11, 12], and N coils [13] to increase the distance and power transfer efficiency. These configurations increased the complexity of the nodes positions and arrangements between the coils.

The use of multiple bands created from many RLC circuits inside the resonator was introduced for single transmitter-receiver in [14]. These systems require a single excitation at the input but use much more elaborate RLC circuits to create multiple frequencies. The implementation was limited to a single receiver by the authors. Recently, the effects of coupling coefficient due to the use of multiple transmitters and receivers were investigated in [15, 16]. This group also successfully demonstrated the use of boosting resonators for higher power transfer in [17]. Our analysis in [18] demonstrated that the splitting frequency creates multiple frequencies in MIMO inductive communication. This paper shows that multichannel communication using splitting can be used for data transmission. However, it is hard to find the idea of using splitting frequencies from transmitter and relays for powering multiple users. Each user could use different splitting frequencies from the collocation between transmitter and relays. Therefore, in this paper the focus is on the use of splitting frequencies to diversify the power transfer

Received 17 July 2020, Accepted 21 December 2020, Scheduled 7 January 2021
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in short range regions. Most wireless power transfer systems involving multiple frequency schemes use single input sources. N distinct frequency sources are generated independently.

The rest of the paper is presented as follows. Section 2 introduces the theory of splitting frequency from circuit theory point of view. It includes the effects of reflected impedance and proposes new antenna structures to present the multiple frequency users. Section 3 proves multiple frequencies models with one and two relays. The last section is our conclusion.

2. MODELS OF MULTIPLE RECEIVERS

An effect of frequency condition for maximizing the power transfer under strong coupling is presented in this section for single magnetic induction systems. The value of reflected impedance to the system according to the frequency change is analyzed. The analysis forms the basis for multiple resonators proposed to power multiple receivers.

2.1. Single Transmitter and Receiver

At the near field, magnetic induction communication and wireless power transfer involving single resonators is equivalent to an RCL circuit in Fig. 1. In the circuit, the transmitter coil (L_1) wirelessly transfers magnetic energy to the receiver coil (L_2) at some short distance away. Components C_1, R_1 and C_2, R_2 are the capacitors and internal/loss resistors of the transmitter and receiver, respectively. The source and load impedances are R_s and R_L.

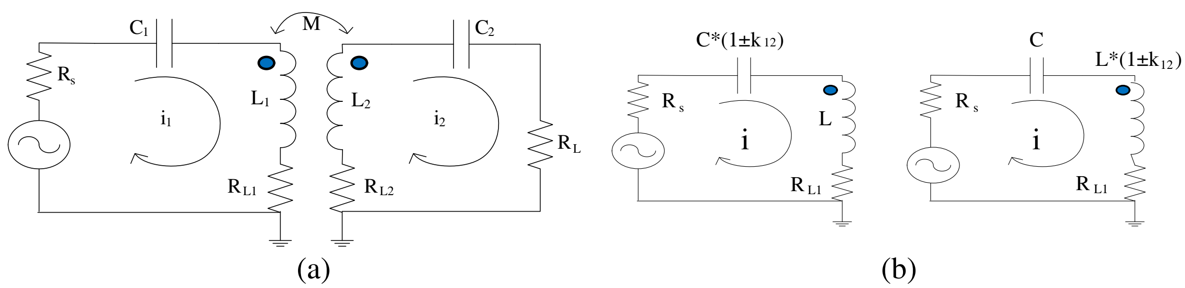


Figure 1. Single resonator and equivalent circuit model.

Based on the Kirchoff's voltage law, the voltage across the transmitting and receiving circuits is expressed as in the following matrix:

$$\begin{bmatrix} Z_{Tx} & j\omega M \\ j\omega M & Z_{Rx} \end{bmatrix} \begin{bmatrix} I_{Tx} \\ I_{Rx} \end{bmatrix} = \begin{bmatrix} V_{Tx} \\ 0 \end{bmatrix} \tag{1}$$

where the impedances of the transmitter and receiver are $Z_{Tx} = (R_s + R_{L1})(1 + jQ_{Tx}\frac{\omega}{\omega_o}(1 - \frac{\omega_o^2}{\omega^2}))$ and $Z_{Rx} = (R_s + R_{L2})(1 + jQ_{Rx}\frac{\omega}{\omega_o}(1 - \frac{\omega_o^2}{\omega^2}))$, respectively. The mutual coupling (M) is calculated as $M = k_{12}\sqrt{L_1 L_2}$ introducing how strong the magnetic coupling between the coils is. When mutual coupling distance is varied, there is a reflected energy from the receiver back to the transmitter. A reflected impedance relating to the coupling change determines the power transfer efficiency of the system. Thus, the reflected impedance at input source is given as:

$$Z_{in} = Z_{Tx} + \frac{\omega^2 M^2}{Z_{Rx}} \tag{2}$$

When a transmitter is coupled to a receiver, the efficiency of a double tuned circuit in Fig. 1(a) can be written as [18]:

$$\frac{P_{Rx}}{P_{Tx}} = \left[\frac{2\omega M}{Z_{Tx} Z_{Rx} + \omega^2 M^2}\right]^2 R_s R_L \tag{3}$$

At weak coupling, the transmitter to receiver efficiency (η) can be derived as the ratio of the transferred power to the total dissipated power [17]:

$$\eta_{\omega_o} == \frac{|I_{Rx}|^2 R_{Rx}}{|I_{Tx}|^2 R_{Tx} + |I_{Rx}|^2 R_{Rx}} = \frac{1}{\left|\frac{I_{Tx}}{I_{Rx}}\right|^2 \frac{R_{Tx}}{R_{Rx}} + 1} = \frac{1}{\frac{1}{k_T^2 Q_{Tx} Q_{Rx}} + 1} \tag{4}$$

where the quality factor of transmitter and receiver is $Q_{Tx} = \omega_o L_{Tx}/R_{Tx}$ and $Q_{Rx} = \omega_o L_{Rx}/R_{Rx}$, respectively. The resistances of the transmitter and receiver are R_{Tx} and R_{Rx}, respectively.

To analyze the effect of frequency condition in the strong coupling mode for maximizing the power transfer with the variation of the coupling between the transmitter and receiver, the received power in Eq. (3) is plotted in Fig. 2. It is observed that when coupling coefficient starts at the value of $k > \frac{1}{\sqrt{Q_{Tx} Q_{Rx}}}$ [3], the receiver power has dropped quickly from the peak at original resonant frequency and has two peaks at two splitting frequencies. This is due to the variation of the input impedance. If we insert the even and odd frequencies [3, 4] $\omega = \frac{\omega_o}{\sqrt{1 \pm k}}$ into Equation (2), the input impedance becomes a pure resistance $Z_{in} = R_s + R_{L1}$. This value of resistance is the same as in the circuit at the loose coupling condition. However, at the original resonant frequency ω_o, the circuit has a high impedance $Z_{in} = (R_s + R_{L1})(1 + k^2 Q_{Tx} Q_{Rx})$. Thus, the term $k^2 Q_{Tx} Q_{Rx}$ is the main reason for the power loss at the receiver. As a result, the receiver has maximized power at the splitting frequencies. This is also the conclusion reached in the work in [3]. Therefore, due to the influences of the coupling coefficient, the system in Fig. 1(a) is converted to the RLC circuit in Fig. 1(b). This circuit indicates that each splitting frequency is equivalent to the product term of a virtual capacitor or inductor denoted with $1 \pm k$. It means that the system can see either more inductance or more capacitance.

It is observed in Fig. 2 that at the coupling coefficient $k_{critical} = \frac{1}{\sqrt{Q_{Tx} Q_{Rx}}}$, the received power has a peak at resonant frequency of 13.56 (MHz). But when $k > k_{critical}$, the circuit creates two splitting frequencies. At the coupling coefficient of $k = 0.3$, 0.5, 0.7, the lower and upper frequencies are specified as 11.94 MHz–16.17 MHz, 11.06 MHz–19.15 MHz, and 10.4 MHz–24.72 MHz, respectively. If the gap is varied, splitting frequencies can be generated. Consequently, this splitting frequency will create multiples of frequency systems, which may be used for multiple receivers.

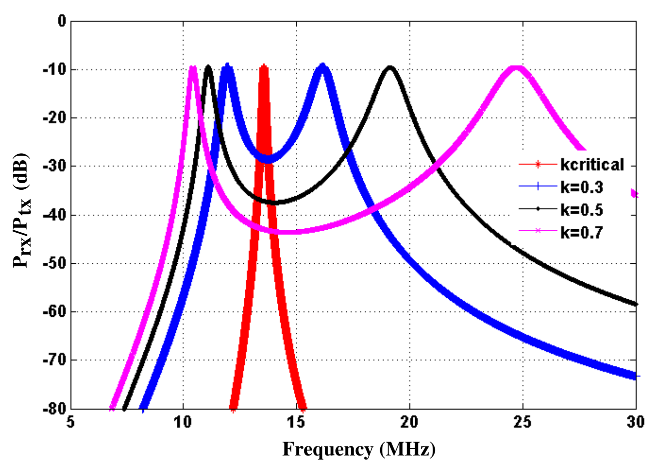


Figure 2. Received power at different couplings.

2.2. Proposed Multiple Receiver Configurations

2.2.1. Single Internal Relay

The system being proposed consists of a transmitter, a relay, and a receiver presented in Fig. 3. The transmitter and relay are close to each other with mutual coupling k. The relay is embedded in the

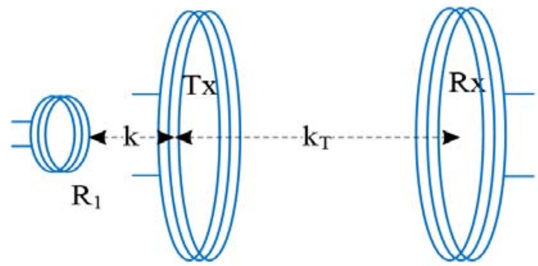


Figure 3. Received power, single relay, and receiver.

transmitter [17, 19] and performs the role of impedance transformation. Two resonators are coupled in strong coupling regime, but they have resonated at weak coupling to the receiver. Unlike other wireless power transfer systems in [11, 12], the receiver operates at resonant frequencies, and in our configuration the receiver has benefited from the splitting frequencies.

When the source voltage V_{Tx} is able to transfer the energy to the receiver, the loop currents at the transmitter, relay, and receiver are induced as I_{Tx}, I_{R1}, and I_{Rx}, respectively. The Kirchhoff Voltage Law (KVL) relating to the loop current is represented by the following matrix:

$$\begin{bmatrix} Z_{Tx} & ka_1 & k_T a_{TxRx} \\ ka_1^{-1} & Z_{R1} & k_T a_{R1Rx} \\ k_T a_{TxRx}^{-1} & k_T a_{R1Rx}^{-1} & Z_{Rx} \end{bmatrix} \begin{bmatrix} I_{Tx} \\ I_{R1} \\ I_{Rx} \end{bmatrix} = \begin{bmatrix} \dfrac{V_{Tx}}{j\omega L_{Tx}} \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

where the impedances of the transmitter, relay, and receiver are $Z_{Tx} = \frac{R_{Tx}}{j\omega L_{Tx}} + (1 - \frac{\omega_o^2}{\omega^2})$, $Z_{R1} = \frac{R_{R1}}{j\omega L_{R1}} + (1 - \frac{\omega_o^2}{\omega^2})$ and $Z_{Rx} = \frac{R_{Rx}}{j\omega L_{Rx}} + (1 - \frac{\omega_o^2}{\omega^2(1 \pm k)})$. The strong coupling between the transmitter and relay is k, and weak coupling from transmitter/relay to receiver is k_T. The inductor factors are defined as $a_1 = \sqrt{\frac{L_{R1}}{L_{Tx}}}$, $a_{R1Rx} = \sqrt{\frac{L_{Rx}}{L_{R1}}}$ and $a_{TxRx} = \sqrt{\frac{L_{Rx}}{L_{Tx}}}$. It is reported in [5, 17] that the factor $\frac{R}{j\omega L}$ becomes negligible because the imaginary part $j\omega L$ of the transformed impedance in resonators greatly exceeds the real part R in magnitude under a critical coupling condition. Simplifying Eq. (5) at the splitting frequencies $\omega = \frac{\omega_o}{\sqrt{1 \pm k}}$ and high-quality factor, it can be rewritten as:

$$\begin{bmatrix} \mp k & ka_1 & k_T a_{TxRx} \\ ka_1^{-1} & \mp k & k_T a_{R1Rx} \\ k_T a_{TxRx}^{-1} & k_T a_{R1Rx}^{-1} & \dfrac{R_{Rx}}{j\omega L_{Rx}} \end{bmatrix} \begin{bmatrix} I_{Tx} \\ I_{R1} \\ I_{Rx} \end{bmatrix} = \begin{bmatrix} \dfrac{V_{Tx}}{j\omega L_{Tx}} \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

Observe the impedance transformations in Equation (6) with changes as $Z_{Tx} = \mp k$, $Z_{R1} = \mp k$ and $Z_{Rx} = \frac{R_{Rx}}{j\omega L_{Rx}}$ along the leading diagonal of the matrix. Solving Eq. (6) for a current ratio between the transmitter/relay and receiver at the splitting frequencies, we have:

$$\begin{aligned} \frac{I_{Tx}}{I_{Rx}} &= \frac{a_1 a_{TxRx} a_{R1Rx} \left(-L_{Rx} k_T^2 \omega \pm jkR_{Rx}\right)}{L_{Rx} k k_T \omega \left(a_{TxRx} \pm a_1 a_{R1Rx}\right)} \\ \frac{I_{R1}}{I_{Rx}} &= \frac{a_{R1Rx} \left(a_1 a_{R1Rx} L_{Rx} k_T^2 \omega + j a_{TxRx} k R_{Rx}\right)}{L_{Rx} k k_T \omega (a_{TxRx} \pm a_1 a_{R1Rx})} \end{aligned} \tag{7}$$

From Equation (7) the current ratio between the transmitter/relay and the receiver is changed if the transmitter operates at even or odd frequency mode. At the odd frequency, this ratio is high to infinity. Consequently, the received power and efficiency will decrease if the higher splitting frequency is used. Another way to explain this is that at the odd frequency, the loop currents in the transmitter and relay are out-of-phase, and at the even frequency, both currents are in-phase [16]. Thus, if the odd frequency is used in a driving source, to get a higher efficiency the switching current direction change is needed at the relay. If the transmitter is used at even frequency, there is no need to change the current direction at the relay. As a result, the current ratio at the even frequency is simplified as:

$$\begin{aligned} \frac{I_{Tx}}{I_{Rx}} &= \frac{-L_{Rx}k_T^2\omega + jkR_{Rx}}{2kk_T\omega\sqrt{L_{Tx}L_{Rx}}} \cong \frac{R_{Rx}}{2k_T\omega\sqrt{L_{Tx}L_{Rx}}} \\ \frac{I_{R1}}{I_{Rx}} &= \frac{L_{Rx}k_T^2\omega + jkR_{Rx}}{L_{Rx}kk_T\omega(a_{TxRx} \pm a_1 a_{R1Rx})} \cong \frac{R_{Rx}}{2k_T\omega\sqrt{L_{Tx}L_{Rx}}} \end{aligned} \tag{8}$$

The approximation in Equation (8) holds when $L_{Rx}k_T^2\omega \ll 1$. This is true since coupling coefficient k_T is loosely coupled. At a first look, Equation (8) is similar to Equations (23) and (24) in [17]; however, the frequency in Eq. (8) is at the even frequency $\frac{\omega_o}{\sqrt{1+k}}$. Applying Eq. (8) to efficiency (η), we have:

$$\begin{aligned} \eta_{\frac{\omega_o}{\sqrt{1+k}}} &= \frac{|I_{Rx}|^2 R_{Rx}}{|I_{Tx}|^2 R_{Tx} + |I_{R1}|^2 R_{R1} + |I_{Rx}|^2 R_{Rx}} = \frac{1}{\left|\frac{I_{Tx}}{I_{Rx}}\right|^2 \frac{R_{Tx}}{R_{Rx}} + \left|\frac{I_{R1}}{I_{Rx}}\right|^2 \frac{R_{R1}}{R_{Rx}} + 1} \\ &= \frac{1}{\frac{(1+k)}{4k_T^2 Q_{Rx}}\left(\frac{1}{Q_{Tx}} + \frac{1}{Q_{R1}}\right) + 1} \end{aligned} \tag{9}$$

where k takes the value between

$$\frac{1}{\sqrt{Q_{Tx}Q_{R1}}} < k < 1 \tag{10}$$

The value k in Equation (10) depicts critical coupling bounds. Equation (9) shows that the efficiency is higher than the single transmitter-receiver in Fig. 1. It means that the interaction of relay has boosted the power of the receiver if the original resonant frequency moves to the splitting frequencies within the transmitter-relay-receiver configuration. This is in agreement with [17]. This conclusion is confirmed by experiments in the later sections of this paper. However, we can see that the coupling between transmitter and relay has also contributed to the received power. The smaller the value of k is, the higher the efficiency of the system is. For example, the received power could increase by 10% if k changes from 0.3 to 0.5.

2.2.2. Double Internal Relays

Figure 4 illustrates one transmitter, two relays, and one receiver. The coupling of relay-transmitters is $k_i (i = 1:2)$; cross coupling of relays is k_{12}; coupling of transmitter/relays to receiver is k_T.

At first sight, it might appear that equal coupling between the relays and receivers and between the transmitter Tx and receiver is hard to achieve. However, understanding that the coupling coefficient is

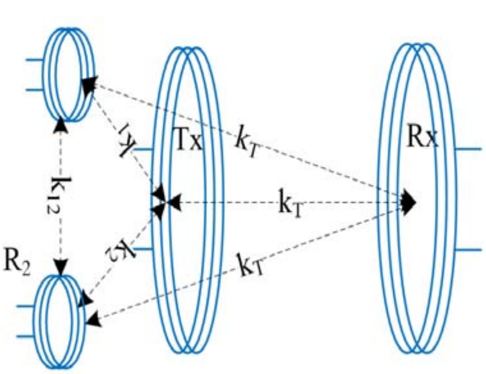


Figure 4. Transmitter, 2 relays and receiver.

a function of the inductances involved, this is achievable by ensuring that the relays (even at smaller radii) have the same inductance as the transmitter. Hence

$$k_T = \frac{M}{\sqrt{L_{Rj}L_{Rx}}} = \frac{M}{\sqrt{L_{Tx}L_{Rx}}} \quad 1 \le j \le N \tag{11}$$

N is the number of relays, and j is a relay index and is the inductance of a relay node. In other words, the relays have more turns. Using the same method in single relay, the matrix resulting from the KVL is:

$$\begin{bmatrix} Z_{Tx} & k_1 a_1 & k_2 a_2 & k_T a_{TxRx} \\ k_1 a_1^{-1} & Z_{R1} & k_{12} a_{12} & k_T a_{R1Rx} \\ k_2 a_2^{-1} & k_{12} a_{12}^{-1} & Z_{R2} & k_T a_{R2Rx} \\ k_T a_{TxRx}^{-1} & k_T a_{R1Rx}^{-1} & k_T a_{R2Rx}^{-1} & Z_{Rx} \end{bmatrix} \begin{bmatrix} I_{Tx} \\ I_{R1} \\ I_{R2} \\ I_{Rx} \end{bmatrix} = \begin{bmatrix} \frac{V_{Tx}}{j\omega L_{Tx}} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

Assume that the components of transmitter and relay are the same as single relay. The receiver changes the capacitor or inductor as the value of splitting frequencies generated between transmitter and relays. The inductor factors are defined as $a_1 = \sqrt{\frac{L_{R1}}{L_{Tx}}}$, $a_2 = \sqrt{\frac{L_{R2}}{L_{Tx}}}$, $a_{12} = \sqrt{\frac{L_{R2}}{L_{R1}}}$, $a_{R1Rx} = \sqrt{\frac{L_{Rx}}{L_{R1}}}$, $a_{R2Rx} = \sqrt{\frac{L_{Rx}}{L_{R2}}}$ and $a_{TxRx} = \sqrt{\frac{L_{Rx}}{L_{Tx}}}$. The transmitter and receiver currents regarding the splitting frequencies between the transmitter and relays are divided as in the following cases.

Case 1 refers to equal coupling of transmitter-relays $k_1 = k_2 = k$ and no cross coupling between relays $k_{12} = 0$. Matrix in Eq. (12) can be rewritten as:

$$\begin{bmatrix} -k & k a_1 & k a_2 & k_T a_{TxRx} \\ k a_1^{-1} & -k & 0 & k_T a_{R1Rx} \\ k a_2^{-1} & 0 & -k & k_T a_{R2Rx} \\ k_T a_{TxRx}^{-1} & k_T a_{R1Rx}^{-1} & k_T a_{R2Rx}^{-1} & \frac{R_{Rx}}{j\omega L_{Rx}} \end{bmatrix} \begin{bmatrix} I_{Tx} \\ I_{R1} \\ I_{R2} \\ I_{Rx} \end{bmatrix} = \begin{bmatrix} \frac{V_{Tx}}{j\omega L_{Tx}} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

As a result, the current ratios from transmitter/relays to receiver are calculated as:

$$\begin{aligned} \frac{I_{Tx}}{I_{Rx}} &= \frac{-2L_{Rx}k_T^2\omega + jkR_{Rx}}{3kk_T\omega\sqrt{L_{Tx}L_{Rx}}} \cong \frac{R_{Rx}}{3k_T\omega\sqrt{L_{Tx}L_{Rx}}} \\ \frac{I_{R1}}{I_{Rx}} &= \frac{L_{Rx}k_T^2\omega + jkR_{Rx}}{3kk_T\omega\sqrt{L_{R1}L_{Rx}}} \cong \frac{R_{Rx}}{3k_T\omega\sqrt{L_{R1}L_{Rx}}} \\ \frac{I_{R2}}{I_{Rx}} &= \frac{L_{Rx}k_T^2\omega + jkR_{Rx}}{3kk_T\omega\sqrt{L_{R2}L_{Rx}}} \cong \frac{R_{Rx}}{3k_T\omega\sqrt{L_{R2}L_{Rx}}} \end{aligned} \tag{14}$$

The splitting frequencies are obtained as $\omega = \frac{\omega_o}{\sqrt{1+k\sqrt{2}}}$ and efficiency at the receiver load:

$$\eta_{\frac{\omega_o}{\sqrt{1+k}}} = \frac{1}{\left|\frac{I_{Tx}}{I_{Rx}}\right|^2 \frac{R_{Tx}}{R_{Rx}} + \left|\frac{I_{R1}}{I_{Rx}}\right|^2 \frac{R_{R1}}{R_{Rx}} + \left|\frac{I_{R2}}{I_{Rx}}\right|^2 \frac{R_{R2}}{R_{Rx}} + 1} = \frac{1}{\frac{(1+k\sqrt{2})}{9k_T^2 Q_{Rx}}\left(\frac{1}{Q_{Tx}} + \frac{1}{Q_{R1}} + \frac{1}{Q_{R2}}\right) + 1} \tag{15}$$

For this case, the higher the Q factors are, the lower the efficiency of power transfer is. Low Q factors favour higher power transfer. Consider the case when $k = 0.5$ and all $Q = 2$, the power efficiency is 63.73%. When the same values are used in Equation (4), the power efficiency is 50%. Using relays has increased power efficiency.

Case 2 is the same as case 1 except for unequal coupling of transmitter-relays $k_1 \ne k_2$. The even frequencies are obtained as $\omega = \frac{\omega_o}{\sqrt{1+\sqrt{k_1^2+k_2^2}}}$, and again efficiency is written as:

$$\eta_{\frac{\omega_o}{\sqrt{1+k}}} = \frac{1}{\left|\frac{I_{Tx}}{I_{Rx}}\right|^2 \frac{R_{Tx}}{R_{Rx}} + \left|\frac{I_{R1}}{I_{Rx}}\right|^2 \frac{R_{R1}}{R_{Rx}} + \left|\frac{I_{R2}}{I_{Rx}}\right|^2 \frac{R_{R2}}{R_{Rx}} + 1} = \frac{1}{\frac{\left(1+\sqrt{k_1^2+k_2^2}\right)}{9k_T^2 Q_{Rx}}\left(\frac{1}{Q_{Tx}} + \frac{1}{Q_{R1}} + \frac{1}{Q_{R2}}\right) + 1} \tag{16}$$

For this case, again considering the case when $k = 0.5$ and all $Q = 2$, the power efficiency is 63.73%. When the same values are used in Equation (4), the power efficiency is 50%.

Case 3 is that there is cross coupling between the relays and coupling $k_1 = k_2 = k_{12} = k$. The splitting frequency is given as $\omega_1 = \omega_o/\sqrt{(1+2k)}$. Thus, the power efficiency is derived as:

$$\eta_{\frac{\omega_o}{\sqrt{1+k}}} = \frac{1}{\left|\frac{I_{Tx}}{I_{Rx}}\right|^2 \frac{R_{Tx}}{R_{Rx}} + \left|\frac{I_{R1}}{I_{Rx}}\right|^2 \frac{R_{R1}}{R_{Rx}} + \left|\frac{I_{R2}}{I_{Rx}}\right|^2 \frac{R_{R2}}{R_{Rx}} + 1} = \frac{1}{\frac{(1+2k)}{9k_T^2 Q_{Rx}}\left(\frac{1}{Q_{Tx}} + \frac{1}{Q_{R1}} + \frac{1}{Q_{R2}}\right) + 1} \tag{17}$$

For this case, again consider the case when $k = 0.5$ and all $Q = 2$, the power efficiency is 60%. When the same values are used in Equation (4), the power efficiency is 50%.

2.2.3. *N Relays*

A system of transmitter, N relays, and receiver is presented in Fig. 5. Each relay is coupled to transmitter by the individual coupling coefficient as $k_i\,(i = 1 : N)$. The cross-couplings between the relays are named as $k_{ij}\,(i = 1 : N,\ j = 1 : N$ and $i \neq j)$. The coupling of transmitter/relay to receiver is k_T.

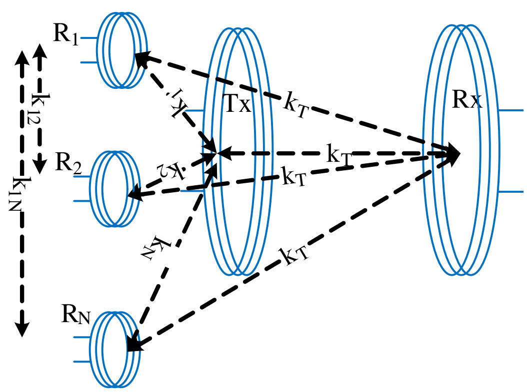


Figure 5. Transmitter, N relays, and receiver.

Applying the equivalent circuit to the configuration, the system can be derived as the matrix equation:

$$\begin{bmatrix} Z_{Tx} & k_1 a_1 & k_2 a_2 & \dots & k_T a_{TxRx} \\ k_1 a_1^{-1} & Z_{R1} & k_{12} a_{12} & \dots & k_{1n} a_{1n} \\ k_2 a_2^{-1} & k_{12} a_{12}^{-1} & Z_{R2} & \dots & k_{2n} a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ k_T a_{TxRx}^{-1} & k_{1n} a_{1n}^{-1} & k_{2n} a_{2n}^{-1} & \dots & Z_{Rx} \end{bmatrix} \begin{bmatrix} I_{Tx} \\ I_{R1} \\ I_{R2} \\ \vdots \\ I_{Rx} \end{bmatrix} = \begin{bmatrix} \frac{V_{Tx}}{j\omega L_{Tx}} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{18}$$

Again, the power ratio between transmitter and receiver regarding to the splitting frequencies is as follows:

Case 1: No cross-coupling within the relays, the coupling of transmitter-relays is identical; even frequency is written as $\omega = \frac{\omega_o}{\sqrt{1+k\sqrt{N}}}$. The efficiency is concluded as:

$$\eta_{\frac{\omega_o}{\sqrt{1+k\sqrt{N}}}} = \frac{1}{\left|\frac{I_{Tx}}{I_{Rx}}\right|^2 \frac{R_{Tx}}{R_{Rx}} + \sum_i^N \left|\frac{I_{Ri}}{I_{Rx}}\right|^2 \frac{R_{Ri}}{R_{Rx}} + 1} = \frac{1}{\frac{\left(1+k\sqrt{N}\right)}{(N+1)^2 k_T^2 Q_{Rx}}\left(\frac{1}{Q_{Tx}} + \sum_i^N \frac{1}{Q_{Ri}}\right) + 1} \tag{19}$$

For $N = 4$, $k = 0.5$, and all $Q = 2$, the power efficiency is 71.43%.

Case 2 is the same as case 1 with the different couplings of transmitter-relays. The splitting frequencies are $\omega = \frac{\omega_o}{\sqrt{1+\sqrt{\sum_i^N k_{Ti}^2}}}$. Thus, the efficiency can be obtained as:

$$\eta_{\frac{\omega_o}{\sqrt{1+\sqrt{\sum_i^N k_{Ti}^2}}}} = \frac{1}{\frac{\left(1+\sqrt{\sum_i^N k_{Ti}^2}\right)}{N^2 k_T^2 Q_{Rx}}\left(\frac{1}{Q_{Tx}}+\sum_i^N \frac{1}{Q_{Ri}}\right)+1} \tag{20}$$

For $N = 4$, $k = 0.5$ and all $Q = 2$, the power efficiency is 61%.

Case 3: the cross-talks of relays are involved in the system. When all couplings are equal, the splitting frequencies are solved as $\omega_1 = \frac{\omega_o}{\sqrt{1+N*k}}$ and $\omega_2 = \frac{\omega_o}{\sqrt{1-k}}$. The efficiency at even frequencies is calculated as:

$$\eta_{\frac{\omega_o}{\sqrt{1+N*k}}} = \frac{1}{\frac{(1+N*k)}{N^2 k_T^2 Q_{Rx}}\left(\frac{1}{Q_{Tx}}+\sum_i^N \frac{1}{Q_{Ri}}\right)+1} \tag{21}$$

For $N = 4$, $k = 0.5$, and all $Q = 2$, the power efficiency is 52%. We have presented the effect of relay interaction on the power transfer between the source and sink. It is obviously seen that using the internal relay will increase the power at the receiver. The improvement is based on the combination of quality factors of resonators and the magnetic flux between transmitter-relays. Additionally, many frequencies are created from the variation of coupling and the number of inserted relays.

3. RESULTS OF MULTIPLE COUPLING MODES FREQUENCIES

The objective of this section is to demonstrate and verify the creation of multiple frequencies from the effect of strong coupling modes between the transmitter and relays. We assume that the circuit elements of the resonators have inductance value of 6.3 μH, capacitive value of 21.9 pF, and resistance value of 5 Ω at transmitter/relay and 50 Ω at receiver. These resonators operate at resonant frequency 13.56 MHz, and the AC analysis in Spice program is used to analyse the frequency response of the circuit.

3.1. Experience with Tx-Rl and Rx

Figure 6 shows the schematic for models of transmitter-single relay and a receiver, which are built in LTspice. Transmitter and relays are coupled in strong coupling using .step command to list the coupling coefficients from 0.2 to 0.7. The coupling coefficients of transmitter/relay to receiver are fixed at the weak coupling of 0.03.

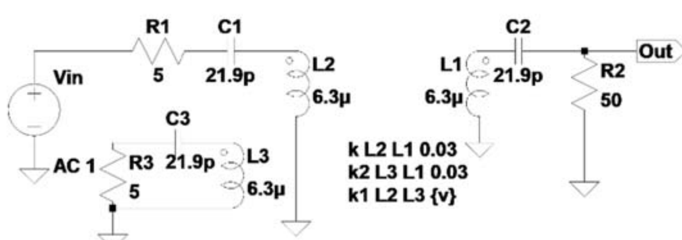


Figure 6. Model of Tx- single relay and Rx.

AC analysis is involved using the frequency sweeping from 5 MHz to 30 MHz to view the response of the circuit with varying the coupling coefficients between the transmitter and relay. The resulting plot in Fig. 7 shows the output voltage of the receiver as a function of frequency sweeping.

It is clear from the plot in Fig. 7 that the received powers always have maximum values at the peak frequency instead of original resonant frequency. Using the cursor to probe the receiver output, the peak

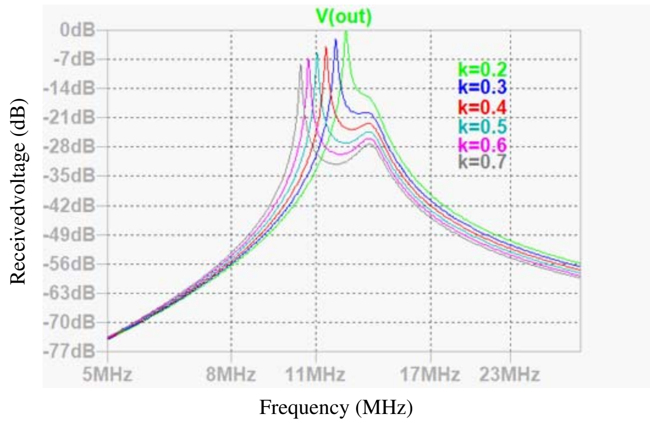


Figure 7. Receiver output at different coupling.

frequency of the circuit can be obtained as 12.3 MHz, 11.87 MHz, 11.43 MHz, 11.06 MHz, 10.69 MHz, and 10.39 MHz at the coupling coefficient of 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7, respectively. Note that this peak frequency can be calculated by using $\omega = \frac{\omega_o}{\sqrt{1+k}}$ which confirms the maximum of the power transfer in Equation (9). It is also observed that the higher the coupling coefficient k is, the lower the received power drops. This is also in agreement with the explanations in Equation (9), Section 2.

3.2. Experience with Tx-2 Rl and Rx

The models of transmitter-double relays and a receiver with and without cross coupling (referring to case 1 and case 3 in item 2 Section 2) are built in Fig. 8. All the coupling coefficients are simulated as in the model of the Tx-single relay in which couplings between transmitter and two relays are equal $k3 = k4 = v$ and cross coupling between relays $k34 = 0$ or $k34 = v$.

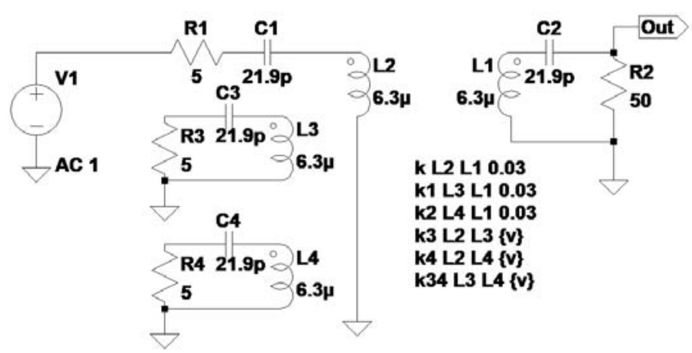
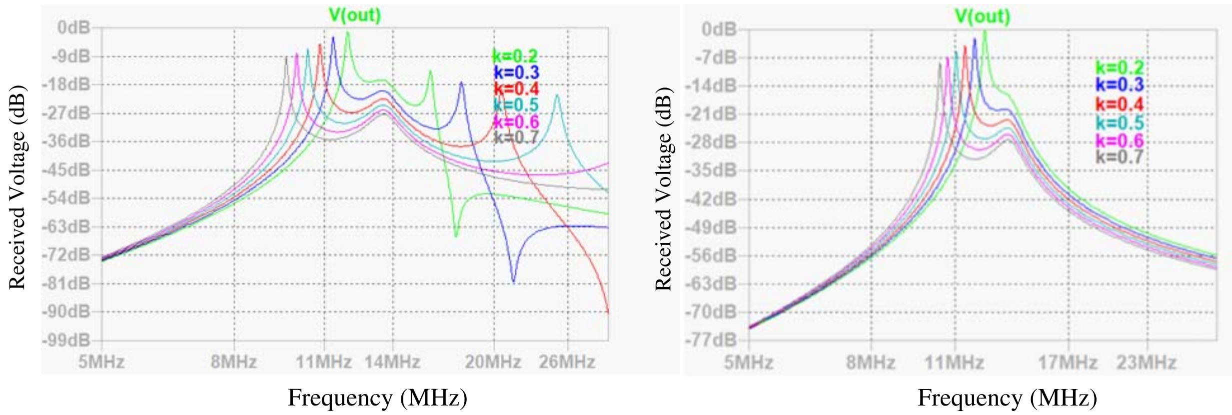


Figure 8. Model of Tx-double relays and Rx.

A similar frequency response showing the received power of the transmitter-double relays and receiver versus frequency is illustrated in Figs. 9(a) and (b). It is seen that the power at the receiver loads still contains a maximum value at splitting frequencies regarding the coupling coefficients between transmitter and relays.

It is noted that the peak frequencies in Figs. 9(a) and (b) can be calculated by using $\omega = \frac{\omega_o}{\sqrt{1+k\sqrt{2}}}$ in Equation (15) and $\omega = \frac{\omega_o}{\sqrt{1+2k}}$ in Equation (17). In detail, at the coupling coefficients of 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7, the frequencies in Fig. 9(a) are equal to 11.9 MHz, 11.32 MHz, 10.8 MHz, 10.4 MHz, 9.95 MHz, and 9.6 MHz, respectively, and the frequencies in Fig. 9(b) are equal to 11.43 MHz, 10.7 MHz, 10 MHz, 9.6 MHz, 9.1 MHz, and 8.7 MHz, respectively.



(a) (b)

Figure 9. Receiver output at different coupling (a) without cross coupling and (b) with cross coupling between transmitter and relays.

3.3. Efficiency of the Receiver

In this section, we use the values of the circuit elements to calculate the efficiency of the single and double relays in Equations (9), (15), (16), and (17). Table 1 shows the comparison of the efficiency of the system regarding the change of the coupling coefficient in Equations (9), (15), (16), and (17). It is observed that the efficiency increases significantly when the double relays are used.

Table 1. Efficiency in single and double internal relays.

k	0.2	0.3	0.4	0.5	0.6	0.7
Efficiency (Equation (9))	0.6335	0.6147	0.5971	0.5803	0.5646	0.5496
Efficiency (Equation (15))	0.9908	0.9954	0.9972	0.9980	0.9988	0.9988
Efficiency (Equation (16))	0.9991	0.9995	0.9997	0.9998	0.9999	0.9999
Efficiency (Equation (17))	0.9990	0.9995	0.9997	0.9998	0.9998	0.9999

4. CONCLUSION

We have proposed and demonstrated innovational multiple inductive relay configurations using single and double relays at the transmitter side in the magnetic inductive communication as a means of increasing the power delivery efficiencies. Increased power efficiencies more than that can be achieved beyond the conventional use of the single resonant frequency systems are demonstrated. The improvement results from the use of splitting frequency and mutual coupling between the relaying coils. Even when cross coupling exists between the relays, power transfer efficiency is still possible up to 50%. This can be used for frequency diversity transmissions as well.

REFERENCES

1. Kurs, A., A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljacic, "Wireless power transfer via strongly coupled magnetic resonances," *Science*, Vol. 317, No. 5834, 83–86, 2007.
2. Cannon, B. L., J. F. Hoburg, D. D. Stancil, and S. C. Goldstein, "Magnetic resonant coupling as a potential means for wireless power transfer to multiple small receivers," *IEEE Transactions on Power Electronics*, Vol. 24, No. 7, 1819–1825, July 2009.

3. Niu, W., J. Chu, W. Gu, and A. Shen, "Exact analysis of frequency splitting phenomena of contactless power transfer systems," *IEEE Transactions on Circuits and Systems I: Regular Papers*, Vol. 60, No. 6, 1670–1677, June 2013.
4. Niu, W. Q., W. Gu, J. X. Chu, and A. D. Shen, "Coupled-mode analysis of frequency splitting phenomena in cpt systems," *Electronics Letters*, Vol. 48, No. 12, 723 –724, June 2012.
5. Nguyen, H. and J. I. Agbinya, "Splitting frequency diversity in wireless power transmission," *IEEE Transactions on Power Electronics*, Vol. 30, No. 11, 6088–6096, November 2015.
6. Agbinya, J. I., "Recursive frequency allocation scheme in wireless power transfer and magnetic induction communication systems," *Wireless Personal Communications*, Vol. 98, No. 1, 213–223, January 2018.
7. Agbinya, J. I. and H. Nguyen, "Principles and applications of frequency splitting in inductive communications and wireless power transfer systems," *Wireless Personal Communications*, April 2019.
8. Kurs, A., R. J. Moffatt, and M. Soljacic, "Simultaneous mid-range power transfer to multiple devices," *Applied Physics Letters*, Vol. 96, 044102, 2010.
9. Ishizaki, M. and A. Kurokawa, "Power transfer system combining wireless resonators and wired three-coil repeater," *2019 IEEE International Symposium on Circuits and Systems (ISCAS)*, 1–4, May 2019.
10. Machnoor, M., E. S. G. Rodríguez, P. Kosta, J. Stang, and G. Lazzi, "Analysis and design of a 3-coil wireless power transmission system for biomedical applications," *IEEE Transactions on Antennas and Propagation*, 1–1, 2018.
11. Duong, T. P. and J. Lee, "Experimental results of high-efficiency resonant coupling wireless power transfer using a variable coupling method," *IEEE Microwave and Wireless Components Letters*, Vol. 21, No. 8, 442–444, August 2011.
12. Sample, A. P., B. H. Waters, S. T. Wisdom, and J. R. Smith, "Enabling seamless wireless power delivery in dynamic environments," *Proceedings of the IEEE*, Vol. 101, No. 6, 1343–1358, June 2013.
13. Lee, C. K., W. X. Zhong, and S. Y. R. Hui, "Effects of magnetic coupling of nonadjacent resonators on wireless power domino-resonator systems," *IEEE Transactions on Power Electronics*, Vol. 27, No. 4, 1905–1916, April 2012.
14. Dionigi, M. and M. Mongiardo, "Multi band resonators for wireless power transfer and near field magnetic communications," *2012 IEEE MTT-S International Microwave Workshop Series on Innovative Wireless Power Transmission: Technologies, Systems, and Applications*, 61–64, May 2012.
15. Ahn, D. and S. Hong, "Effect of coupling between multiple transmitters or multiple receivers on wireless power transfer," *IEEE Transactions on Industrial Electronics*, Vol. 60, No. 7, 2602–2613, July 2013.
16. Ahn, D. and S. Hong, "A study on magnetic field repeater in wireless power transfer," *IEEE Transactions on Industrial Electronics*, Vol. 60, No. 1, 360–371, January 2013.
17. Ahn, D. and S. Hong, "A transmitter or a receiver consisting of two strongly coupled resonators for enhanced resonant coupling in wireless power transfer," *IEEE Transactions on Industrial Electronics*, Vol. 61, No. 3, 1193–1203, March 2014.
18. Nguyen, H., J. I. Agbinya, and J. Devlin, "Fpga-based implementation of multiple modes in near field inductive communication using frequency splitting and mimo configuration," *IEEE Transactions on Circuits and Systems I: Regular Papers*, Vol. 62, No. 1, 302–310, January 2015.
19. Ahn, D., M. Kiani, and M. Ghovanloo, "Enhanced wireless power transmission using strong paramagnetic response," *IEEE Transactions on Magnetics*, Vol. 50, No. 3, 96–103, March 2014.