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Analysis of a Rectangular Waveguide Fed Compound Slot by Image Method

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Abstract—A compound or an offset inclined slot fed by a rectangular waveguide has been analysed using the image method for the evaluation of internal admittance in the Method of Moments framework. The internal admittance has been evaluated from the 2D infinite planar array equivalent image representation of a slot in a waveguide. The advantages offered by this method include the freedom to work in slot coordinates rather than the waveguide coordinates, the reuse of external admittance for air filled waveguides, and the flexibility in the choice of the mutual admittance evaluation technique. Unlike the conventional mode method, the proposed technique does not run into difficulties while evaluating the fields from longitudinal magnetic current for points in the same transverse plane. The θ-algorithm has been used for the convergence acceleration of the series of mutual admittances in the internal admittance evaluation and has been shown to yield better results than other convergence acceleration algorithms investigated. The results obtained from this method have been shown to agree within 0.5% average with those from other theoretical techniques and within 1% average with measurements. The proposed method is useful in the design of compound slot arrays and can be extended to other configurations where the image representation is possible and for various slot aperture distributions.

1. INTRODUCTION

Arbitrarily inclined offset slots or compound slots in a rectangular waveguide are often used where the independent control of amplitude and phase excitation is desired such as in missile fuze applications [1, 2]. The spacing between the compound slot elements can be made less than centre inclined slots, thereby avoiding the formation of grating lobes [3]. Compound slots have an additional degree of freedom in their tilt from the longitudinal axis, which can be exploited to compensate the detuning of slot elements due to mutual coupling in an array environment. Compound broadwall slot radiators are more general radiating elements than the longitudinal or transverse slots, and an analysis of compound slots helps in understanding all types of broadwall slot radiators [4].

Compound slots have been studied by Watson [5] and have been analysed using various techniques such as analytical techniques assuming a halfwave slot [1, 6, 7], Method of Moments (MoM) using waveguide Green's function [2], FDTD [8], and equivalent circuit model [3]. Compound coupling slots have often been utilised for feeding large arrays of longitudinal slots [9, 10], and an equivalent circuit model for centre inclined coupling slots has been found in [11]. The MoM is an accurate and suitable method for such problems. The method involves matching the tangential magnetic field at the internal and external interfaces using the waveguide and half space Green's function, respectively. The waveguide Green's function involves a double infinite summation over all the waveguide modes, and the evaluation using MoM and waveguide Green's function has been designated as the mode method in this paper. The evaluation of internal admittance of a compound slot using the mode method presents difficulties which have been simplified in [12] by modifying the slot geometry to that of a parallelogram instead of

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a rectangle. Alternative Green's functions have also been proposed [13, 14] to deal with the singularity in internal admittance due to a longitudinal current source in a waveguide for source and observation points in the same transverse plane. These Green's functions use the waveguide coordinate system and give alternative representation for the fields generated by a point source within the waveguide, which then need to be integrated over the slot aperture.

Recently, the internal admittance of a longitudinal slot has been evaluated by replacing the waveguide fed slot with its images formed due to reflections from the perfectly conducting waveguide walls, leading to an equivalent 2D infinite array of slots [15–19]. The magnetic field at the reference slot due to all the magnetic current images has been evaluated by adding the mutual admittances due to all the image currents. This method has been designated as the image method in this paper. In this method, the choice of mutual admittance evaluation technique depends upon the separation between the image and the reference slot, and suitable simplifying approximations may be employed depending upon the separation. Further, the external admittance gets reused as the self admittance of the slot for air filled waveguides and consequently the formulation does not land into any difficulties owing to singularity due to longitudinal magnetic current. Further, various convergence acceleration techniques may be used for accelerating the convergence of the series. The evaluation is intuitively simple as the mutual admittances are evaluated in the slot coordinates. The technique affords flexibility in the choice of mutual admittance evaluation technique best suited for the concerned slot image depending upon its contribution and overall impact on the total internal admittance as well as depending upon the specific slot geometry such as rectangular, elliptical slots and slot aperture distribution, e.g., with edge condition and different basis functions. An additional advantage is that the knowledge regarding the nature or the cutoff frequency of the waveguide modes is not needed in the analysis.

Because of all these advantages, the compound slots have also been analysed using the image method in this paper. The mutual admittance for the nearest slots has been evaluated using the single dipole or the double dipole approximation whereas the farther admittances have been evaluated using the simpler point dipole approximation [20, 21]. The θ-algorithm has been used for convergence acceleration of the summation of image admittances, and the effect of various algorithms on the convergence of resonant length with respect to the number of images has been studied. The results calculated using this method have been shown to agree well with other theoretical and measured results [1, 2, 7].

2. THEORY

The geometry of the slot is shown in Fig. 1. The slot is of length $2L$, width W, and the tilt with respect to the longitudinal axis of the waveguide is ψ. The offset from the centre line of the waveguide

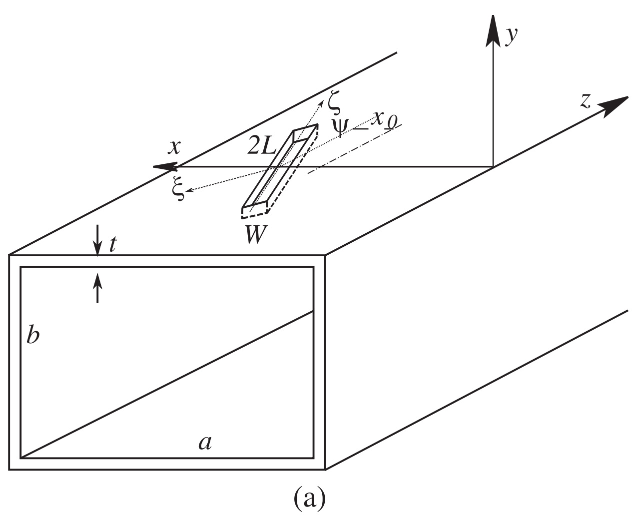

Figure 1. Geometry of the problem. (a) Waveguide fed compound slot. (b) Top view of the waveguide with compound slot.

is x_0, and the slot is assumed to be narrow, i.e., $W \leq L/5$. The waveguide walls are of thickness t and assumed to be perfect electric conductors. The slot electric field is along the ξ direction and has been assumed constant along the width of the slot for simplicity, although the transverse electric field distribution with the edge condition may also be incorporated just as easily into the analysis as done in [15]. The electric field has been assumed to be equiphase over the slot and has no component along the ζ direction. Proceeding as per the MoM formulation [2, 12, 19], the geometry has been divided into three regions: one internal to the waveguide, second within the wall thickness region of the slot that is also a small section of a waveguide cavity with cross-section $2L \times W$, and third, the free space region external to the slot. The unknown electric field $\mathrm{E}_{\mathrm{i(e)}}$ within the slot aperture at the internal (external) interface is expanded in P global sinusoidal basis functions with coefficients $e_{\mathrm{i(e)}}$.

$$\mathrm{E}_{\mathrm{i(e)}}(\mathbf{p}) = \sum_{\mathbf{p}=\mathbf{1}}^{\mathbf{P}} \mathbf{e}_{\mathrm{i(e)}} \sin\left[\frac{\mathbf{p}\pi}{\mathbf{2L}}(\mathbf{L}+\zeta)\right] \hat{\xi} \tag{1}$$

where $\hat{\xi}$ is a unit vector along ξ direction. The enforcement of tangential magnetic field continuity at the internal and external interfaces leads to coupled equations that when being solved they yield the solution to the unknown coefficients $e_{\mathrm{i(e)}}$. The magnetic field is found from the magnetic current $\mathrm{M}_{\mathrm{i(e)}} = \mathrm{E}_{\mathrm{i(e)}} \times \hat{\mathbf{n}}$, where $\hat{n}$ is the outward normal at each interface, and the corresponding Green's function for the internal, waveguide wall thickness cavity, or the external region needs to be employed for finding the magnetic field at each interface [2, 12]. The incident magnetic field for a dominant TE_{10} mode traveling along $\pm\hat{z}$ is given by

$$\mathbf{H}_{\mathrm{inc}} = \frac{\pi/a}{\mathrm{j}\eta k}\left[(\pi/a)\cos(\pi x/a)\,\hat{z} + \mathrm{j}\beta_{10}\sin(\pi x/a)\,\hat{x}\right] e^{\mp \mathrm{j}\beta_{10} z} \tag{2}$$

where $\eta = 120\pi$ is the free space wave impedance, k the free space wavenumber, and $\beta_{10} = \sqrt{k^2 - (\pi/a)^2}$ the propagation constant of the fundamental TE_{10} mode. The incident field component along the slot $\mathbf{H}_{\mathrm{inc}}^{\zeta}$ is given by the dot product

$$\mathbf{H}_{\mathrm{inc}}^{\zeta} = \mathbf{H}_{\mathrm{inc}}.(\hat{z}\cos\psi + \hat{x}\sin\psi) \tag{3}$$

Denoting the $\hat{\zeta}$ directed internal (external) magnetic fields at the slot aperture by $\mathbf{H}_{\mathrm{i(e)}}^{\zeta}$ and those within the wall thickness cavity at the internal (external) interface by $\mathrm{H}_{\mathrm{ci(e)}}^{\zeta}$, a pair of coupled equations are obtained

$$\mathrm{H}_{\mathrm{inc}}^{\zeta} + \mathrm{H}_{\mathrm{i}}^{\zeta} = \mathrm{H}_{\mathrm{ci}}^{\zeta} \tag{4}$$

$$\mathrm{H}_{\mathrm{e}}^{\zeta} = \mathrm{H}_{\mathrm{ce}}^{\zeta} \tag{5}$$

The unknown coefficients $e_{\mathrm{i(e)}}(p)$ are obtained by applying Galerkin's procedure and taking the inner product with S global sinusoidal testing functions $w_s = \sin[s\pi/(2L)(L+\zeta)]$. The inner product of H with w_s is denoted by $\langle H, w_s \rangle$ [2]. For a zero thickness slot, $\mathbf{E}_{\mathrm{i}} = \mathbf{E}_{\mathrm{e}}$ and the electric field coefficients are given by

$$[h_{\mathrm{inc}}] = [Y_{\mathrm{e}} - Y_{\mathrm{i}}]\,[e_{\mathrm{i(e)}}] \tag{6}$$

where $h_{\mathrm{inc}}(s) = \langle H_{\mathrm{inc}}^{\zeta}, w_s \rangle$, and $Y_{\mathrm{i(e)}}(p,s) = \langle H_{\mathrm{i(e)}}^{\zeta}, w_s \rangle$ is the internal (external) admittance element. $Y_{\mathrm{i(e,c)}}(p,s)$ for internal (external, cavity) admittance is given by

$$Y_{\mathrm{i(e,c)}}(p,s) = \iint_{\mathrm{slot}} \sin\left[\frac{s\pi}{2L}(L+\zeta)\right] \mathrm{d}\zeta\,\mathrm{d}\xi \iint_{\mathrm{slot}} G_{\mathrm{i(e,c)}} \sin\left[\frac{p\pi}{2L}(L+\zeta')\right] \mathrm{d}\zeta'\mathrm{d}\xi' \tag{7}$$

where $G_{\mathrm{i(e,c)}}$ is the internal (external, cavity) Green's function for magnetic field due to magnetic current. The details of analysis and the expressions for $Y_{\mathrm{e}}, Y_{\mathrm{c}}$ along with the external and cavity Green's function have been given in [2, 12]. The internal Green's function is the waveguide Green's function or the Stevenson's Green's function expressed as a double summation of the modes of waveguide [2, 12]. Since the Green's function is in waveguide coordinates, the ζ directed magnetic current within the slot aperture is split into its x and z components, and each component in turn generates both x and z

components of magnetic field. These are then combined to yield the total ζ component of the magnetic field below the slot aperture. The H_z component of magnetic field generated due to the M_z component of magnetic current involves the operation $\partial^2/\partial z^2[e^{-j\gamma_{mn}|z-z'|}]$. The evaluation of Y_{i} presents difficulties because of the singularity at $z = z'$ due to the double differentiation, and the slot region has to be divided into three regions as done in [2] or the slot geometry modified to a parallelogram [12]. This problem becomes more serious as one considers various aperture distributions along the slot transverse dimension or different slot shapes such as elliptical slots and crossed slots. To overcome this problem, alternative representations have been proposed using a virtual cavity [14] or taking Fourier transform in the direction along the guide and a Fourier series in one of the transverse directions [13].

Recently, the longitudinal slot in a waveguide has been analysed by using the equivalent image representation which replaces the original slot in a waveguide with infinite images of the slot in the homogeneous medium filling the waveguide (free space for air filled waveguides). These images are formed due to successive reflections in the perfectly conducting waveguide walls [15–19]. The magnetic field at the reference slot is then due to the mutual admittance from all the image slots. This technique allows working in the slot coordinates as one is no longer constrained to work in the waveguide coordinates for the evaluation of internal admittance. Also, an appropriate method for the evaluation of mutual admittance such as numerical integration or employing simple but accurate approximations [20–23] may be adopted for a particular slot geometry such as rectangular, elliptical, or crossed one, and for a particular slot aperture distribution. A suitable convergence acceleration algorithm helps in expediting the convergence of the summation of mutual admittances.

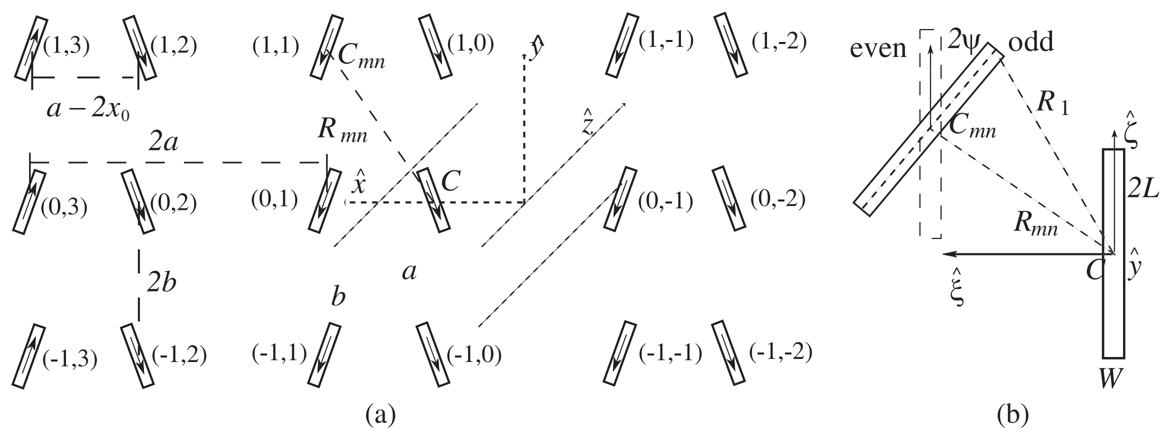

Figure 2. Image representation of a waveguide fed compound slot and geometry for mutual admittance evaluation. (a) Equivalent image representation of the slot. (b) Geometry for the evaluation of mutual admittance between the reference and image slot in an n = even column (dotted) and n = odd column (solid).

The image method has been adopted for the analysis of a waveguide fed compound slot in this paper. The image representation for a compound slot is shown in Fig. 2(a). The reference slot, waveguide, and slot images are shown in Fig. 2(a). The image number (m, n) has been written in parentheses beside the image slot, and the direction of magnetic current in the internal region has been shown for the reference slot and its images. Images within the same row (m = constant) are coplanar and parallel to the xz plane with the separation between each row being $2b$. The internal admittance is then the summation of the self admittance of the slot in free space, Y_{00}, and the mutual admittance between the reference slot and all the images.

$$Y_{\mathrm{i}}(p,s) = Y_{00} + \sum_{m=-\infty}^{\infty'} \sum_{n=-\infty}^{\infty'} Y_{mn} \tag{8}$$

where $Y_{mn}(p,s)$ is the mutual admittance between the image slot with centre at $C_{mn} \equiv (\xi_{mn}, y_{mn}, \zeta_{mn})$ in the slot coordinates and having aperture distribution $\sin[s\pi/2L(L+\zeta)]$, and the reference slot with centre $C \equiv (0, 0, 0)$ at the origin of the slot coordinate system and aperture distribution $\sin[p\pi/2L(L+\zeta')]$. The $(')$ symbol on the summations in Eq. (8) indicates that $m = n = 0$ corresponding to Y_{00} has been omitted from the summation as it has been added separately. For air filled waveguides Y_{00} is nothing but the external admittance $Y_e(p, s)$, and no separate evaluation is necessary. The evaluation of external admittance $Y_e(p, s)$ takes care of the singularity at the source using well known spectral [24] or spatial domain [2] techniques and does not lead to any difficulty in the internal admittance evaluation. The geometry used for the evaluation of mutual admittance Y_{mn} is shown in Fig. 2(b). The mutual admittance is calculated by using the half space Green's function and substituting for G in Eq. (7) by [20]

$$G = \frac{1}{j\eta k}\left(k^2 + \frac{\partial^2}{\partial\zeta'^2}\right)\frac{e^{-jkR}}{2\pi R} \tag{9}$$

where R is the distance from any point $(\xi', 0, \zeta')$ on the reference slot to any point (ξ, y, ζ) on the image slot. The reference slot and its image are parallel for $n =$ even columns, e.g., $n = 0, \pm 2, \pm 4 \ldots$, with the image slot centre $C_{mn} \equiv (na, 2mb, 0)$ in the slot coordinates of Fig. 2(b), and their axes are co-planar with angle $\psi' = 0$ between them. For $n =$ odd columns, e.g., $n = \pm 1, \pm 3 \ldots$, the reference slot and image axes are generally non-coplanar for $m \neq 0$ with the image slot centre $C_{mn} \equiv \{[(n-1)a + 2x_0]\cos\psi, 2mb, [(n-1)a + 2x_0]\sin\psi\}$ in the slot coordinates, and the angle between the slot and the projection of its image on xz plane is $\psi' = 2\psi$ as shown in Fig. 2(b). The mutual admittance may be evaluated by considering one slot along the ζ axis and the other tilted by 2ψ for an odd column slot or by 0 (parallel slots neglecting the effect of width for narrow slots) for even column slot images. Thus, the analysis becomes independent of the waveguide coordinate system but depends upon the slot co-ordinates, thereby simplifying the analysis. One can also see that one does not need to find the mode pattern or the cutoff frequency for modes when the mutual admittances are simply added as in Eq. (8). The mutual admittance has been evaluated using the more accurate single dipole approximation or double dipole approximation (for co-planar slots) [19, 20] for images with centre-centre separation of R_{mn} satisfying the criteria [19]

$$\frac{(L/p)^2}{\lambda\, R_{mn}} \geq \mu \ \text{ and } \ \frac{(L/s)^2}{\lambda\, R_{mn}} \geq \mu \tag{10}$$

where λ is the free space wavelength, and μ is a parameter whose value is chosen to optimise accuracy and speed. A smaller value of μ implies more images whose mutual admittances are evaluated using more accurate but slower techniques such as numerical double integration, single dipole approximation, and double dipole approximation [20]. The double dipole approximation may be employed for images in even columns and odd column images in the $m = 0$ row, where the reference and image slots become parallel to each other. For farther images, where the criteria in Eq. (10) are not met, the faster point dipole approximation [19, 20] has been employed. The results calculated in this work are with $\mu = 0.03$ and are shown to give satisfactory results. Mutual admittances have been added for images up to M rows and N columns on either side of the reference slot. The convergence of the sum of mutual admittances has been accelerated using various algorithms in the next section. It is shown in the next section that the results obtained using the θ-algorithm [25] converge faster with the number of images than other convergence acceleration algorithms such as Shanks' transformation [15], Chebyschev-Toeplitz algorithm [16], and Levin's t-transform [18]. The θ-algorithm has been shown to have better convergence properties than Shanks' transformation for free space periodic Green's function [25]. Hence, the θ-algorithm has been used for convergence acceleration in this paper. A brief overview of the θ-algorithm is given next.

For a series $a_1, a_2, \ldots a_n$, if S_i represents the partial sum of the first i terms of the series, a modified series S_i^θ having accelerated convergence is given by the θ-algorithm. The terms S_i^θ for a particular iteration are given by

$$S_i^\theta = S_{i+1} + \frac{(S_{i+2} - S_{i+1})(r_2 - r_1)}{r_2 - 2r_1 + r_0} \tag{11}$$

$$r_j = r_{j+1}^* + \frac{1}{S_{i+j+1} - S_{i+j}} \tag{12}$$

This algorithm can be used iteratively, and r_{j+1}^* terms are the corresponding r_{j+1} terms from the previous iteration, being 0 for the first iteration. For the compound slot analysis, it was also seen that better accuracy is obtained by summing the images in even columns (n = even) and the images in odd columns (n = odd) separately and then summing up the two. It may be noted that the summation of even column admittances is independent of slot tilt ψ and needs to be calculated only once for a particular slot length. The results obtained using the present work have been compared with theoretical results obtained using the modified analytical formulation [7], results from the mode method [2], and measured results. The present method is shown in the next section to yield results that are in excellent agreement with all these results.

3. NUMERICAL RESULTS

The results obtained using the image method presented in this work and the θ-algorithm for convergence acceleration have been compared in Fig. 3 with Shanks' transformation, Levin's t-transform, and the Chebyschev-Toeplitz algorithm for convergence acceleration. The resonant length has been calculated using these convergence acceleration algorithms for a compound slot in a standard X-band waveguide with a tilt angle $\psi = 20$ degrees with respect to the axis of the waveguide at a frequency of 9.3 GHz and relative slot offset $x_0/a = 0.11$. The parameter $\mu = 0.03$ has been taken for all the results in this paper. This parameter value results in about 36 mutual admittance evaluations using the single dipole approximation for a half height waveguide and nine basis functions in the slot aperture ($P = S = 9$). This more rigorous evaluation satisfying the criteria in Eq. (10) is for some images in row or column $m(n) \leq 3$ and for basis (testing) function $p(s) \leq 2$. All other mutual admittances have been evaluated with the point dipole approximation leading to accurate results as shown in the results to follow. For the results in Fig. 3, the resonant length obtained by taking 96 rows and columns on either side of the slot have been taken as reference for each technique, and the absolute error for various maximum image row or column numbers has been calculated with respect to this reference resonant length. It has been shown that the θ-algorithm converges faster as the number of images increases and gives better accuracy than the other techniques. The Shanks' and Levin's t-transforms give identical error for the first iteration of the respective algorithm. The Chebyschev-Toeplitz algorithm gives the worst convergence of all the techniques studied. Hence, the θ algorithm has been used in the subsequent results presented using the proposed image method.

The proposed image method has been validated by comparing the waveguide fed compound slot results obtained using this method with those obtained from theoretical results such as the mode method [2] and analytical formulation [7] as well as with measured results [1] for various offsets and tilt angles. The resonant length calculated from this technique for a compound slot in a full height and a half height waveguide is compared in Fig. 4 with measured results [1] and resonant length calculated

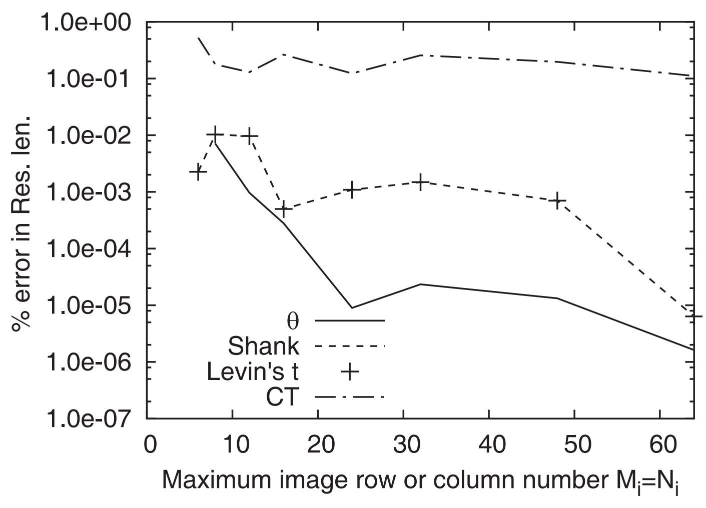

Figure 3. % error in resonant length with the maximum number of image row or column for various convergence acceleration algorithms.

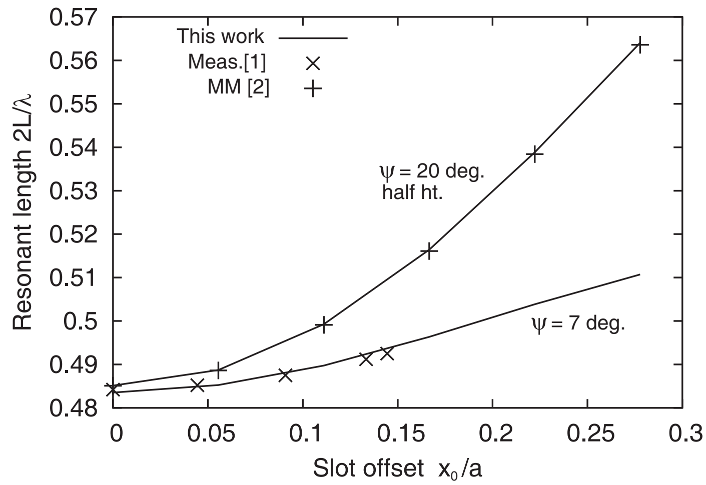

Figure 4. Resonant length with slot offset compared with measured [1] and mode method (MM) results [2].

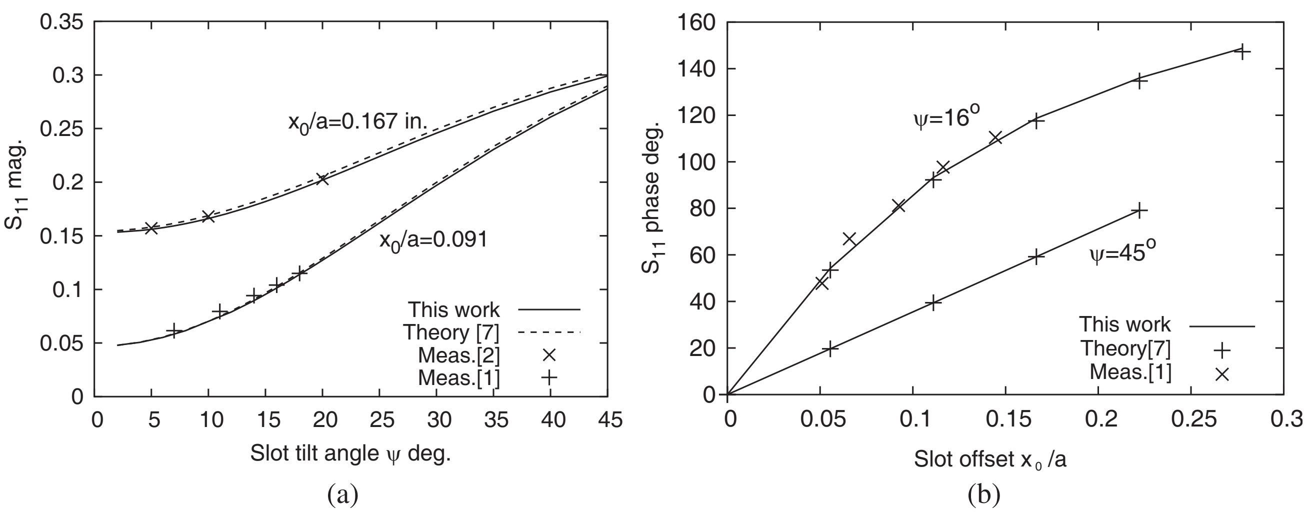

Figure 5. Comparison of magnitude and phase of backscattered wave at resonance with modified analytical theory [7] and measurement. (a) Magnitude of backscattered wave at resonance with tilt angle. (b) Phase of backscattered wave at resonance with tilt angle.

from the mode method [2]. The slot in a full height standard X-band waveguide was tilted at an angle $\psi = 7$ degrees, and the resonant length calculated from this method shows excellent agreement with Maxum's measured data for $\psi = 7°$ [1]. Maxum's measured data are for rounded end slots, and the equal area approximation [24] has been applied by considering a rectangular slot with its length shortened by 0.215 W for mutual admittance evaluation. The resonant length calculated from this method is also compared in Fig. 4 with resonant length calculated from the mode method [2] for a 20 degree tilted slot in a half height waveguide with thickness $t = 0.03$ in., $b = 0.2$" and other dimensions as per standard X-band waveguide. Both the results agree very well with each other as shown in Fig. 4.

The magnitude $|S_{11}|$ and phase $\angle S_{11}$ of the backscattered wave from the slot discontinuity at resonance are plotted in Fig. 5(a) and Fig. 5(b), respectively. The magnitude has been plotted against the tilt angle for a resonant slot in a standard X-band waveguide at a relative offset of $x_0/a = 0.91$, and the S_{11} magnitude from this work has been compared with that from [7] using modification to Maxum's analytical formulation as well as compared with Maxum's measured results [1]. The S_{11} magnitude has also been compared with the measured results from [2] and with the modified analytical

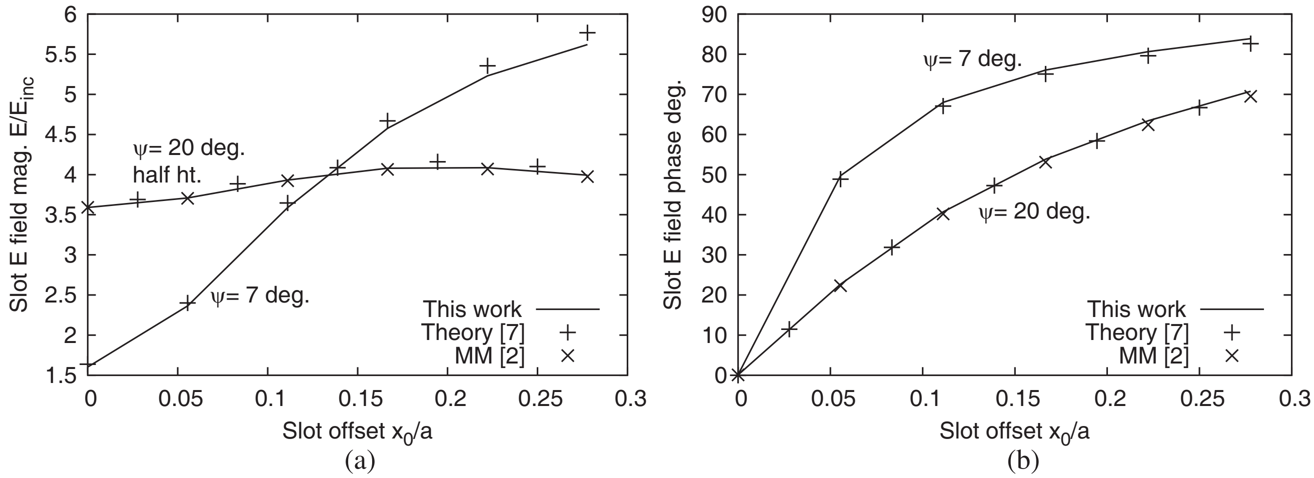

Figure 6. Variation of slot aperture electric field with slot offset. Comparison with modified analytical method [7] and mode method (MM) results [2]. (a) Aperture electric field magnitude with slot offset. (b) Aperture electric field phase with slot offset.

method results [7] for slots at a relative offset $x_0/a = 0.167$ in a standard X-band waveguide. The S_{11} magnitude from the proposed method is seen in excellent agreement with the theoretical as well as measured results. S_{11} phase obtained from this method is compared in Fig. 5(b) with the results from modified analytical method [7] for a tilt of 45° and with measured results [1] for a tilt angle of 16° in a standard X-band waveguide. The agreement in both the cases is seen to be excellent.

The aperture electric field used for calculating the radiated power and the radiation pattern of an array has been calculated using the present method, and the aperture field normalised with respect to the incident field is compared in Fig. 6(a) with the results from modified analytical method [7] and the mode method [2] for a slot tilt of 7 degrees in a standard X-band waveguide and for $\psi = 20$ degrees tilt in a half height waveguide. The phase from this method for a 7 degree tilted slot and a 20 degree tilted slot has been compared with the phase calculated from modified analytical formulation results [7] and the mode method results [2] in Fig. 6(b). The aperture electric field magnitude and phase calculated from this method are in excellent agreement with that calculated from other techniques, thus validating the proposed image method.

4. CONCLUSION

The image method has been proposed for the analysis of arbitrarily inclined compound slots fed by a rectangular waveguide. The method relies on calculating the internal admittance by adding up the mutual admittances between the reference slot and its images and is inherently efficient and flexible as the most suitable method for mutual admittance evaluation may be selected depending upon the separation between the slot and the image as well as depending upon the particular slot shape. It has been shown that the self admittance of the slot for an air filled waveguide is the same as the external admittance of the slot and need not be calculated again. It has also been shown that unlike the mode method, this method does not run into any issues encountered with mode method analysis when calculating fields from longitudinally directed magnetic currents for points in the same transverse plane. In the proposed method, the mutual admittance between the slot and its image has been calculated in slot coordinates and not in waveguide coordinates, thereby simplifying the analysis. The method does not presuppose any knowledge of the waveguide modes or their propagation constant thereby providing an efficient solution for configurations permitting image formation but requiring tedious calculations for propagation constant, such as inhomogeneously filled waveguides. The θ-algorithm has been used for accelerating the convergence of the series of mutual admittances and has been shown to yield superior convergence properties compared to other techniques investigated. The results calculated using this method for various slot offsets and tilts have been shown to have excellent agreement with measured results and those calculated from analytical formulation and from the mode method. The proposed method can be used in the efficient design of waveguide fed compound slot arrays with better control of aperture excitation for shaped beam applications and fuze antennas. The method can be extended to other slot geometries such as elliptical inclined slots, crossed slots, compound slot couplers, and for various slot aperture distributions such as with edge singularity in the transverse direction.

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