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Nonuniform Electromagnetic Field at the Interface between Dielectric and Conducting Media

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Abstract—The study of the electromagnetic field, taking into account eddy currents in the conductive half-space, is based on the exact analytical solution of the general three-dimensional quasi-stationary problem. The mathematical model includes an approximate solution using asymptotic expansion in the case of strong skin effect. Analytical expressions are obtained for the electric and magnetic fields at a flat interface in the form of limited asymptotic series, each term of which is expressed through a known field of external sources. The expressions take into account the nonuniformity of the field near the surface, since they contain its derivatives with respect to the coordinate. The series expansion was carried out according to a small parameter, which is proportional to the ratio of the field penetration depth to the distance between the interface and the sources of the external field. The found expressions generalize the approximate boundary impedance condition for the case of the penetration of nonuniform electromagnetic field into conductive medium.

1. INTRODUCTION

At high frequencies and fast pulse processes, a strong skin effect occurs in conductive bodies, when the current and electromagnetic field are concentrated in a thin surface layer. Despite the considerable amount of research devoted to the study of the strong skin effect, the practical needs for the development of devices with specific field distribution conditions still cause interest in this problem [1–4].

In the case of strong skin effect, the field distribution on the interface is important due to the possibility of a justified construction of mathematical models. The electromagnetic field on this surface defines such characteristics as the energy flow of the electromagnetic field into the conductive body, the surface density of Joule heat release in the surface layer, and the magnetic pressure on the surface of the body.

More than 70 years ago, Leontovich formulated an approximate impedance boundary condition, which is valid with a strong skin effect [5]. This condition corresponds to the assumption that locally an electromagnetic field penetrates into a metal body in the same way as a uniform field penetrates a conductive half-space. On the surface, only the tangent components of the electric and magnetic field intensities are non-zero [6]. Deviation from the obtained results may be due to various factors: nonuniformity of penetrating electromagnetic field, non-planar surface of the conducting body, limited body size, and the presence of inhomogeneities in metal body, identification of which is also a research problem [7, 8].

In [9, 10], the theory of surface impedance was constructed for surface of bodies with a double curvature. A detailed review of works using the concept of surface impedance in electrodynamics problems, where the geometric and physical properties of real boundary surfaces are taken into account, is given in [11].

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In most of the cited papers, the problem of diffusion of a nonuniform electromagnetic field into a conducting medium was formulated with significant restrictions on the depth of field penetration. The exact solution of the problem for nonuniform field is presented in [6] for a special case of a field excited by a thin rectilinear conductor with current oriented parallel to the interface of the media. For the considered nonuniform field, the constraints are shown under which the impedance boundary condition is valid.

A feature of this study is that it is based on the exact analytical solution obtained for the general problem of a three-dimensional quasi-stationary field [12, 13]. In particular, the main property of the formation of electromagnetic field with a flat interface has been established. It consists in the fact that in a conducting medium the current density and electric field intensity do not contain components perpendicular to the boundary surface for any system of initial currents and arbitrary dependence of currents on time. The exact solution allows to also get other general justified results.

The aim of this work is to study the formation of electromagnetic field taking into account its nonuniformity near flat interface between dielectric and conducting media, obtain explicit expressions for the electric and magnetic field intensities, and generalize the impedance boundary condition to the case of diffusion of nonuniform field.

2. MATHEMATICAL MODEL

2.1. Analytical Solution of the General Three-Dimensional Problem

An analytical solution to the linear problem of determining the electromagnetic field generated in a system containing an arbitrary current contour and a conducting body with a flat surface modeled by a conducting half-space was obtained in [12, 13]. The solution was obtained without restrictions on the geometry and orientation of the contour, the electrophysical properties of the medium (electrical conductivity γ_i and relative magnetic permeability μ_i), and the frequency of the field ω. Expressions for complex-value vector and scalar potentials, electric and magnetic field intensities in both areas are presented: in the dielectric half-space $\dot{\mathbf{A}}_e$, $\dot{\varphi}_e$, $\dot{\mathbf{E}}_e$, $\dot{\mathbf{H}}_e$, where the closed contour l with current $\dot{I}_0$ is located, and in the conducting half-space $\dot{\mathbf{A}}_i$, $\dot{\varphi}_i$, $\dot{\mathbf{E}}_i$, $\dot{\mathbf{H}}_i$, where the eddy current with density $\mathbf{j}_i = \gamma_i \dot{\mathbf{E}}_i$ flows. (As it is often used, the complex-value amplitudes are marked with a dot over the corresponding symbols). The problem easily extends to the general case of an arbitrary system of contours, that is, an arbitrary external field and to an arbitrary relation of current as a function of time $I_0(t)$ using a Fourier transform over time.

Expressions for the complex-value amplitudes of potentials in Lorentz's calibration and the vectors of field at an arbitrary point of the dielectric half-space are presented as contour integrals

$$\dot{\mathbf{A}}_e = \frac{\mu_0 \dot{I}_0}{4\pi} \oint_l \left(\frac{\mathbf{t}}{r} - \frac{\mathbf{t}_1}{r_1} - \mathbf{t}_1 \frac{\partial \dot{G}_e}{\partial z} \right) dl, \tag{1}$$

$$\dot{\varphi}_e = i\omega \frac{\mu_0 \dot{I}_0}{4\pi} \oint_l (\mathbf{t}_1 \cdot \mathbf{e}_z)\, \dot{G}_e dl, \tag{2}$$

$$\dot{\mathbf{E}}_e = -i\omega \frac{\mu_0 \dot{I}_0}{4\pi} \oint_l \left(\frac{\mathbf{t}}{r} - \frac{\mathbf{t}_1}{r_1} - \mathbf{e}_z \times \left[\mathbf{t}_1 \times \nabla \dot{G}_e \right] \right) dl, \tag{3}$$

$$\dot{\mathbf{H}}_e = -\frac{\dot{I}_0}{4\pi} \oint_l \left[\frac{\mathbf{t} \times \mathbf{r}}{r^3} - \frac{\mathbf{t}_1 \times \mathbf{r}_1}{r_1^3} - \mathbf{t}_1 \times \nabla \left(\frac{\partial \dot{G}_e}{\partial z} \right) \right] dl. \tag{4}$$

Potentials and field intensities are determined by single function $\dot{G}_e$

$$\dot{G}_e = \frac{2}{\sqrt{i}} \int_0^\infty \frac{\exp\left(-\chi \cos \beta_1 / \varepsilon_1\right) J_0\left(\chi \sin \beta_1 / \varepsilon_1\right)}{w_1(\chi)} d\chi. \tag{5}$$

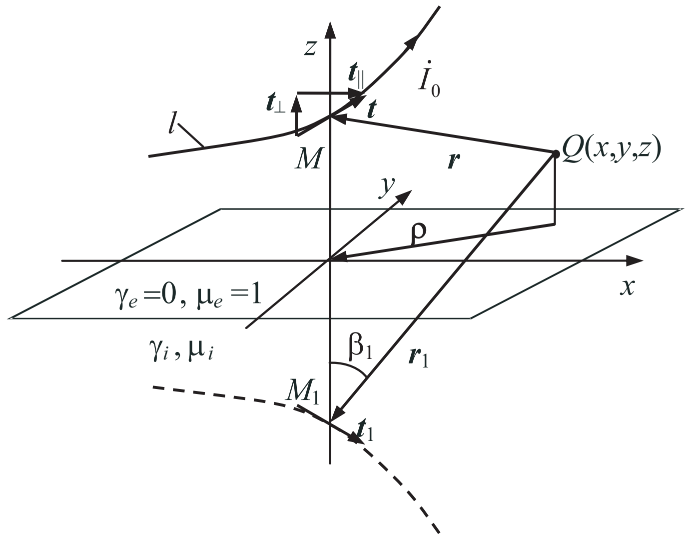

Figure 1. Geometrical parameters for the element of arbitrary contour l with current $\dot{I}_0$ located near conducting half-space, and the mirror reflection of this contour element from the interface between the dielectric and the conducting media.

To explain the geometric quantities included in expressions (1)–(5), in Fig. 1 the solid line shows the element of the initial contour with current I_0 located in the upper half-space $z > 0$. In the lower half-space $z < 0$, the dashed line shows the mirror reflection of this contour element from the flat interface. The vectors $\mathbf{t}$ and $\mathbf{t}_1$ are unit tangent vectors to the initial contour at point M and to the mirror reflected contour at point M_1. Positions of points M and M_1 relative to the observation point Q are determined by vectors $\mathbf{r}$ and $\mathbf{r}_1$, respectively. The vector ρ is the projection of vectors $\mathbf{r}$ and $\mathbf{r}_1$ onto interface surface xy. The angle β_1 shows the orientation of the vector $\mathbf{r}_1$ relative to the vertical axis.

Expressions (1)–(5) contain the following notation: $\varepsilon_1 = \mu_i/\big(r_1\sqrt{\omega\mu_i\mu_0\gamma}\big) = \mu_i\delta/\big(\sqrt{2}r_1\big)$, where δ is the penetration depth of a homogeneous field, $J_0(\cdot)$ the Bessel function of the first kind of zero order, $\mathbf{e}_z$ the unit vector in the z-axis direction, μ_0 the permeability of vacuum, and i the imaginary unit. The dimensionless function $w_1(\chi)$ in the denominator of the integrand is as follows

$$w_1(\chi) = \chi/\sqrt{i} + \sqrt{1 + \left[\chi/(\mu\sqrt{i})\right]^2}. \tag{6}$$

The above expressions allow us to determine an electromagnetic field at an arbitrary observation point at $z \geq 0$.

2.2. Asymptotic Approximation for Strong Skin-Effect

Consider the simplification of analytical expressions (1)–(5) in the case of strong skin effect in its extended formulation, when the penetration depth δ is small not only with respect to the characteristic body sizes, but also to the size of the entire electromagnetic system, including the distance from the surface of body to the sources of the external field. In this case, the entered parameter ε_1 is small for non-magnetic media, and with the additional assumption it can be considered small at $\mu > 1$. Under these conditions, the function $\dot{G}_e$ can be represented by an asymptotic series bounded by a certain number of its terms N [14]

$$\dot{G}_e \approx \sum_{n=0}^{N} \dot{G}_n = \sum_{n=0}^{N} 2(-1)^n a_n(\mu_i)\left(\frac{\varepsilon_1}{\sqrt{i}}\right)^{n+1} r_1^{n+1}\frac{\partial^{(n)}}{\partial z^n}\left(\frac{1}{r_1}\right), \tag{7}$$

where $a_n(\mu_i)$ are the Taylor series coefficients of the function $1/w_1 = \sum_{n=0}^{\infty} a_n(\mu_i)(\chi/\sqrt{i})^n$.

Taking into account Eq. (7), expressions (3) and (4) for the electric and magnetic field intensities are as follows

$$\dot{\mathbf{E}}_e = -\frac{\dot{I}_0}{4\pi}\left[i\omega\mu_0 \oint\limits_l \left(\frac{\mathbf{t}}{r} - \frac{\mathbf{t}_1}{r_1}\right) dl - \varsigma \sum_{n=0}^{N} (-1)^n 2a_n(\mu_i)\left(\frac{\mu_i}{p}\right)^n \frac{\partial^{(n)}}{\partial z^n}\mathbf{e}_z \times \oint\limits_l \frac{\mathbf{t}_1 \times \mathbf{r}_1}{r_1^3} dl\right], \tag{8}$$

$$\dot{\mathbf{H}}_e = -\frac{\dot{I}_0}{4\pi}\left[\oint\limits_l \left(\frac{\mathbf{t}\times\mathbf{r}}{r^3} - \frac{\mathbf{t}_1 \times \mathbf{r}_1}{r_1^3}\right) dl - \sum_{n=0}^{N} (-1)^n 2a_n(\mu_i)\left(\frac{\mu_i}{p}\right)^{n+1} \frac{\partial^{(n+1)}}{\partial z^{n+1}} \oint\limits_l \frac{\mathbf{t}_1 \times \mathbf{r}_1}{r_1^3} dl\right]. \tag{9}$$

Here, $\varepsilon_1 r_1/\sqrt{i} = \mu_i/p$, where $p = \sqrt{i\omega\mu_i\mu_0\gamma_i}$ is the propagation constant, and $\varsigma = p/\gamma_i$ is the surface impedance.

Despite the limitations associated with the presence of a small parameter, the approximate solution in the form of asymptotic series makes it possible to take into account the influence of the nonuniformity of the external electromagnetic field when it interacts with the conducting medium.

3. NONUNIFORM ELECTROMAGNETIC FIELD AT THE INTERFACE

At points on the surface ($z = 0$), expressions (1)–(4) are simplified. An even greater simplification can be obtained in the case of a strong skin effect, when the external field sources are far from the surface of the conducting body, and accordingly, the parameter ε_1 is small. Moreover, the final results allow us to draw conclusions of some general nature.

3.1. Electric and Magnetic Intensities

The distribution of the electromagnetic field (8), (9) on the surface when ε_1 is small is not limited to members of the asymptotic series, for which the law of penetration of uniform field is valid, which is usually considered when constructing mathematical models.

First, consider the electric field intensity. On the interface in the dielectric half-space, the first term in Eq. (8) has only the normal component, which is completely determined by the induced electric field of external sources [10]. On the contrary, the second term has only the tangential component $\dot{\mathbf{E}}_{\|}(z = 0)$.

In turn, the terms of asymptotic series $\dot{\mathbf{E}}_{\|}(z = 0)$ in Eq. (8) contain derivatives of the expression for the magnetic field intensity created by the current contour mirrored from the interface. We transform the expression in such a way as to have the magnetic field intensity of the initial current flowing in dielectric medium. Taking into account the relations between the values for the initial and mirror reflected contours $\mathbf{t} = \mathbf{t}_{\|} + \mathbf{t}_{\perp}$, $\mathbf{t}_1 = \mathbf{t}_{\|} - \mathbf{t}_{\perp}$, $\mathbf{r}(z = 0) = \rho + z_M\mathbf{e}_z$, $\mathbf{r}_1(z = 0) = \rho - z_M\mathbf{e}_z$ we have

$$(-1)^n \left\{\frac{\partial^{(n)}}{\partial z^n}\mathbf{e}_z \times \oint\limits_l \frac{\mathbf{t}_1 \times \mathbf{r}_1}{r_1^3} dl\right\}\Bigg|_{z=0} = -\left\{\frac{\partial^{(n)}}{\partial z^n}\mathbf{e}_z \times \oint\limits_l \frac{\mathbf{t}\times\mathbf{r}}{r^3} dl\right\}\Bigg|_{z=0}. \tag{10}$$

Note that the expression

$$\dot{\mathbf{H}}_0 = -\frac{\dot{I}_0}{4\pi}\oint\limits_l \frac{\mathbf{t}\times\mathbf{r}}{r^3} dl \tag{11}$$

represents the magnetic field intensity of the initial current contour. The tangential component of the field to the surface is $\dot{\mathbf{H}}_{0\|} = -\mathbf{e}_z \times [\mathbf{e}_z \times \dot{\mathbf{H}}_0]$.

As a result, the tangent component of the electric field intensity at the interface can be written by expressing it through the tangent component of the known field of external sources and derivatives of the field with respect to the coordinate z.

$$\dot{\mathbf{E}}_{\|}(z = 0) = \varsigma \sum_{n=0}^{N} 2a_n(\mu_i)\left(\frac{\mu_i}{p}\right)^n \left\{\frac{\partial^{(n)}}{\partial z^n}\mathbf{e}_z \times \dot{\mathbf{H}}_{0\|}\right\}\Bigg|_{z=0}. \tag{12}$$

Based on the principle of superposition, expression (12) will also be valid in the case of an external magnetic field $\dot{\mathbf{H}}_0$ created by a system of sources, with the restriction associated with their distance from the interface.

Similarly, the magnetic field intensity $\dot{\mathbf{H}}_e = \dot{\mathbf{H}}_{e1} + \dot{\mathbf{H}}_{e2}$ (9) at the interface can be represented as a limited sum, including the external magnetic field and its derivatives with respect to the coordinate z.

The first term $\dot{\mathbf{H}}_{e1}$ gives the magnetic field in the dielectric half-space at ideal skin effect, when $\delta \to 0$ and accordingly $\dot{\mathbf{H}}_{e1}$ has a zero value of the normal component of the magnetic field intensity at the interface

$$\dot{\mathbf{H}}_{e1}(z=0) = \frac{\dot{I}_0}{2\pi}\mathbf{e}_z \times \oint\limits_l \frac{\mathbf{e}_z \times (\mathbf{t} \times \mathbf{r})}{r^3} dl = 2\dot{\mathbf{H}}_{0\|}\,(z=0)\,. \tag{13}$$

The second term $\dot{\mathbf{H}}_{e2}$ depends on the electrophysical properties of the medium and field frequency. In contrast to the electric field intensity, the term $\dot{\mathbf{H}}_{e2} = \dot{\mathbf{H}}_{2\|} + \dot{\mathbf{H}}_{2\perp}$ has non-zero both tangential $\dot{\mathbf{H}}_{2\|} = -\mathbf{e}_z \times (\mathbf{e}_z \times \dot{\mathbf{H}}_{e2})$ and normal $\dot{\mathbf{H}}_{2\perp} = (\mathbf{e}_z \cdot \dot{\mathbf{H}}_{e2})\mathbf{e}_z$ components to the interface.

We rewrite $\dot{\mathbf{H}}_{e2}$ in Eq. (9) in such a way that instead of the magnetic field created by the current of the mirror reflected contour, we have the magnetic field intensity of the initial current contour. As before, we take into account the relations between the quantities characterizing the points of the initial and mirror-reflected contours. As a result, the expression for the tangential component of the magnetic field intensity takes the form

$$\dot{\mathbf{H}}_{\|}\,(z=0) = -\sum_{n=0}^{N+1} 2a_{n-1}\,(\mu_i)\left(\frac{\mu_i}{p}\right)^n \left\{\frac{\partial^{(n)}\dot{\mathbf{H}}_{0\|}}{\partial z^n}\right\}\Bigg|_{z=0}. \tag{14}$$

Here, the terms $\dot{\mathbf{H}}_{e1}(z=0)$ in Eq. (13) and $\dot{\mathbf{H}}_{2\|}(z=0)$ are combined. The numbers n of the asymptotic series terms are replaced by $n+1$; accepting $a_{-1} = -1$.

The expression for the normal component of the magnetic field intensity on the surface in the dielectric half-space takes the form

$$\dot{\mathbf{H}}_{e\perp}\,(z=0) = -\sum_{n=0}^{N} 2a_n\,(\mu_i)\left(\frac{\mu_i}{p}\right)^{n+1} \left\{\frac{\partial^{(n+1)}\dot{\mathbf{H}}_{0\perp}}{\partial z^{n+1}}\right\}\Bigg|_{z=0}. \tag{15}$$

Expressions (12), (14), and (15) allow, without solving additional equations, to directly find the electromagnetic field at the interface, having only the known distribution of the field of external sources at the boundary. The presence of derivatives indicates that the electromagnetic field on the surface not only is determined by the value of the field of external sources, but also depends on the nonuniformity of the external field. This feature is related to the dependence of the distribution law of the induced field and currents in the conducting medium on the degree of remoteness of the external field sources compared to the depth of field penetration.

3.2. Impedance Boundary Conditions

Let us now consider the impedance boundary condition, which establishes the relation between the tangential components of the electric and magnetic field intensities at the interface. We will not impose a limiting condition for the uniformity of the field at the boundary, which is usually used in mathematical models.

Let us establish the desired relationship for each term of the asymptotic series of electric $\dot{\mathbf{E}}_{\|}(z=0) = \sum\limits_{n=0}^{N} \dot{\mathbf{E}}_{\|n}$ and magnetic $\dot{\mathbf{H}}_{\|}(z=0) = \sum\limits_{n=0}^{N} \dot{\mathbf{H}}_{\|n}$ fields. Note that the number of the series terms n simultaneously indicates the degree of small parameter $\varepsilon^n = [\mu_i\delta/(\sqrt{2}\lambda)]^n = [\mu_i/(|p|\lambda)]^n$, for estimating the value of which it is expedient to choose the minimum distance λ from external field sources to the interface.

Comparison of the series terms (12) and (14) with the same number n shows that they are interconnected by the following relation

$$a_{n-1}\,(\mu_i)\,\dot{\mathbf{E}}_{\|n} = -a_n\,(\mu_i)\,\varsigma\mathbf{e}_z \times \dot{\mathbf{H}}_{\|n}. \tag{16}$$

The first several expansion coefficients a_n in the power series of the function $1/w_1$ defined in Eq. (6) have the values $a_{-1} = -1,\ a_0 = 1,\ a_1 = -1,\ a_2 = 1 - 1/(2\mu_i^2), \ldots$. Given these values for the first three terms of the series, we obtain

$$\begin{aligned} n &= 0\left(\varepsilon^0\right): \quad \dot{\mathbf{E}}_{\parallel 0} = \varsigma \mathbf{e}_z \times \dot{\mathbf{H}}_{\parallel 0}, \\ n &= 1\left(\varepsilon^1\right): \quad \dot{\mathbf{E}}_{\parallel 1} = \varsigma \mathbf{e}_z \times \dot{\mathbf{H}}_{\parallel 1}, \\ n &= 2\left(\varepsilon^2\right): \quad \dot{\mathbf{E}}_{\parallel 2} = \left[1 - 1/\left(2\mu_i^2\right)\right] \varsigma \mathbf{e}_z \times \dot{\mathbf{H}}_{\parallel 2}, \ldots \end{aligned} \tag{17}$$

It can be seen that the Leontovich approximate impedance boundary condition [9] is valid up to the first two terms of the asymptotic series. Starting from $n = 2(\varepsilon^2)$, the ratio changes. Nevertheless, the relationship between the terms of the series for the tangent components of the field intensities at the interface still exists. In this regard, it can be argued that expression (16) generalizes the impedance boundary condition to wider domain of nonuniformity electromagnetic field diffusing into the conducting body.

In the approximate impedance boundary condition, it is assumed that the normal field component is equal to zero. As can be seen from Eq. (15), the requirement for fulfilling this assumption is more stringent. It holds only for the zero term of the asymptotic series, and it is violated already at ε^1.

4. CONCLUSION

In the case of a strong skin effect, explicit expressions are found for the electric and magnetic fields as functions of the known field of the external sources. Unlike commonly used models, expressions, in addition to the local value of vector fields, contain their derivatives with respect to the coordinate perpendicular to the surface between dielectric and conducting media, thereby determining the effect of field nonuniformity near the surface.

The expressions found in the asymptotic approximation generalize the Leontovich's impedance boundary condition to the case when an inhomogeneous electromagnetic field penetrates into the conducting medium. The approximate ratio between the electric and magnetic field intensities is true up to the terms containing the introduced small parameter in the first degree.

Further theoretical work involves the investigation of the reduction of nonuniform electromagnetic field in conducting body and the effect of field nonuniformity at the interface. The development of the theory is also possible in the direction of a more general description of nonuniform field of the sources at the interface, not limited the sources in the form of current contours. The results obtained allow their practical use for the formulation of boundary value problems taking into account nonuniform field at the interface, which, as shown, can be considered known.

REFERENCES

1. Babutsky, A., A. Chrysanthou, and J. Ioannou, "Influence of pulsed electric current treatment on corrosion of metals," *Strength of Materials*, Vol. 41, No. 4, 387–391, 2009.
2. Batygin, Yu. V., S. F. Golovashchenko, and A. V. Gnatov, "Pulsed electromagnetic attraction of nonmagnetic sheet," *Journal of Materials Processing Technology*, Vol. 214, No. 2, 390–401, 2014.
3. Acero, J., R. Alonso, J. M. Burdio, L. A. Barragan, and D. Puyal, "Analytical equivalent impedance for a planar induction heating system," *IEEE Transaction on Magnetics*, Vol. 42, No. 1, 84–86, 2006.
4. Rudnev, V., D. Loveless, R. Cook, and M. Black, *Handbook of Induction Heating*, Taylor & Francis Ltd, London, 2017.
5. Leontovich, M. A., "On the approximate boundary conditions for electromagnetic field on the surface of highly conducting bodies," *Propagation of Electromagnetic Waves*, USSR Academy of Sciences Publ., Moscow-Leningrad, USSR, 1948 (in Russian).
6. Fridman, B. E., "Skin effect in massive conductors used in pulsed electrical devices: I. Electromagnetic field of massive conductors," *Technical Physics*, Vol. 47, No. 9, 1112–1119, September 2002.

7. Cacciola, M., F. C. Morabito, D. Polimeni, and M. Versaci, "Fuzzy characterization of flawed metallic plates with Eddy current tests," *Progress In Electromagnetics Research*, Vol. 72, 241–252, 2007.
8. Versaci, M., "Fuzzy approach and Eddy currents NDT/NDE devices in industrial applications," *IET Electronics Letters*, Vol. 52, No. 11, 943–945, 2016.
9. Rytov, S. M., "Calculation of skin effect by perturbation method," *Journal of Experimental and Theoretical Physics*, Vol. 10, No. 2, 180–190, 1940 (in Russian).
10. Kravchenko, A. N., *Boundary Characteristics in Electrodynamics Problems*, Naukova Dumka, Kyiv, 1989 (in Russian).
11. Berdnik, S. L., D. Y. Penkin, V. A. Katrich, Yu. M. Penkin, and M. V. Nesterenko, "Using the concept of surface impedance in problems of electrodynamics (75 years later)," *Radio Physics and Radio Astronomy*, Vol. 19, No. 1, 57–80, 2014.
12. Vasetskyi, Yu. M. and K. K. Dziuba, "An analytical calculation method of quasi-stationary three-dimensional electromagnetic field created by the arbitrary current contour that located near conducting body," *Technical Electrodynamics*, No. 5, 7–17, 2017 (in Russian).
13. Vasetsky, Yu. M. and K. K. Dziuba, "Three-dimensional quasi-stationary electromagnetic field generated by arbitrary current contour near conducting body," *Technical Electrodynamics*, No. 1, 3–12, 2018.
14. Vasetsky, Yu., I. Mazurenko, and K. Dziuba, "Conditions for application of asymptotic method to electromagnetic field analysis in system of "a current loop — An electroconducting body"," *Computational Problems of Electrical Engineering*, Vol. 4, No. 1, 91–96, 2014.