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Variation of the Shape Parameter of K-Distribution for Sea Clutter with the Spatial Correlation of Sea Surface

Junghoon Park[1], Donghoon Kim[2], Donghwan Kim[3], and Sanghoek Kim[4, *]

Abstract—In this study, the physical relationship between the shape parameter ν of the K-distribution and the spatial correlation of a sea clutter signal received with a radar is demonstrated through simulation results. The spikiness of the sea clutter is well modeled by the shape parameter ν of the K-distribution. According to a well-known empirical formula, the shape parameter ν changes with the radar resolution based on a constant power-law relation. However, as with most empirical findings, this finding is valid only for the environmental conditions under which the formula was developed. In other words, the existing power-law models for the shape parameter of the K-distribution for sea clutter do not consider the relative ratio of the cross-range resolution R_c to the spatial decorrelation length R_{dec} of the sea surface. Our study investigates this relation using statistical simulations based on the principle of superposition for backscattered signals that represent sea clutter within a resolved area on the sea surface. This study shows that the constant factor α in the power-law relation must be modified to a function of the ratio R_c/R_{dec}. The findings of this study will be useful for the evaluation of detection performance in designing radar systems operating in the maritime environment.

1. INTRODUCTION

The local power of sea clutter does not follow the Rayleigh distribution as land clutter does, but follows the gamma distribution instead [1]. Statistically, this indicates that the power of sea clutter far beyond the mean value appears more occasionally, or *spikier*, than that of land clutter. Consequently, the false alarm rate is higher in the sea environment with the same threshold criteria [1]. Thus, it is necessary to adjust the threshold criteria of the constant false alarm rate according to the radar configuration and clutter environment, based on a precise modeling of sea clutter, to suppress the false alarm rate in the sea environment. In addition to the gamma distribution modeling of the local power, a *speckle* component of sea clutter is accommodated to model the sea clutter with the K-distribution [1]. The shape parameter ν of the K-distribution is a statistical parameter that describes the spikiness of the sea clutter environment. It is very useful to characterize the property of sea clutter. Generally, a smaller value of ν indicates a spikier sea clutter condition.

Intuitively, the shape parameter ν should depend on the radar resolved area A_c. For a larger radar resolved area A_c, the sea clutter signal over such a larger area is summed up in each range bin of the radar. As an extreme example, if the radar resolved area A_c becomes very large, the central limit theorem states that the clutter signal summed up as such should resemble the Rayleigh distribution, which is a special case of the K-distribution for which ν is infinite. This suggests that ν would increase with the increase in the radar resolved area A_c.

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* Corresponding author: Sanghoek Kim (sanghoek@khu.ac.kr).
[1] Department of Electronic Engineering, Kyung Hee University, Gyeonggi-do, Republic of Korea. [2] Department of Electronic Engineering, Kyung Hee University, Gyeonggi-do, Republic of Korea. [3] Agency for Defense Development, Republic of Korea.
[4] Department of Electronic Engineering, Kyung Hee University, Gyeonggi-do, Republic of Korea.

Indeed, empirical models for the shape parameter of the K-distribution for sea clutter [2–4] demonstrate that ν increases in a power-law relation with the radar resolved area $(A_c)^{\alpha}$ with α being a constant. Similarly, the dependencies of ν on other environmental parameters are empirically derived from the measurement data. For example, for a low grazing angle ($\theta_{gr} < 10°$), one of the most popular models of the shape parameter ν is

$$\log_{10}(\nu) = \frac{2}{3}\log_{10}(\theta_{gr}) + \frac{5}{8}\log_{10}(A_c) - k_{pol} - \frac{\cos(2\phi_{sw})}{3} \tag{1}$$

where θ_{gr} is the grazing angle in degrees, A_c the radar resolved area, k_{pol} a polarization-dependent parameter, and ϕ_{sw} the aspect angle with respect to the swell direction in radians [2]. The radar resolved area A_c is the product of the range resolution R_r and the cross-range resolution R_c; $A_c = R_r R_c$ (Refer to Fig. 1). From this equation, we can observe that the shape parameter is proportional to the cross-range resolution R_c with a constant power-law relationship as

$$\nu \propto (R_c)^{5/8} \tag{2}$$

For instance, the factor α is 5/8 in the model of [2]. In other reports, different values of α such as 0.49 and 0.79 were proposed [3]. A more recent model of the shape parameter included the effect of the wind speed or sea state [4]. However, all the models proposed so far suggested a constant power-law relation between the cross-range resolution R_c and the shape parameter ν.

This work suggests that the factor α in the power-law relation is not a constant, but rather changes with the relative ratio between the cross-range resolution R_c and the spatial decorrelation length R_{dec}. By definition, the decorrelation length is the point at which the autocorrelation reduces to $1/e$. This can

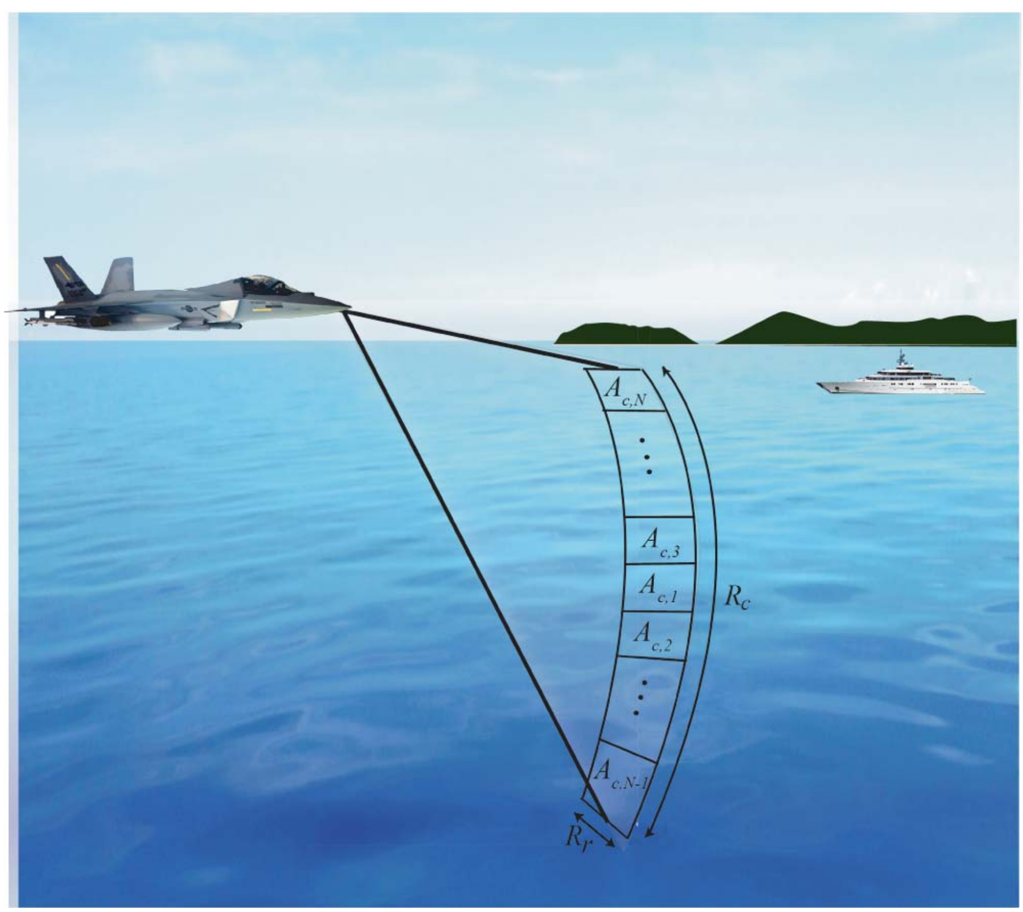

Figure 1. Generated spatially and temporally correlated patches.

be intuitively argued from two extreme conditions. For the first extreme case, suppose that the cross-range resolution R_c is much larger than the decorrelation length R_{dec}. If we further double the range $R'_c = 2R_c$, the clutter signal from A'_c is the summation of two *independent* clutter signals, each from an area of A_c and with the shape parameter ν. As the summation of two identical and independent gamma random variables, each having the shape parameter ν, has the shape parameter ν' twice as large as ν, i.e., $\nu' = 2\nu$, the shape parameter ν must be simply linearly proportional to R_c. That is, α should be unity. In the other extreme case, if the cross-range resolution R_c is much smaller than the decorrelation length R_{dec}, the statistical property of the clutter signal would rarely change with a change in the radar resolved area A_c. Therefore, the shape parameter ν should not vary with A_c, and hence, α should be zero.

This work will rigorously demonstrate the variation of the power factor α according to the relative value of R_c to that of R_{dec} from a computationally generated sea clutter model.

2. GENERATING CLUTTER SIGNALS WITH VARIOUS VALUES OF A_c

Figure 1 illustrates the case where a radar is looking down the sea surface. The radar resolved area A_c is divided into multiple patch areas $A_{c,i}$. The clutter signal in a range bin at a moment t for the radar resolved area A_c, denoted by $s(A_c; t)$, is the summation of the clutter signals from each patch area $A_{c,i}$ expressed as follows:

$$s(A_c; t) = \sum_{i=1}^{N_{patch}} s(A_{c,i}; t) \tag{3}$$

where N_{patch} is the total number of patches within A_c.

The clutter signals from each patch area $s(A_{c,i}; t)$ follow the gamma distribution in their local power. Moreover, they are spatially and temporally correlated. The process of generating temporally correlated gamma-distributed signals for sea clutter is presented in [5]. The process of generating both temporally and spatially correlated gamma-distributed signals is illustrated in Fig. 2. First, given the total number of patches N_{patch}, the number of time samples N_{time}, the spatial decorrelation length R_{dec}, and the temporal decorrelation length T_{dec}, we determine a desired correlation function to generate the correlated gamma signals. Among the several existing correlation models for sea clutter [4, 6, 7], we

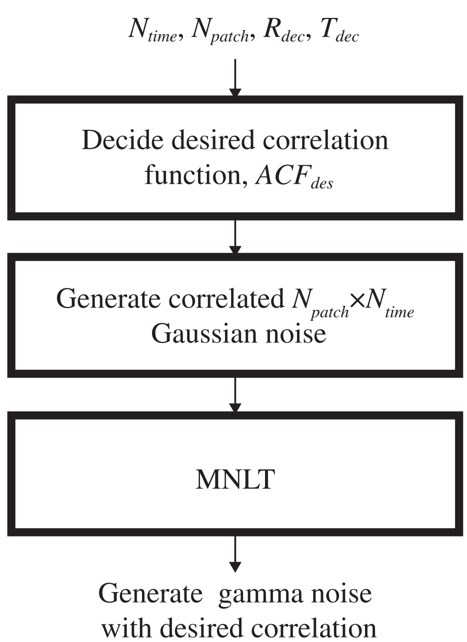

Figure 2. Block diagram of the generation of correlated signals.

choose an exponential model to represent the correlation as

$$ACF_T(t) = \exp\left(-\frac{t}{T_{dec}}\right) \tag{4}$$

$$ACF_R(x) = \exp\left(-\frac{x}{R_{dec}}\right), \tag{5}$$

in time and space domains, respectively. When identical and independent Gaussian noise of size N_{time} following $N(0, \sigma^2 = 1)$ passes through a one-dimensional filter designed using the method presented in [5], temporally correlated Gaussian signals are obtained. The memoryless nonlinear transformation (MNLT) is then applied to this Gaussian signal to obtain a gamma distribution with the desired autocorrelation function of (4). This work extends the dimension of the filter to two to generate gamma variables that are temporally and spatially correlated. As the identical and independent Gaussian noise of size $N_{time} \times N_{patch}$ following $N(0, \sigma^2 = 1)$ passes through the filter and the MNLT, the gamma-distributed clutter signal with the desired autocorrelation function is obtained.

We checked the autocorrelation of the generated clutter signals to verify whether the generated signals follow the desired autocorrelation. The decorrelation length R_{dec} is 10 m with a total number of patches $N_{patch} = 10{,}000$ and a total number of time samples $N_{time} = 2^{15}$. The generated signals follow the desired autocorrelation function well, both spatially and temporally, as shown in Fig. 3.

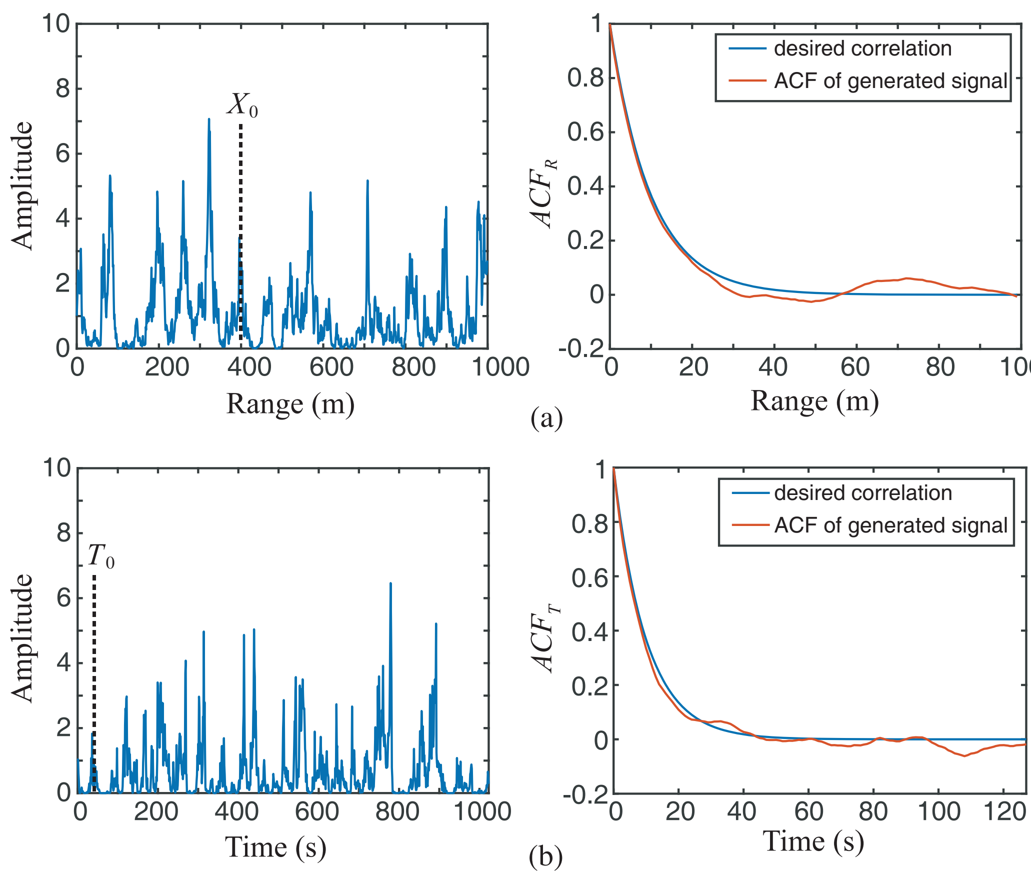

Figure 3. (a) Signal generated at a specific moment T_0; (b) Signal generated at a specific range X_0.

From Eq. (3), the clutter signal in a range bin can be obtained by the summation of the clutter signals from these patches. Assuming the patch area to be a fixed small value, the clutter signal with an arbitrary radar resolved area A_c can be synthesized by the summation accordingly. Regarding the empirical works [2–4], the shape parameter of this clutter signal is known to follow a constant power-law relationship with the cross-range resolution $\nu \propto R_c^\alpha$. However, in the following section, we demonstrate that the power factor α is not a constant, but that it changes due to the relative ratio of the cross-range resolution to the decorrelation length.

3. SIMULATION RESULTS

After generating the patches, that is, the clutter signals following the gamma distribution with the desired spatial and temporal correlation function, we add the signals and check the variation of the shape parameter. The MATLAB built-in function, *gamfit*, is employed to determine the shape parameter of the generated clutter signals. The resultant shape parameters are presented in Fig. 4 for various values of R_{dec}, ν, and R_c. As R_c/R_{dec} increases, the dependency of the shape parameter on R_c changes. In other words, the exponential factor α, which indicates the slope of the nonlinear curve, varies for the different values of R_c and R_{dec} and is not constant as indicated in the previous studies. We fit the nonlinear curve into three-piecewise linear asymptotic curves via least square differences to quantify this dependency. Each linear section is set to have the fixed slopes 0, 5/8, and 1. The slope of 5/8 is included to account for the previous empirical results [2].

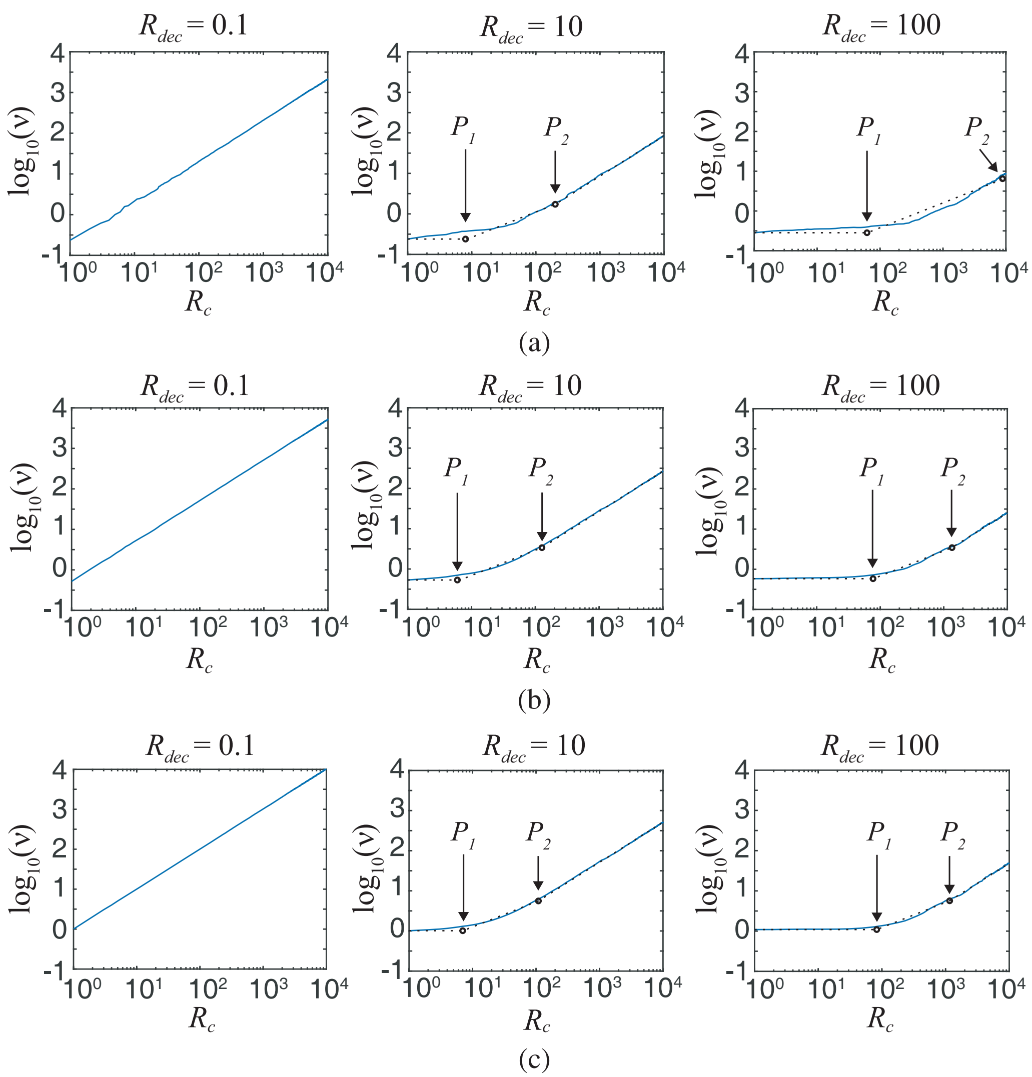

Figure 4. (a), (b), (c) Shape parameter variation with R_{dec} and R_c for the shape parameters $\nu = 0.1$, $\nu = 0.5$, and $\nu = 1$, respectively.

In the figure, P_1 is the point below which the slope of the plot is near-zero. P_2 denotes the point beyond which the slope is near-unity. From the results, the points P_1 and P_2 change according to the decorrelation length R_{dec}. When $R_{dec} = 10$, P_1 lies in the range $6 \sim 8$ and P_2 lies in the range $100 \sim 200$. When $R_{dec} = 100$, P_1 and P_2 lie in the ranges $60 \sim 80$ and $1500 \sim 9000$, respectively. Combining these two results, when the normalized R_c values with R_{dec} are in the ranges $[R_c/R_{dec} < 0.6 \sim 0.8]$, $[0.6 \sim 0.8 < R_c/R_{dec} < 10 \sim 90]$, and $[R_c/R_{dec} > 10 \sim 90]$, the slopes roughly correspond to 0, 5/8, and 1, respectively.

This is consistent with the experimental condition under which the power factor of 5/8 in the empirical formula [2] was proposed. Considering that the R_c value of the radar used in [2] was $100 \sim 800\,\mathrm{m}$, and the typical range of the sea wave R_{dec} for vertical polarization at a low grazing angle is $7 \sim 15\,\mathrm{m}$ [7], the normalized R_c/R_{dec} was in the range $[6 < R_c/R_{dec} < 110]$. Another empirical result reported the power factors of 0.46 and 0.79 when $[10 < R_c/R_{dec} < 60]$ [3]. The range of R_c/R_{dec} for the experimental data in [2, 3] falls into the region in which the slope is greater than zero in our numerical results. However, as the radar cross-range resolution decreases in modern radar technology, a new formula for the shape parameter is required in the future. This formula should account for the fact that the dependency of the shape parameter on the radar resolution varies according to the relative value of R_c to R_{dec}.

4. CONCLUSION

We analyzed the variation in the shape parameter of the K-distribution for sea clutter with spatial correlation. The clutter signals following the gamma distribution with a specific spatial and temporal autocorrelation function were generated. From their linear superposition, the clutter signal with an arbitrary radar resolved area was generated. Consequently, in contrast to the conventional empirical models, the shape parameter did not follow the constant power-law relation with the radar resolution. Instead, the power factor α changed with the relative ratio of the cross-range resolution R_c to the spatial decorrelation length R_{dec}. This shows the limitations of the existing empirical results and necessitates the development of a new formula for the shape parameter to accommodate broader environmental and radar conditions for sea clutter.

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