

High Quality Factor Microwave Multichannel Filter Based on Multi-Defectives Resonators Inserted in Periodic Star Waveguides Structure

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Abstract—The transmission spectrum of electromagnetic waves through a one-dimensional photonic star waveguides (SWGs) structure is obtained using the Green function method (GFM). The proposed structure is composed of a finite number of periodic cells; each cell contains a segment of length d_1 grafted in its extremity by resonators of length d_2 . This periodic system is altered by insertion of defects resonators located in three different sites which have lengths d_{02} , d_{04} , and d_{06} different from that of the perfect resonators. The properties of such a multi-defects structure are suitable for tunable very narrow band multichannel filter applications in microwave domain. The perfect photonic structure demonstrates large microwave photonic band gaps (PBGs) in which the propagation of electromagnetic waves is prohibited. For the sake of comparison, we first introduce single defect resonators located in one site; this defect creates hence two transmission peaks (defect modes) in the gaps. These induced modes of noticeable transmissions and good quality factors permit our proposed SWGs to behave like a single narrow band channel filter. Several localized modes appear in the band gaps when we introduce three resonators defects of different lengths and located in three different sites. With an appropriate choice of the geometrical parameters $d_1 = 1$ m, $d_2 = 0.5$ m, $d_{02} = d_{04} = d_{06} = 2$ m, our thrice defectives system can create, in the microwave frequencies range, till eight transmission peaks in the gaps when the defects are located in three consecutive sites. These peaks have very high values of transmission rates and important quality factors reaching $Q = 610000$. Therefore, our proposed system acts as very narrow band multichannel filters for which defects modes shift towards the low frequencies when the defects resonators lengths increase. The results obtained demonstrate the dependence of the defect modes frequencies, transmission coefficient, and the quality factor on the resonators defects lengths, their numbers, and their positions inside the defective structure.

1. INTRODUCTION

Filter design based on the electromagnetic waveguides structure is of great interest to the optical, microwave, and photonic communities. Recently, photonic crystal and electromagnetic waveguides filters with the tunable feature have attracted much attention because they are of more practical use in the optical and microwave signal processing [1]. In the last few years, the filters are much desired for multiplicity applications, specifically in periodic waveguides structure [2]. The introduction of defects into the periodic structure generates localized narrow bands, which can be utilized for the construction of filters with an exceptionally narrow transmission band [3]. In most studies, filters are focused on the photonic band gaps (PBGs) manipulation, i.e., all defects modes are designed to set within PBGs. Newly, a new class of filters passband based on a perfect periodic waveguides structure has also aroused

Received 2 May 2020, Accepted 28 July 2020, Scheduled 20 August 2020

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researcher's considerable attention. In comparison to conventional filters using the structure modes of very low quality factor, current filters based on defects modes process show a huge quality factor [4]. In previous years, researchers started to have interest in the filters based on the different types of new artificial materials called "metamaterials", due to the unusual physical properties, for instance, a negative or pure imaginary refractive index in such mediums, a Poynting vector directed opposite to the propagation wave vector, the reversal of Doppler and Cerenkov effects, and the inverse of the laws of refraction of Snell and Descartes. Because of the absence of naturally existing metamaterials, the realization experiment of such media is based on the relative permittivity $\varepsilon(\omega)$ and relative permeability $\mu(\omega)$ simultaneously which depend on the frequency of the electromagnetic wave [5].

A multichannel electromagnetic filter is an important periodic or quasi-periodic photonic crystal device used in frequency division multiplexing applications, which can filter [6] or separate multiple frequencies simultaneously [7–10]. In recent years, one-dimensional electromagnetic star waveguides structures (SWGs) have attracted close attention of several researchers. This is associated with wide possibilities of their practical use in the creation of numerous devices to guide, control, and manipulates electromagnetic waves in various frequencies ranges [11–14]. These wired SWGs systems based on coaxial cables are composed of periodicity of cells, and each cell is constituted by segments and grafted in each site by a finite number of resonators. Electromagnetic star waveguides structure is of a particular interest because of very wide PBGs [15, 16]. The width and position of these PBGs can be adjusted by the number of periods (number of sites) of the structure, the number of resonators in each site, and the boundary condition at the limit interface of the resonators. The introduction of defects inside the SWGs structure or other periodic structures, gives rise to well-defined transmission peaks corresponding to defect modes inside the band gaps in the transmission spectra [17–24]. For many practical applications, an important task is the predicted rearrangement of the electromagnetic spectrum, which is related to the correct choice of the mediums constituting the structure and nature of the defects introduced inside it. Recently, our research team has investigated the effect of the simultaneous presence of two different resonators defects located in arbitrary sites. This study shows the possible existence of four defects modes in the gaps, which overlap between them when the defects move away from each other [16]. These modes keep a maximum transmission when the number of defects in each site increases at the same time. With the development of the frequency division multiplexing (FDM) technology, it has been necessary to propose a structure which behaves as a multi-channel filter in a high-capacity communication system. Moreover, defective SWGs compound structure has become an ideal choice to achieve multi-channel filtering, due to their simple structures constituted by coaxial cables and their large gaps.

In this paper, the resonators defects located in three different sites are introduced to construct a very narrow band multichannel filter. This multichannel filter is composed of the periodicity of the segments of length $d_1 = 1$ m and grafted in its extremity by a finite number of resonators of length $d_2 = 0.5$ m. We create inside this structure the resonators defects located in three different sites, namely J for the first resonators defects of lengths d_{02} , J' for the second resonators defects of lengths d_{04} , and J'' for the third resonators defects of lengths d_{06} . The number of resonators defects in sites J , J' , and J'' are denoted by N'_{02} . The transmittance versus frequency of considered periodic star waveguides structure is analyzed for potential application in a tunable multichannel filter at the microwave region.

Our paper is organized as follows. We present a general introduction in Section 1. In Section 2, we present our theoretical expressions of the dispersion relation through an infinite system and the transmission coefficient through a finite system containing the resonators defects located in three different sites such as these three sites containing a number N'_{02} of defective resonators. Section 3 shows the numerical results for a defective structure constituted of N sites (Fig. 1) and embedded between two semi-infinite guides. The conclusion is presented in Section 4.

2. MODEL AND FORMALISM

2.1. Dispersion Relation of Infinite Star Waveguides Structure

In this part, the structure is formed by an infinite number of segments of length d_1 in the x direction. Each segment has two free interfaces; the interface domain consists of all the connection points between the backbones. Each connection point (called site) is defined by an integer n , and each site n is grafted

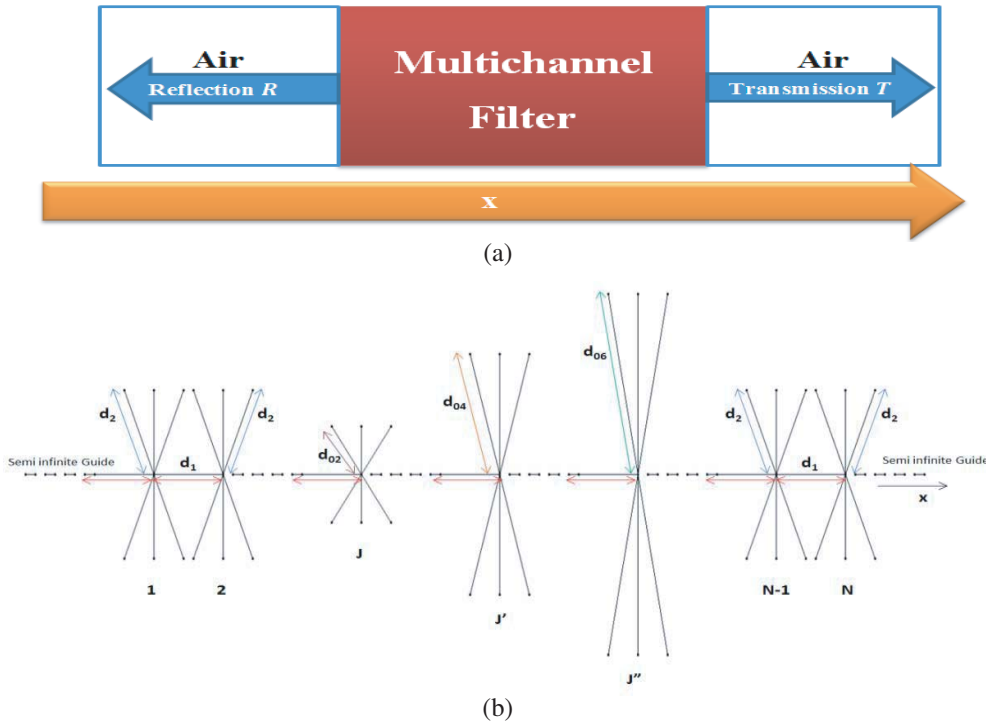


Figure 1. (a) The model structure for a multichannel narrow band filter based on the resonators defects in SWGs. The transmittance T will be the quantity of interest calculated for the purpose of investigating tunable properties. x is the direction of the propagation of the electromagnetic waves inside the multichannel filter. (b) Illustration of a multichannel filter containing N sites and delimited by two semi infinite guides. This multichannel filter contains the resonators defects of lengths d_{02} , d_{04} and d_{06} located respectively in the sites J , J' and J'' .

by side branches (resonators) of length d_2 . The position x in a cell between sites n and $n + 1$ is represented by the pair (n, x) where x is a local coordinate with $0 \leq x \leq d_1$, and d_1 represents the period of the structure. However, to calculate the Green function of the infinite periodic structure, we need to know the Green function of a segment of length $d_1 (i = 1)$ and the Green function of resonator of length $d_2 (i = 2)$. The element of a segment is denoted by $\overleftarrow{g}_1^{-1}(M_1, M_1)$, which is a matrix (2×2) in the interfaces space $M_1 = \{0; d_1\}$ and the element $\overleftarrow{g}_2^{-1}(0; 0) = \overleftarrow{g}_2^{-1}(d_2; d_2)$.

The inverse matrix of a segment is given by [19]:

$$\overleftarrow{g}_{si}^{-1}(0; d_1) = \begin{pmatrix} -\frac{F_1 C_1}{S_1} & \frac{F_1}{S_1} \\ \frac{F_1}{S_1} & -\frac{F_1 C_1}{S_1} \end{pmatrix} \quad (1)$$

while $\overleftarrow{g}_2^{-1}(0; 0) = \overleftarrow{g}_2^{-1}(d_2; d_2)$ depends on the choice of the boundary conditions on the end of the resonators. This quantity is given by:

$$\overleftarrow{g}_2^{-1}(0; 0) = -\frac{S_2 F_2}{C_2} \text{ for the magnetic field } H = 0 \quad (2)$$

with:

$$C_i = \cosh(\alpha_i d_i) \text{ and } S_i = \sinh(\alpha_i d_i) \quad (2a)$$

$$\alpha_i = j \frac{\omega}{c} \sqrt{\epsilon_i \mu_i} \quad (2b)$$

$$j = \sqrt{-1} \quad (2c)$$

periodically grafted with a spacing period d_1 in N sites on a finite line, N and N' being integers. For the sake of simplicity, it is assumed that the two semi-infinite guides and the segments are constituted by the same physical properties. The defects are created inside the structure at the resonators levels. The first defect changes the N' resonators of length d_2 located in site J by N'_{02} resonators of lengths d_{02} , and the second defects of lengths d_{04} are located in site J' and the third defects of lengths d_{06} are located in site J'' . Therefore, the disturbed states are: $M_s = \{-1, 0, J, J', J'', N, N + 1\}$. The cleavage operator is a matrix (7×7) defined in the interfaces domain M_s : $\vec{V}(M_s, M_s) = \vec{g}_t^{-1}(M_i, M_i) - \vec{g}_\infty^{-1}(M_i, M_i)$ [28] where $\vec{g}_\infty^{-1}(M_i, M_i)$ of this system is given in Equation (3a).

$$\vec{V}(M_s, M_s) = \begin{pmatrix} \frac{F_1 C_1}{S_1} & -\frac{F_1}{S_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{F_1}{S_1} & \frac{F_1 C_1}{S_1} + N' \frac{S_2 F_2}{C_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_1 F_1}{S_1} & -\frac{F_1}{S_1} \\ 0 & 0 & 0 & 0 & 0 & -\frac{F_1}{S_1} & \frac{F_1 C_1}{S_1} \end{pmatrix} \quad (8a)$$

With:

$$\alpha = N' \frac{S_2 F_2}{C_2} - N'_{02} \frac{S_{02} F_{02}}{C_{02}}, \quad \beta = N' \frac{S_2 F_2}{C_2} - N'_{02} \frac{S_{04} F_{04}}{C_{04}} \quad \text{and} \quad \gamma = N' \frac{S_2 F_2}{C_2} - N'_{02} \frac{S_{06} F_{06}}{C_{06}} \quad (8b)$$

Knowledge of the elements of the response function in the interfaces space M_s of the infinite star waveguide structure $\vec{g}(M_s M_s)$ and those of the cleavage operator $\vec{V}(M_s, M_s)$ allow us to deduce the elements of the interface response operator [12]:

$$\vec{A}(M_s, M_s) = \sum_{M_s} \vec{V}(M_s, M_s) \vec{g}(M_s, M_s) \quad (9)$$

From Equation (7a), $\vec{g}(M_s, M_s)$ is written as follows:

$$\vec{g}(M_s, M_s) = \frac{S_1}{F_1} \frac{t}{t^2 - 1} \begin{pmatrix} 1 & t & t^{J+1} & t^{J'+1} & t^{J''+1} & t^{N+1} & t^{N+2} \\ t & 1 & t^J & t^{J'} & t^{J''} & t^N & t^{N+1} \\ t^{J+1} & t^J & 1 & t^{J'-J} & t^{J''-J} & t^{N-J} & t^{N+1-J} \\ t^{J'+1} & t^{J'} & t^{J'-J} & 1 & t^{J''-J'} & t^{N-J'} & t^{N+1-J'} \\ t^{J''+1} & t^{J''} & t^{J''-J} & t^{J''-J'} & 1 & t^{N-J''} & t^{N+1-J''} \\ t^{N+1} & t^N & t^{N-J} & t^{N-J'} & t^{N-J''} & 1 & t \\ t^{N+2} & t^{N+1} & t^{N+1-J} & t^{N+1-J'} & t^{N+1-J''} & t & 1 \end{pmatrix} \quad (10)$$

The parameter t is given in Equation (7b).

The interface response function $\vec{A}(M_s, M_s)$ allows us to get the matrix $\vec{\Delta}(M_s, M_s)$, which is given by the following relation:

$$\vec{\Delta}(M_s, M_s) = \vec{I}(M_s, M_s) + \vec{A}(M_s, M_s) \quad (11)$$

with $I(M_s M_s)$ being the identity matrix in the interfaces space M_s .

After calculating the operator $\vec{\Delta}(M_s, M_s)$, we will write this operator in the interfaces space $M_0 = \{0, J, J', J'', N\}$.

The Green's function of surface $\vec{d}(M_0, M_0)$ for an infinite star waveguides structure is defined in the interfaces space M_0 by the following equation:

$$\vec{d}(M_0, M_0) = \vec{g}(M_0 M_0) \vec{\Delta}^{-1}(M_0 M_0) \quad (12)$$

with $\overleftarrow{\Delta}^{-1}(M_0M_0)$ being the inverse of the operator $\overleftarrow{\Delta}(M_0M_0)$.

We deduce the truncated matrix $\overleftarrow{d}_{tr}(M_0, M_0)$ in the interfaces space $M'_0 = \{0, N\}$. The inverse of that matrix is written as follows:

$$\overleftarrow{d}_{tr}(M'_0, M'_0) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \quad (13)$$

where d_{11} , d_{12} , d_{21} , and d_{22} are the function of elements (1, 1), (1, 5), (5, 1), and (5, 5) of the inverse matrix of $\overleftarrow{d}(M_0, M_0)$.

As mentioned above, the Green's function used here enables us to determine not only the transmissions peaks of the defect modes within the gaps, but also the resonant modes lying within the allowed bands. In this case, the Green function of finite defective SWGs containing defects located in three different sites and delimited by two semi-infinite guides of inverse Green function $-F_1$:

$$\overleftarrow{d}_h^{-1}(M'_0, M'_0) = \begin{bmatrix} d_{11} - F_1 & d_{12} \\ d_{21} & d_{22} - F_1 \end{bmatrix} \quad (14)$$

where:

$$\overleftarrow{d}_h(M'_0, M'_0) = \frac{1}{(d_{22} - F_1)(d_{11} - F_1) - d_{21}d_{12}} \begin{bmatrix} d_{22} - F_1 & -d_{21} \\ -d_{12} & d_{11} - F_1 \end{bmatrix} \quad (15)$$

The transmission rate through the finite SWGs system is written as follows:

$$T = \left| -2F_1 \overleftarrow{d}_h(s, e) \right|^2 = \left| 2F_1 \frac{1}{(d_{22} - F_1)(d_{11} - F_1) - d_{21}d_{12}} d_{12} \right|^2 \quad (16)$$

with e : the input interface between the first semi-infinite guide and the SWGs and s : the output interface between the second semi-infinite guide and the SWGs.

Let us notice that our numerical calculations are done with the help of the FORTRAN compile. Equations (1) to (11) are analytically obtained; however, we have numerically calculated the $\overleftarrow{d}(M_0, M_0)$ matrix, given by Equation (12). The last is the product of the two matrix $\overleftarrow{g}(M_0, M_0)$ and $\overleftarrow{\Delta}^{-1}(M_0, M_0)$. We truncate this matrix $\overleftarrow{d}(M_0, M_0)$ in matrix (2×2) and limit our structure by two semi-infinite guides. Finally, we numerically deduce the relation of transmission rate in Eq. (16).

3. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will study a star waveguides (SWGs) structure composed by the periodicity of segments of length $d_1 = 1$ m and grafted in its extremity by resonators of length $d_2 = 0.5$ m. The resonators and segments are composed by the same material, i.e., $\varepsilon_1 = \varepsilon_2$. The introduction of geometrical resonators defects in these SWGs generates narrow transmission peaks inside the band gaps. The frequencies of these peaks are called defects modes frequencies. We will study the effect of the numbers of the resonators defects, their positions, and their lengths on the behavior of the defects modes frequencies. We consider SWGs with seven sites ($N = 7$), and the resonators defects are located in three different positions, namely J (for the first defects of length d_{02}), J' (for the second defects of length d_{04}), and J'' (for the third defects of length d_{06}). We take $N' = 1$, where N' indicates the number of undisturbed resonators of length $d_2 = 0.5$ m in each site, this choice is because these resonators play a role of obstacles to the propagation of the frequencies of defects modes.

The transmission T of the star waveguides structure without defects (perfect structure) is shown in Fig. 2(a) where the lengths of resonators defects are $d_{02} = d_{04} = d_{06} = 0.5$ m. We notice the existence of three passbands (permitted bands), such that each two passbands are separated by a band gap where no propagating modes are present. Hence, this perfect structure does not behave as a very narrow multichannel MHz filter. To realize this kind of tunable filter which is the crucial components for future communication, it is necessary to introduce the defects into the 1D SWGs to break the periodicity. In this context, we purpose to disturb these PBGs to realize a transmittance of a narrow band multichannel

filter based on the addition of resonators defects. Figs. 2(b)–(d) show the transmission spectra for multi-defective SWGs structure, obtained for different numbers of the resonators defects. From Figs. 2(b)–(d), the filtering effect strongly depends on the numbers of resonators defects, while the band gaps are independent of the variation of the defects numbers. We consider only one resonator defect, and Fig. 2(b) shows the presence of two very narrow transmissions Dirac peaks in each gap with very important values of transmission rate. So, by using only a single resonator defect inside the structure, our proposed structure filters two frequencies modes, which permit it to behave like a two channel filter.

In the aim of qualitative comparison with previous work, we have to mention that the authors in [29] have demonstrated the existence of two defects modes in the gaps by introducing two cavities layers (two defects layers) in a 1D multilayer system including two planar cavities. Such twice defective SWGs work better for the realization of a multichannel filter. Till now, we have just considered the case of a single-channel filter in the MHz frequency range. However, dual-channel, three-channel, or even multi-channel filters are required in many communication systems. The multi-channel filters can be realized by increasing a number of defects inside the structure. In the case where we consider two different resonators defects inside the structure, we observe through Fig. 2(c) the appearance of four narrow defects modes in each gap with low transmission. Fig. 2(d) describes the case where three resonators defects are introduced in the structure, and one observes eight defects modes in each gap with very high values of transmission. Particularly, with three resonators defects, a few sharp transmission peaks are found in the band gaps. In this situation, the structure works as a multi-channel microwave filter with very high performance. According to Fig. 2, we can achieve the filtering frequencies and transmittance amplitude at different numbers of resonators defects. Also, the filtering frequencies gradually shift in the center of the large gaps. From Fig. 2, it is also clear that if we increase the number of resonators defects in the structure, the number of transmission peaks in gaps increases. The structure works as a double/multi-channel filter depending upon the number of defects.

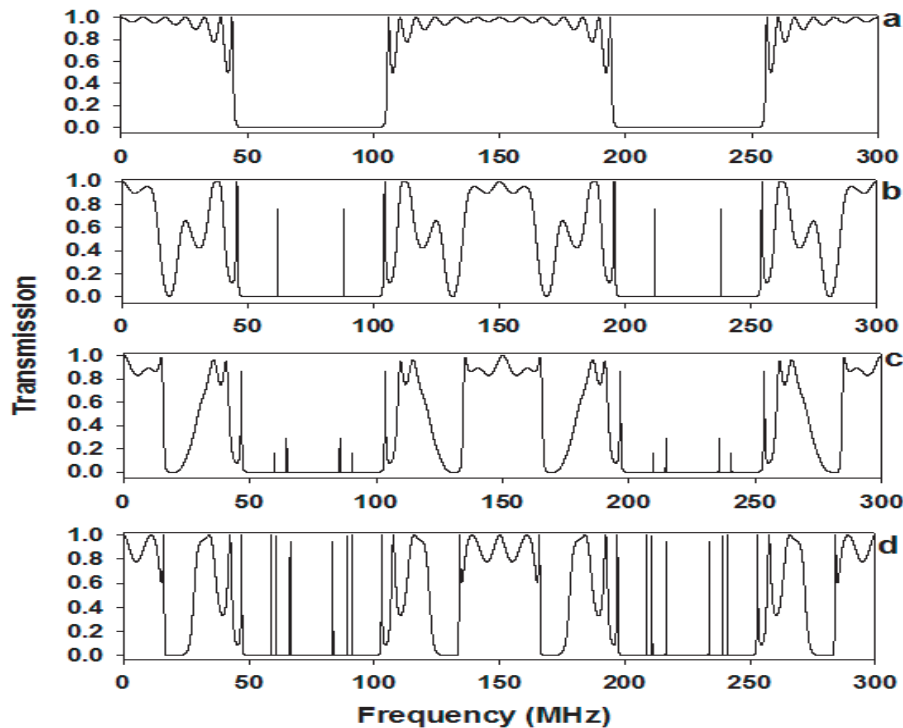


Figure 2. Variation of the transmission rate for designed multichannel filters in microwave range as a function of the frequency (MHz) for different numbers of resonators defects namely $d_{02} = d_{04} = d_{06} = 0.5$ m (case a), $d_{02} = 2$ m ($J = 4$) and $d_{04} = d_{06} = 0.5$ m (case b), $d_{02} = 2$ m ($J = 3$), $d_{04} = 2$ m ($J' = 4$) and $d_{06} = 0.5$ m (case c), and $d_{02} = 2$ m ($J = 3$), $d_{04} = 2$ m ($J' = 4$) and $d_{06} = 2$ m ($J'' = 5$) (case d). We take $N'_{02} = 1$.

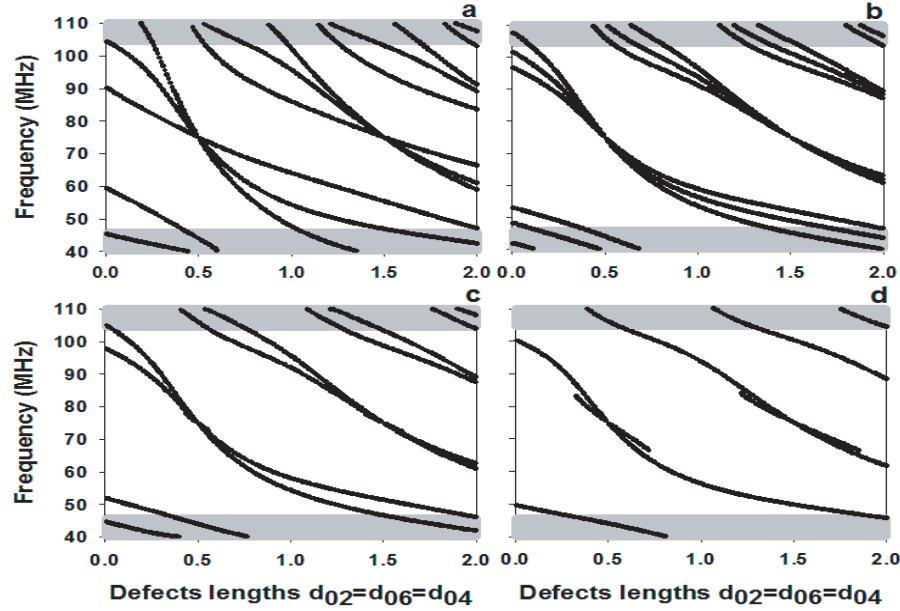


Figure 3. Variation of frequency (MHz) as a function of the defects lengths $d_{02} = d_{04} = d_{06}$ for different values of J , J' and J'' , namely $J = 3$, $J' = 4$ and $J'' = 5$ (case a), $J = 2$, $J' = 4$ and $J'' = 6$ (case b), $J = 1$, $J' = 4$ and $J'' = 6$ (case c), and $J = 1$, $J' = 4$ and $J'' = 7$ (case d). We take $N'_{02} = 1$.

Now, we study the effect of the positions and lengths of the defects at the resonators levels on the frequencies of the modes located inside the band gaps, by keeping the other parameters constant as $d_1 = 1$ m and $d_2 = 0.5$ m. Fig. 3 shows the variation of the frequency as a function of the defects lengths $d_{02} = d_{04} = d_{06}$ for some values of J , J' , and J'' , namely $J = 3$, $J' = 4$, and $J'' = 5$ (case a); $J = 2$, $J' = 4$, and $J'' = 6$ (case b); $J = 1$, $J' = 4$, and $J'' = 6$ (case c); and $J = 1$, $J' = 4$, and $J'' = 7$ (case d). The gray areas represent the passbands, and the white areas represent the band gaps of perfect infinite star waveguides structure. The black branches located inside the band gaps represent the defect modes. The frequencies of the defect modes decrease with increase of the defect length $d_{02}(d_{04}, d_{06})$ within gaps. These results correlate with that obtained in the previous works, for instance by Ghadban et al. when they studied the presence of two cavities layers in 1D photonic multilayer structure [29], or by Khaled et al. when they introduced two defects layers in 1D phononic crystal [30], or by Bouzidi et al. with the creation of one defect in photonic crystal [21]. The system studied here presents an advantage: all the segments and the resonators present in the periodic structure are made of the same material instead of being constructed by two different materials. Moreover, the defects modes frequencies decrease is due to the interaction between the Eigen modes of the resonators defects and the frequencies of the incoming electromagnetic wave propagating in the structure. As shown in Fig. 3(a), where $J = 3$, $J' = 4$, and $J'' = 5$ (the defects are close to each other), and we notice the appearance of several defects modes. These modes are very far apart, but for very precise values of the defects ($d_{02} = d_{04} = d_{06} = 0.47$ m, and $d_{02} = d_{04} = d_{06} = 1.5$ m), we obtain the phenomenon of superposition between the defects modes due to the high interaction between the three different defects branches. This exchange process becomes smaller after these particular values of the three resonators defects lengths (0.47 m and 1.5 m) and the localized branches refined their initial behavior. From Fig. 3(b), where the defects begin to move away ($J = 2$, $J' = 4$, and $J'' = 6$), we find that the defects branches observed in the first case (3a) begin to get closer to each other (the frequency shift between two defects branches begins to decrease). As we continue to move these three defects $J = 1$, $J' = 4$, and $J'' = 6$ (Fig. 3(c)), we observe that the corresponding branches are almost superimposed, and when we move these defects $J = 1$, $J' = 4$, and $J'' = 7$ more away (Fig. 3(d)), the defects branches become totally superimposed; therefore, the structure behaves like a single resonator defect comb-like structure [17]. Due to the large number of defects modes, we deduce that when the resonators defects are situated in three consecutive sites ($J = 3$, $J' = 4$, and $J'' = 5$), our system acts as an MHz multichannel filter which allows to separate a significant number of frequencies

reaching till eight filtered frequencies. Moreover, our proposed multichannel filter can be employed in a frequency division multiplexing (FDM) application used in microwave telecommunications [31, 32].

In order to obtain the adequate geometrical parameters leading to a better performance of our multichannel filter with very high transmission rates of the filtered frequencies, we plot a color map of the transmission rate of the defects branches observed in the center of the gap of Fig. 4 as a function of both defects lengths d_{02} and d_{04} , with $J = 3$, $J' = 4$, $J'' = 5$, $d_{06} = 2$ m, and $N'_{02} = 1$. The red, orange, and purple colors indicate, respectively, the high, average, and low values of the transmission rate. The map is obtained by keeping, for every point of the defects lengths d_{02} and d_{04} plane, the maximum of transmission within the first gap. According to this figure, we observe a high transmission for high values of d_{02} and d_{04} , i.e., $d_{02} = d_{04} = 2$ m (red areas). Similarly, the transmission decreases when d_{02} and d_{04} decrease. This shows that the propagating energy of the electromagnetic wave starts to attenuate as long as the lengths d_{04} and d_{06} of the two defects begin to decrease, and consequently the defects play the role of perturbations during the propagation of electromagnetic waves in the defectives SWGs structure. Conclusively, the results in Fig. 4 reveal that a multichannel transmission filter can be obtained with a very high transmission when the resonators defects lengths are four times of the length of the undisturbed resonators ($d_{02} = d_{04} = d_{06} = 4 d_2 = 2$ m).

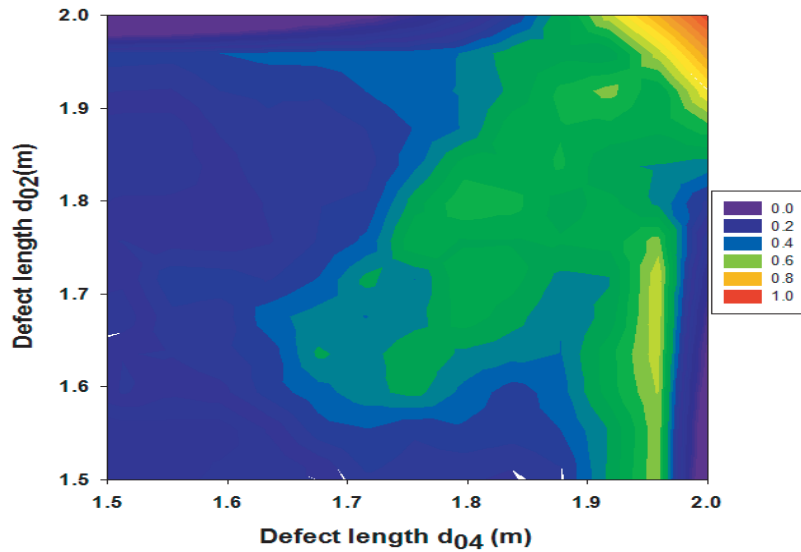


Figure 4. Color map of the transmission rate as a function of both defect lengths d_{04} and d_{06} . We take $J = 3$, $J' = 4$, $J'' = 5$, $N'_{02} = 1$ and $d_{02} = 2$ m.

In order to examine the filtering quality of our multichannel filter based on the defects modes, we take the defect branch located between the frequencies [83 MHz–88.5 MHz] situated in the center of the gap of Fig. 3(a) when $d_0 = d_{02} = d_{04} = d_{06} = 2$ m, and we study, in Fig. 5, the variation of transmission rate T and quality factor Q (where Q is given by $Q = f_{pic}/\Delta f$ with f_{pic} being the frequency of the defects peaks and Δf the width at half-height of the peak) of this defect branch versus the defect lengths $d_0(d_{02}, d_{04}, d_{06})$ and the frequency (MHz) with $J = 3$, $J' = 4$, $J'' = 5$, and $N'_{02} = 1$. From Fig. 5, where the defects lengths are in the range varying from 1.8 m to 2 m, the defect mode has very high transmission and shifts to lower frequency. Consequently, the characteristics of the proposed filter can be tuned by operating on defects lengths. This change in the position of the transmission peak is due to the geometrical properties of the resonators defects. This defect mode has a very high quality factor which reaches $Q = 145000$, and is well confined in the structure thanks to the important number of defects inserted inside the structure. The above analyses show that a very narrow MHz multichannel filter with very high performance can be achieved by defectives star waveguides structure. The frequencies, transmission rate, and quality factors of the defects modes can be tuned by varying the resonators defects lengths.

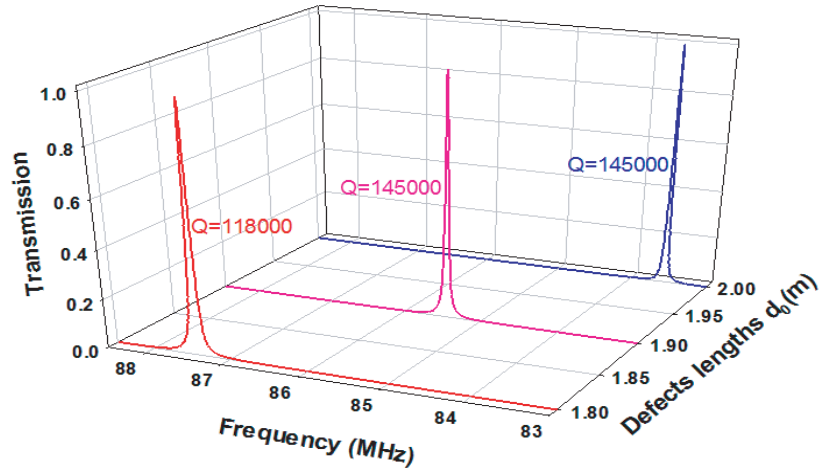


Figure 5. Variation of the transmission rate versus the frequency (MHz) and the defects lengths $d_0 = d_{02} = d_{03} = d_{06}$. We take $N'_{02} = 1$, $J = 3$, $J' = 4$, and $J'' = 5$.

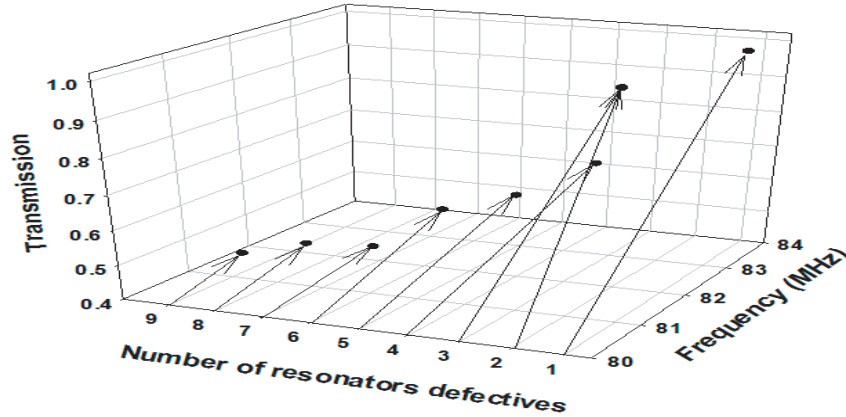


Figure 6. Variation of the transmission rate versus the number resonators defects N'_{02} and the frequency (MHz). We take $J = 3$, $J' = 4$, $J'' = 5$, and $d_{02} = d_{04} = d_{06} = 2$ m.

In the above numerical simulations, we keep the number of the resonators defects $N'_{02} = 1$ in the defects sites J , J' , and J'' . Fig. 6 presents the simulated transmittance spectra of the defect branches of frequencies located in the interval [80 MHz–84 MHz] for different numbers of the resonators defects N'_{02} ranging from 0 to 9, with defects lengths fixed at 2 m, and the defects are located in three consecutive sites, namely $J = 3$, $J' = 4$, and $J'' = 5$. From Fig. 6, it can be seen that the defects branches have an almost maximum transmission when $N'_{02} = 1$, and these values of the transmission rate decrease with the increase of N'_{02} . Such a result is similar to that obtained in our previous work [13] when we have introduced resonators defects located in one site inside the SWGs made of metamaterials of type double negative (the segments have a relative permittivity and a magnetic permeability simultaneously positive, and the resonators have a relative permittivity and a magnetic permeability simultaneously negative). Besides, this result shows that the increase of the defects number in each site amplifies the reflection of the electromagnetic wave happening at each defect interface. These resonators defects hence disposed act like multi-obstacles against the propagation of the incoming electromagnetic wave. From the above analysis, one can conclude that a single resonator defect ($N'_{02} = 1$), situated in sites $J = 3$, $J' = 4$, and $J'' = 5$, gives rise to a very narrow band multichannel filter.

The variation of the frequencies of the defects modes for SWGs structure versus the number of the defects in each site N'_{02} is calculated and shown in Fig. 7. It can be seen that when N'_{02} increases, four defects modes, of frequencies belonging to the interval [60.7 MHz–89.9 MHz], approach each other,

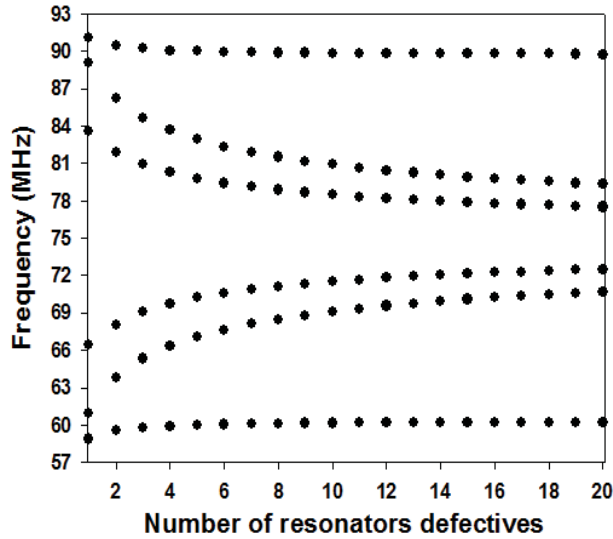


Figure 7. Defects branches frequencies (MHz) versus the number of defectives resonators N'_{02} . The other parameters are $d_{02} = d_{04} = d_{06} = 2$ m, $J = 3$, $J' = 4$ and $J'' = 5$.

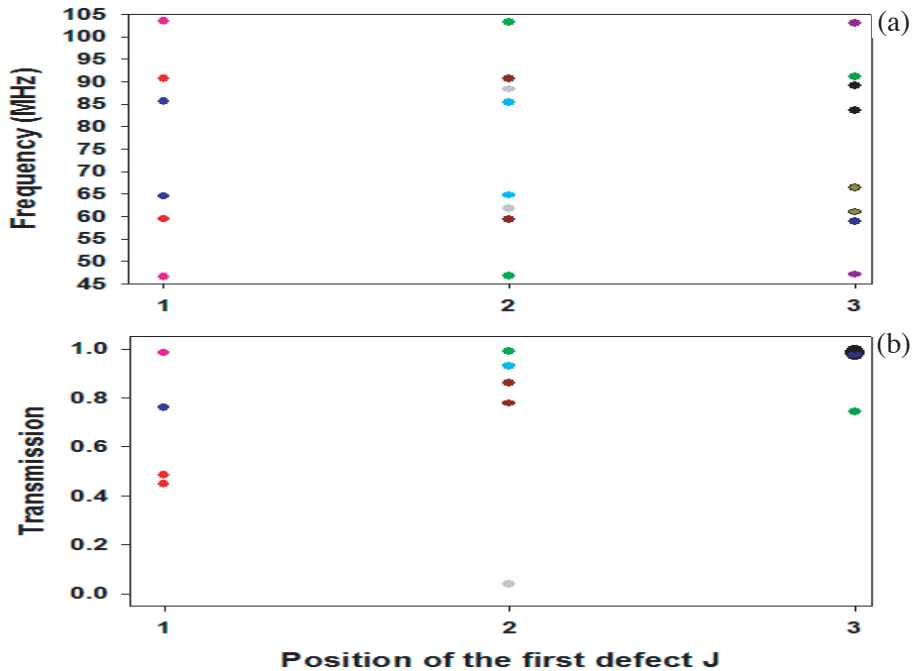


Figure 8. (a) Defects branches frequency (MHz) versus the position of the first defect J . (b) Transmission coefficient of the defects branches observed in Fig. (a). The other parameters are $d_{02} = d_{04} = d_{06} = 2$ m, $J' = 4$, and $J'' = 5$.

while the others two modes of frequencies located either in [57 MHz–60 MHz] or in [90 MHz–92 MHz] are almost independent of N'_{02} . This shows that there is a strong interaction between the defects modes induced by two resonators defects, in contrary the other defect which generates defect modes of frequencies almost constant versus N'_{02} . Likewise, we note that the frequency shift between the defects modes is very important, which permits to obtain multi-channel filter with high sensitivity avoiding any noise problem during filtering.

Beyond the methods just outlined, we can control the frequencies positions of the filter by changing the positions of defects in the 1D SWGs. In this part, we plot the frequencies (Fig. 8(a)) and transmission rate (Fig. 8(b)) of the defects modes observed in Fig. 3(a) when $d_{02} = d_{04} = d_{06} = 2$ m for distinct positions of the first defect J by keeping the other defects positions $J' = 4$ and $J'' = 5$. It can be noticed that when $J = 1$, six transmission peaks appear at frequencies $f_1 = 46.54$ MHz and $f_2 = 103.65$ MHz in the edge of the gap (Pink color) which have a maximum transmission and the other four peaks located in the center of the gap. The other four peaks are located in the center of the gap. The red peaks of frequencies $f_3 = 59.4$ MHz and $f_4 = 90.61$ MHz have transmission rates, respectively, $T = 0.44$ and $T = 0.48$, and the blue peaks of frequencies $f_5 = 64.5$ MHz and $f_6 = 85.5$ MHz have $T = 0.76$. When $J = 2$, we clearly observe the appearance of eight peaks in the gap, and these peaks have very high transmission rate, i.e., the green modes of frequencies $f_1 = 47.13$ MHz and $f_2 = 103.65$ MHz have $T = 0.99$, and the brown modes of frequencies $f_3 = 59.3415$ MHz and $f_4 = 90.66$ MHz have, respectively, $T = 0.777$ and $T = 0.86$. The gray modes of frequencies $f_5 = 61.38$ MHz and $f_6 = 88.32$ MHz have $T = 0.1$, while the sky peaks of frequencies $f_7 = 64.72$ MHz and $f_8 = 85.27$ MHz have respectively $T = 0.93$ and $T = 0.928$. For $J = 3$, the existence of eight peaks of defects can be seen very well, which have very high transmission amplitude. The Dk. pink peaks in the edge of the gap at frequencies $f_1 = 47.57$ MHz and $f_2 = 103.43$ MHz have a maximum transmission; the Dk. green peak of frequency $f_3 = 91.28$ MHz has a transmission $T = 0.742$; the Dk. blue peak of frequency $f_4 = 58.86$ MHz has a transmission $T = 0.9759$; the black peaks of frequencies $f_5 = 89.29$ MHz and $f_6 = 83.77$ MHz have respectively transmission $T = 0.98$ and $T = 0.989$; and the Dk. yellow peaks of frequencies $f_7 = 61.04$ MHz and $f_8 = 66.78$ MHz have maximum transmission. The results also show that a very important frequency shift exists between each two defects modes, which avoids the noise problem during filtering. The results of these figures show that our multichannel filter which can filter until high frequencies can be realized with very high performance, when the first defect located in the site $J = 3$, the second defect located in the site $J' = 4$, and the third defect located in the site $J'' = 5$.

Let us recall that the above-analyses given in Fig. 3 are done in the case where the three defects have the same length. Now, we turn our attention to the case where two defects have the same lengths which are fixed at $d_{04} = d_{06} = 2$ m, and we vary the length d_{02} of the first defect. In Fig. 9, we plot the frequencies of the defects modes versus the defect length d_{02} varying from 0 to 2 m. The gray areas indicate the passbands, and the white areas represent the band gap where the defects branches are localized. The defects modes which are almost insensitive to d_{02} correspond to the modes of two others defects of lengths d_{04} and d_{06} , while the decreasing modes (versus frequency) are induced by the first defect resonator of length d_{02} , situated in site $J = 3$. We note that for some specific values of d_{02} (in particular, $d_{02} = 0.78$ m around 60 MHz, $d_{02} = 1.14$ m and $d_{02} = 0.29$ m around 90.05 MHz), the different localized branches caused by each resonator defect interact and show an exchange behavior between them giving rise to degeneration lift phenomenon. This process describes the transfer of energy between the defects modes frequencies during the propagation of electromagnetic waves inside the structure. In what follows, we take $d_{02} = 2$ m and illustrate, according to Fig. 9(b), the transmission rate and quality factor Q of the defects modes. From Fig. 9(b), it is found that the defects modes have very high transmission rate and very important values of quality factors. This shows that our proposed structure makes it possible to filter till eight frequencies with very high-quality factors (very good signal transfer), i.e., $f_1 = 74.01$ MHz with $T = 1$ and $Q = 110000$, $f_2 = 58.94$ MHz with $T = 0.79$ and $Q = 395000$, $f_3 = 61$ MHz with $T = 1$ and $Q = 276670$, $f_4 = 66.35$ MHz with $T = 1$ and $Q = 300000$, $f_5 = 83.44$ MHz with $T = 1$ and $Q = 560000$, $f_6 = 88.998$ MHz with $T = 0.98$ and $Q = 478000$, $f_7 = 90.855$ MHz with $T = 1$ and $Q = 610000$, and $f_8 = 102.986$ MHz with $T = 1$ and $Q = 394880$.

The results in Fig. 9 illustrate that eight channels with very high performance can be obtained by fixing the length of three defects d_{02} , d_{04} , and d_{06} at four times of the length of the perfect resonator of length $d_2 = 0.5$ m.

Moreover, based on the defective photonic periodic structure proposed in this paper and by selecting the suitable defects resonators parameters (N'_{02} , J , J' , J'' , d_{02} , d_{04} , d_{06}), a very narrow band optimized tunable multichannel filter may be realized in the microwave region. The principal advantage of the electromagnetic SWGs structure presented in this paper is linked to the existence of several transmission peaks of the defects modes depending on the different resonators defects lengths, their numbers, and their positions. The obtained results demonstrate that our proposed filter based on multi-defects modes

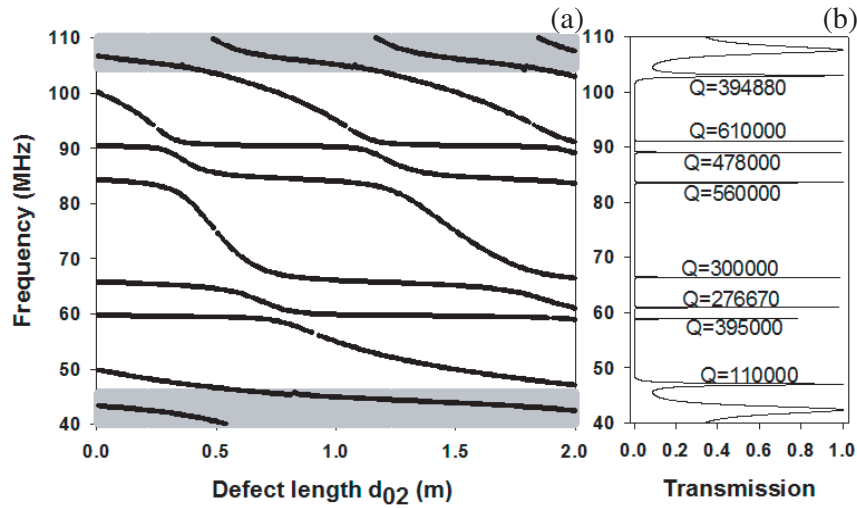


Figure 9. (a) Variation of frequency (MHz) as a function of the defect length d_{02} . (b) Transmission spectrum when $d_{02} = 2$ m. We take $N'_{02} = 1$, $d_{04} = d_{06} = 2$ m, $J = 3$, $J' = 4$ and $J'' = 5$.

has a very important transmission coefficient and very high quality factor compared to no-periodic acoustic or photonic waveguides structure system [33, 34]. We believe that this paper brings a new piece of work in the field of electromagnetic wave transport in 1D waveguide structures containing different types of metamaterials [35–40].

4. CONCLUSION

Based on the Green function method, we calculate the microwave spectra of a periodic star waveguides structure with multi-resonators defects used to design a narrow band multichannel filter in the microwave region. As we have seen, this multichannel filter based on defects modes is tunable with resonators defects lengths, their positions, and their number in the structure. We have found that the two transmission peaks of the defect modes can appear in the gaps when one resonator defect is introduced. On the other hand, we have inserted different resonators defects simultaneously in three sites and demonstrated that eight transmission peaks of the defects modes are shifted towards lower frequencies when the defects lengths increase. Furthermore, as the number of resonators defects increases, the transmissions of these defects modes decrease while their quality factors become very high reaching $Q = 600000$. We have also studied the disposition of defects modes when the resonators defects move away from each other. In this case, the structure behaves almost as containing one defect at the resonators level located in just one site, because the modes of each defect fall in the same frequency (superposition phenomena). Also, we have investigated the interactions between the defects modes due to different resonators defects, by varying only one defect length and keeping the others constant. The defects modes induced by the fixed defect length and the variable defect length interact between each other, giving rise to degeneration lifting. After this interaction, we obtain the change in the behavior of these defects modes.

Moreover, our theoretical model is, in principle, universal and then also valid in other frequency domains of the electromagnetic spectrum. Indeed, the frequency domains in such a structure depend on the size of the segment and resonator. The manufacturing of such waveguides structures could be very useful in guiding, filtering, multiplexing, demultiplexing, and other electromagnetic devices. It would even be more interesting for integrated structures working at optical frequencies. We must observe that for optical frequencies, the periodicity and resonators lengths must be in the order of magnitude of micrometer. Recent works show clearly that the manufacturing of such star waveguides structures at a sub-micrometric scale becomes realizable with the new technological developments using high-resolution electron beam lithography [41, 42].

5. DECLARATION OF COMPETING INTEREST

The authors of this paper declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

ACKNOWLEDGMENT

We thank Professor Hayat Lbekkouche for correcting the English and for making sure that our paper would be sent properly.

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