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Accurate Vortex Beam Mode Measurement Based on Rotational Antenna Method

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Abstract—For the application of electro-magnetic (EM) wave with orbital angular momentum (OAM), which is also called the vortex beam, it is essential to determine the real OAM mode of the transmit antenna, i.e., accurately measure the OAM mode of the manufactured antenna with systematic error. In this paper, an accurate OAM measurement of EM wave based on rotational antenna is proposed. Specifically, the EM beam with helical phase fronts can be well measured via frequency shift detection by rotating the OAM wave at the transmitter. At the same rotation speed, different OAM beams will produce significantly different frequency shifts. The simulation results show that the rotational detection method allows the Root-Mean-Square Error (RMSE) to drop rapidly with no error floor. In addition, this method will not be affected by the different installation environments in practical applications, and has higher detection accuracy and robustness.

1. INTRODUCTION

During the propagation of electromagnetic (EM) waves, Orbital Angular Momentum (OAM) brings paraxial beams with helical phase fronts. Laser light with a Laguerre-Gaussian amplitude distribution was discovered to have a well-defined orbital angular momentum by Allen in 1992 [1]. This discovery has led to a wide range of research of OAM in optic band besides fiber transmission, such as quantum state manipulation [2] and micromanipulation [3]. Until now, researches on OAM wave characteristics are still emerging.

In 2016, the kinetic theory of Landau damping of ion acoustic twisted modes is developed in the presence of OAM of the helical electric field in plasmas with Kappa distributed electrons and Maxwellian ions [4]. In 2017, the authors further studied the kinetic description of the instability of electrostatic dust twisted modes in non-thermal dusty plasmas [5]. Furthermore, in non-thermal dusty plasma, quasi electrostatic modes are investigated using non-gyrotropic Kappa distribution in the presence of helical electric field, which has greatly promoted the research of OAM waves in various fields [6].

In recent decade, the low frequency EM wave with OAM invokes a great interest, e.g., radio wave, microwave, and millimeter wave. After the first radio OAM evaluation performed by Thidé in 2007 [7], a considerable number of contributions have been delivered in this field. Up to now, there are three kinds of commonly-used generators to produce EM waves with OAM based on antenna, i.e., using the Holographic Plate (HP) [8], Spiral Phase Plate (SPP) [9], or antenna array [10].

HP mask is a common way to generate OAM waves. It is simulated with a central dislocation as a phase grating in horizontally separated patterns. After the input signal passes through the disturbance in the middle of the horizontally/vertically separated HP mask, diffraction is formed to the input signal, thus generating an OAM state [8]. SPP is another to generate OAM beams through changing the phase of an incoming radio signal around a 2π circle. It is an alternative way of generating an

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OAM. In theory, an infinite amount of OAM is possible. The power of the incoming radio signal will not be reduced utilizing the SPP [9]. Generally speaking, regardless of HP or SPP, each plate can only generate OAM beam of a specific mode. Comparied to these kinds of methods, the antenna array can generate different multiplexing OAM beams through an antenna array aligned in a circle. The phases of all antenna elements change sequentially. Obviously, the implementation of the antenna array is more costly than HP and SPP [10]. However, all these traditional schemes always have some errors due to the imperfections in the manufacturing process. These errors limit the ability to correctly generate OAM waves. Thus, we have to check the accuracy of the OAM generator and make sure that it meets the required standard of precision, especially for the cases of HP and SPP.

The angular momentum can, in a beam geometry, be decomposed into a polarization-dependent intrinsic rotation and an extrinsic rotation [10]. The extrinsic rotation is referred to as OAM with mode numbers denoted by l. In order to measure the accuracy of OAM generator, the OAM mode parameters l should be checked. Mohammadi et al. proposed two methods in 2010 [11]. One is Single-Point Detection (SPD) method, which estimates the magnetic field from the electric field based on the OAM far-field approximation. The other method is Phase Gradient (PG) method, which utilizes the helical phase structure and estimates the OAM mode by measuring the two-point phase gradient in the transverse plane vertical to the propagation direction. However, the SPD method is only suitable for collimated beams. The approximation formula does not work sufficiently well because of the approximate treatment of the EM field measurements. Moreover, the PG method is easily affected by many factors, e.g., the spatial fabricated deviation between two antennas and the antenna coupling issues. These issues degrade the accuracy of the measurement of the OAM generator in EM wave.

In this paper, an accurate OAM measurement scheme based on a rotational antenna is proposed. Generally speaking, the traditional methods measure the OAM mode based on the OAM far-field approximation or the phase gradient in the transverse plane. Instead, according to the idea of transform domain, the detection method of the rotating antenna essentially transforms the spatial detection of OAM modes into the frequency domain. Through a long-term cumulative observation, the OAM modes detection can be realized more accurately. Section 2 will illustrate the detection method and device, while Section 3 shows the measurement error analysis, and conclusion is drawn in Section 4.

2. DETECTION METHOD AND DEVICE

Briefly speaking, the detection method is to rotate the OAM radio wave at the transmitter and receive it with a single antenna in a fixed location. With this method, the OAM modes can be well detected in the frequency domain. The rotation of the OAM wave in our scheme does not change the polarization, i.e., only the OAM parameters l can be detected in the receiving side. With the rotational antenna method, a different OAM mode brings a different frequency shift in the receiving end, which finally maps different OAMs into different frequency shifts and obtains the orthogonal reception in the time domain [12].

With the rotation of the OAM generator, the rotating helical phase front yields the frequency shift at the receiving end, which can be considered as an angular Doppler effect. Note that in optical OAM transmission, the mathematical formulas has been deduced from Schrodinger equations [13]. However, to the best of our knowledge, the similar OAM modes detection method in millimeter waves has not been reported.

Mathematically, at the same rotating speed, the EM waves with different OAM modes will bring different frequency shifts. Moreover, the different frequency shifts caused by different OAMs with integer mode (l) are expected to be mutually orthogonal. If the OAM generator does not rotate, the received signal at the receiving end can be expressed as $A\cos(2\pi f_0 t + \phi_0)$, where A is the amplitude of the received signal, f_0 the carrier frequency, and ϕ_0 the initial phase. If the OAM generator rotates at a speed of Ω (r/s), the phase of the received signal changes $2\pi l \times \Omega$ in each circle of rotation. Then the received signal can be denoted as,

$$A\cos(2\pi f_0 t + 2\pi l \times \Omega t + \phi_0) = A\cos[2\pi(f_0 + l \times \Omega)t + \phi_0] \tag{1}$$

Hence, the rotational speed Ω of the electric motor, the OAM mode number l, and the resulting signal frequency shift f_s have the following relationship, which has been experimentally verified in [12](Page

4).

$$f_s = l \times \Omega \tag{2}$$

Therefore, if the rotation direction of SPPs is the same as the phase increment direction of OAM wavefront, the frequency offset is positive; otherwise, the frequency offset is negative.

The conceptual structure of the EM wave OAM mode measurement equipment with the rotational antenna detection method is shown in Fig. 1. Meanwhile, the antenna is used to generate OAM waves by deploying the SPP or HP in front of it. The signal generator generates a plane wave through the horn antenna. After passing through the SPP or HP, the plane wave becomes OAM wave with helical phase front. Per rotation manipulation which is achieved by rotating the SPP or the HP with electric motor, the wavefront of OAM wave is rotated, and thereby the rotational OAM wave is obtained. At the receiving side, only a portion of the energy ring is received which is sufficient to detect the OAM, i.e., a common antenna such as a dipole antenna or a waveguide antenna can be used to receive the signal and feed it to a spectrum analyser.

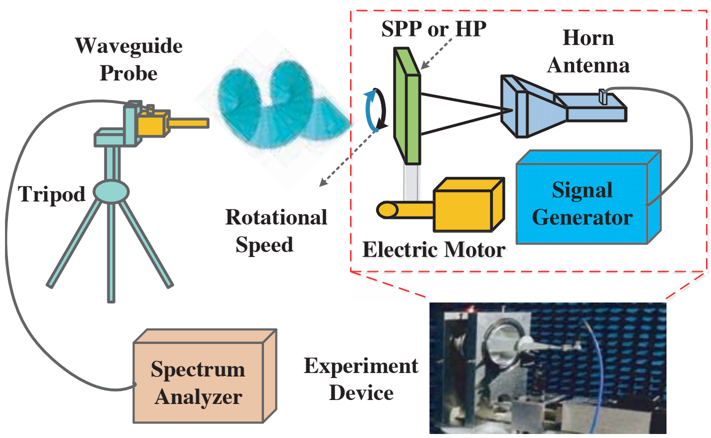

Figure 1. The OAM mode measurements experiment: principle and prototype.

According to Eq. (2), the following facts are clear: 1) frequency shifts generated by the two OAM waves with an opposite mode are equal and opposite in direction with the same rotational speed; 2) with the increase of the OAM mode, the frequency shift will increase with equal proportion; 3) for a specific OAM wave, the frequency shift increases with the rotational speed of the SPP; 4) the frequency shift is caused by the rotational OAM wave rather than changes in the frequency of the transmitted EM wave, i.e., this frequency shift is similar to the Doppler effect of the radio wave in the linear movement; 5) with the same rotation speed, these frequency shifts caused by different OAM modes will be orthogonal to each other. In order to estimate the detection error of the OAM modes, the measurement error equation can be given as

$$\Delta l = \begin{bmatrix} \frac{1}{\Omega} & -\frac{f_s}{\Omega^2} \end{bmatrix} \begin{bmatrix} \Delta f_s \\ \Delta\Omega \end{bmatrix} \tag{3}$$

where Δf_s and $\Delta\Omega$ are the observation errors, which lead to the estimation error Δl of OAM modes.

3. EVALUATION RESULTS OF THE DETECTION METHOD

In order to evaluate the correctness and effectiveness of the rotational OAM (ROAM) detection method, we established the prototype measurement system in a microwave anechoic chamber, which is also shown in Fig. 1. Without loss of generality, in the experiment, the SPPs are employed to generate EM waves with different OAM modes. The SPPs are defined by their thickness, which varies azimuthally and results in the maximum thickness difference of

$$\Delta h = l\lambda/(n-1) \tag{4}$$

where n is the refractive index of the plate material; parameter l denotes the OAM mode; and λ is the wavelength of the EM wave [14].

The ROAM EM waves with different OAM modes are generated through the combination of a horn antenna and SPPs. The electric motor is used to rotate the SPP ahead of the horn antenna, which is controlled by a variable-frequency driver for adjusting the rotational speed. In the experiment, the signal generator is connected to a horn antenna with a centre frequency of 35 GHz and an antenna gain of 25 dB. The rectangular waveguide probe is installed on the scanning frame to receive the rotational OAM wave and feed it to the spectrum analyser for frequency observation. The structures of the SPPs are the same in [12]. The measurement location is deployed on the transverse plane vertical to the propagation axis, 80λ away from the SPP which is a typical distance illustrated in [11]. The major parameters are listed in Table 1.

Table 1. Simulation parameters.

Center frequency	OAM modes	Rotational speed	Antenna gain
35 GHz	$l = \pm 1, 2, 4$	$\Omega = 7.2\,\mathrm{r/s}$	25 dB

The carrier frequency is chosen as 35 GHz because this is a typical millimeter wave frequency band commonly used in the industry. The SPPs are fabricated with high density polyethylene (HDPE) through 3D printing. Different modes of SPPs can produce different frequency shifts when SPPs are rotated. Theoretically, the faster the rotation speed is, the greater the frequency shift is, and 7.2 r/s is a relatively stable speed of the motor in our experiment. Within the range allowed by the receiving end, we can choose an antenna with any gain in our experiment.

According to the experiment results, different OAM modes can generate different frequency shifts under the same rotation speed. The correctness has also been verified in the previous work in [12]. In order to verify the effectiveness of the ROAM detection method, in this paper, we compare the experiment data with the simulation data of the SPD method and PG method. The evaluation results are shown in Fig. 2 and Fig. 3. Fig. 2 evaluates the Root-Mean-Square Error (RMSE) of the OAM

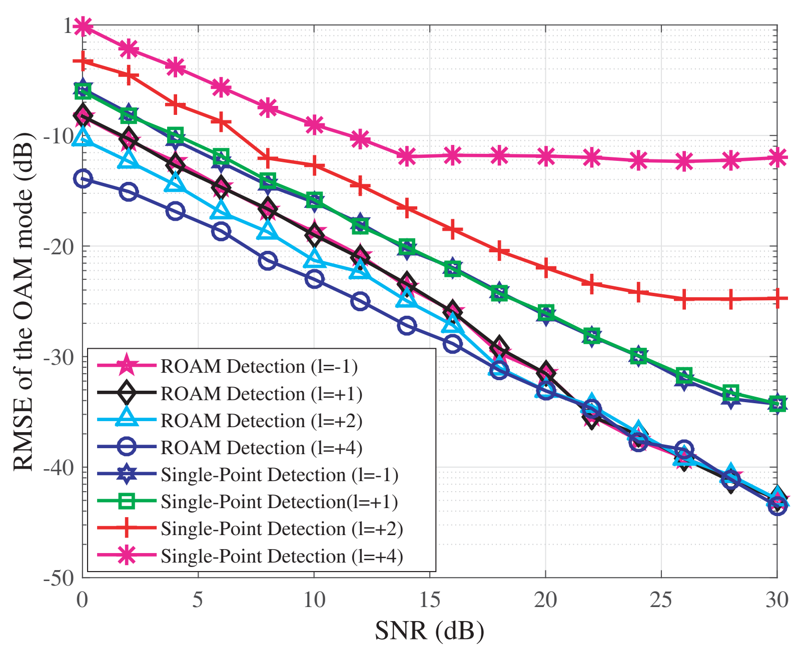

Figure 2. RMSE of the OAM modes versus different SNR.

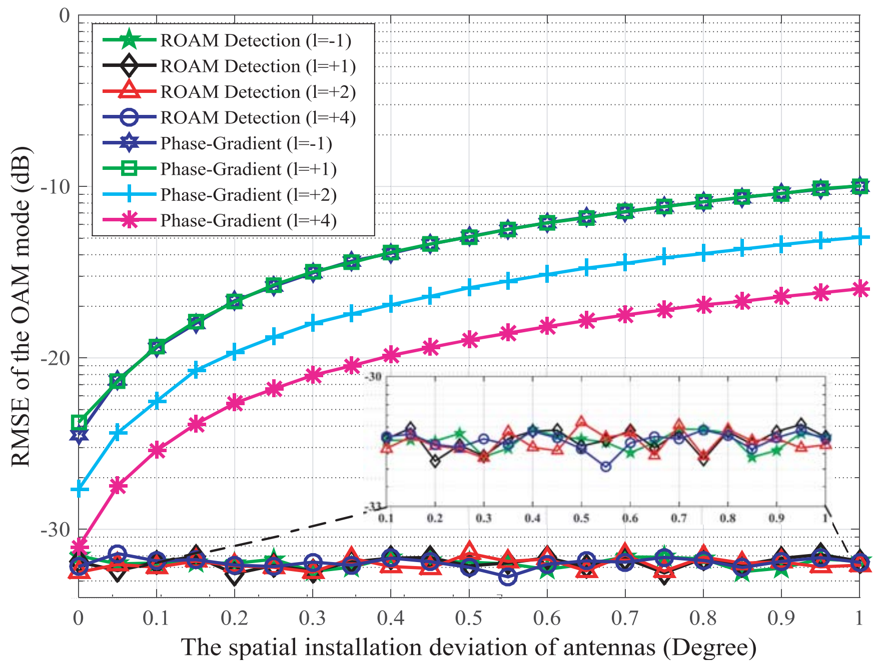

Figure 3. RMSE of the OAM modes versus different spatial fabricated deviation of antennas (SNR $= 30\,\text{dB}$).

mode numbers versus SNR for the proposed rotational antenna detection method and SPD method. The ROAM detection method always outperforms the SPD method which has some error floor even in high SNR. The reason can be analyzed as follows.

The SPD method only works better for collimated beams. However, for a certain antenna, the beam angle increases with higher OAM mode. Evidently, the approximated formula in SPD method fails to estimate the precise OAM mode unless the beam is highly collimated. As shown in Fig. 2, as the OAM modes increase, the beam angle becomes larger, and the efficiency of the approximate formula becomes worse. Even in the case of high SNR, due to the residual error in the approximation, the SPD method still cannot precisely estimate the right OAM modes with an error floor. This effect becomes outstanding with the increase of the OAM modes. In contrast, the proposed ROAM detection method has no such an effect. With the increase of SNR, the measurement RMSE decreases rapidly.

Figure 3 shows the RMSE evaluation results between the proposed rotational antenna detection method and the PG method. The RMSE of rotational antenna detection is less than that of the PG method. For the PG method, the OAM wave is detected by two separated antennas spanned on the energy circle vertical to the propagation axis. Thus, the phase difference between the two received signals can be recorded and used to calculate the OAM mode. However, the spatial installation deviation, which refers to the difference of the angle between the connection of the two antennas and main propagation axis, can greatly affect the RMSE of the measurement results. In other words, if the two antennas are not installed in the same circle sector of a transverse plane at a vertical distance, it will lead to the severe measurement error in the phase gradient. Moreover, to the small OAM mode, the phase gradient between the two antennas may not be obvious for PG detection, which will also affect the measurement accuracy. Fortunately, the rotational antenna detection method only needs to measure the phase change of a point in space, i.e., frequency, so there will be no such problems revealed in PG method.

In order to improve the estimation accuracy, the estimation error Δl can be reduced by the method of averaging multiple observations. Checking the product manual of the inverter (VD300A high-performance general-purpose vector inverter), the rotation speed error $\Delta\Omega$ is less than 0.5%. The measurement results of the SPPs are listed in Table 2. It can be seen that the average frequency shift measurement error is about 1 Hz, which can also be verified by the product manual of the spectrum analyser (Agilent E4446A).

Table 2. Frequency measurement data of the SPPs.

rotation speed	0 r/s	2.4 r/s	4.8 r/s	7.2 r/s
modes	frequency shifts measurement data (Hz)			
$l = 0$	35000002441 ± 0.9	35000002441 ± 0.8	35000002441 ± 0.9	35000002441 ± 1.3
$l = +1$	35000002441 ± 1.0	35000002438 ± 1	35000002435 ± 1.1	35000002433 ± 1.1
$l = -1$	35000002441 ± 1.1	35000002444 ± 0.9	35000002447 ± 1.3	35000002449 ± 1.2
$l = +2$	35000002441 ± 0.9	35000002446 ± 0.7	35000002451 ± 0.5	35000002456 ± 1.4

According to Eq. (3), the OAM mode measurement error is estimated under different rotation speeds and observation times N in Fig. 4. It can be seen that the OAM mode measurement error decreases with the increase of the observation times and the rotation speed. Hence, the measurement accuracy of the ROAM method can be further improved by increasing the rotation speed and the number of observation times.

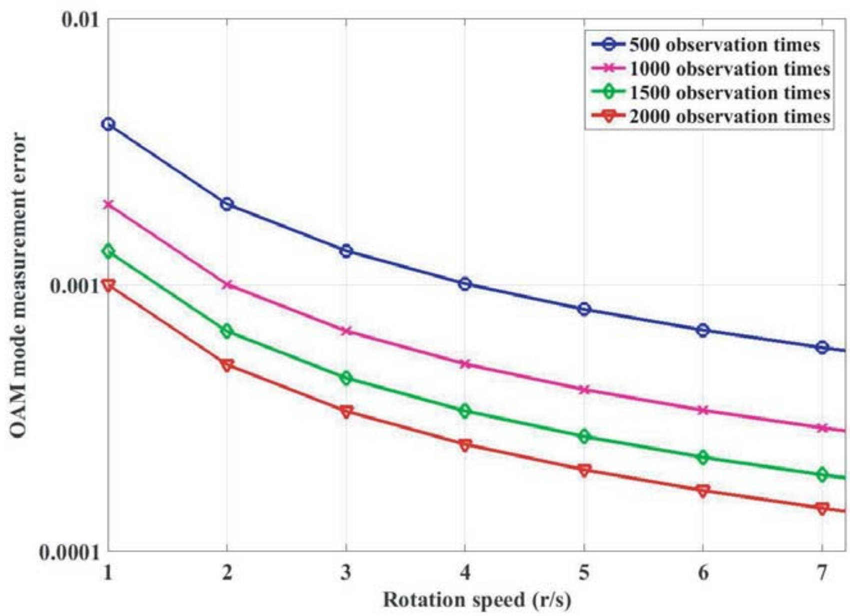

Figure 4. OAM modes measurement error versus different rotation speeds and observation times.

Last but not least, the machining precision and mechanical vibrations are less than 0.1 mm. We established the simulation with the corresponding errors added on the coordinates of the transmitting antenna and found that these errors had almost no effect on the measured phases and frequency shifts, which is shown in the supplementary materials of [12]. Moreover, the proposed method is realized by rotating an SPP. However, the method can also be implemented for HP or integrated antenna array.

4. CONCLUSION

In this paper, we propose an accurate measurement method for OAM modes detection of EM wave based on rotational antenna method. Compared with the traditional measurement schemes, our detection method is more practical. OAM modes can be accurately detected by measuring the frequency shift at the receiving end with a single commonly-used antenna, which is promisingly used to determine the machining accuracy of different OAM generators. Specifically, our detection method measures the OAM mode number in the frequency domain, essentially using the statistical average results after multiple measurements in the time domain. Moreover, because it only needs to receive and detect part of the helical phase front, our detection method will not be affected by the placement of the transceiver. Hence, in our prototype experiment, the measurement accuracy achieves results much better than SPD and PG methods

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