Dispersion of Elastic Waves in an Asymmetric Three-Layered Structure in the Presence of Magnetic and Rotational Effects

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Abstract—The present paper investigates the propagation and dispersion of elastic surface waves in an asymmetric inhomogeneous isotropic three-layered plate in the presence of magnetic field and rotational effects. The skin layers are exposed to an external magnetic field force while the core layer is assumed to be in a rotational frame of reference, which are perfectly bounded together with free-ends conditions. The resultant displacements and shear stresses in the respective layers are derived analytically together with the general dispersion relation. Further, the general dispersion relation is analyzed for some physical cases of interest. Finally, the effects of the magnetic field, rotation, and electric field on the propagation and dispersion of the present model are presented graphically.

1. INTRODUCTION

Wave propagation in elastic bodies has been extensively investigated long ago owing to its various applications in many fields of sciences and engineering, including civil engineering, geology, earthquakes, modern aerospace, automotive industries, and meta-materials among others, see [1–3]. Besides, some of these studies were incorporated with external forces that affect the propagation of waves in the respective bodies under consideration. The external forces can be due to the presence of magnetic field forces [4–6], thermal or temperature presence [7–9], and gravitational force [10]. Furthermore, other effects that play a part in an elastic wave propagation include the rotational effects [11, 12], thermal stress [13], initial stress [14], voids presence [15], damping effects [16], and various structural and material discontinuities [17, 18] just to mention a few. Of particular interest, multilayered elastic structures [19, 20] have also been greatly examined in the literature due to their various applications such as in sandwich plates, composite rods, layered laminates, photovoltaic panels, and beams considerations. For instance, the harmonic wave determination in elastic sandwich plates was presented by Lee and Chang [21], while the dispersion of elastic waves in a three-layered laminate which was considered to be inhomogeneous was given by Kaplunov et al. [22]. Other studies comprise theories governing the propagations in layered photovoltaic panels and laminated glass [23], buckling and bending analysis of vibrating laminated composite and sandwich beams presented by Sayyad and Ghugal [24], the determination of lowest motion modes of an elastic beam with varying layers given by Sahin et al. [25], a layer-wise finite element analysis for laminated and composite plates by Belarbi et al. [26] and for sandwich five-layered composite plate by Shishehsaz et al. [27]. Furthermore, regarding the influence of some external forces on the propagation of waves in layered media, Jiangong et al. examined the propagation of waves in magneto-electro-elastic inhomogeneous hollow cylinders in [28], and the same investigation was carried out by Bin et al. [29] on elastic plates. In [30] by Mandi et al., an analytic solution of Love wave travelling in a double layered media placed over an inhomogeneous half-space

layer was determined in addition to the dispersion relation analysis, see also [31–36] and the references therein for relevant studies.

However, in the present paper, the propagation and dispersion of elastic surface waves in an asymmetric inhomogeneous isotropic three-layered plate in the presence of magnetic field and rotation are investigated. The model is to be governed by an anti-plane motion with appropriate prescribed interfacial and boundary conditions within and outside the plate under consideration. The skin layers will be assumed in the presence of a magnetic field with the core layer in a rotational frame of reference. The respective displacements, shear stresses, and general dispersion relation will be determined and analyzed. Some physical cases of interest will be investigated from the general dispersion relation. Additionally, the arrangement of the paper goes as follows. Section 2 presents the problem formulation, and the problem-solution is given in Section 3. Section 4 reveals the Rayleigh-Lamb dispersion relation, and Section 5 gives numerical results and discussion. Section 6 concludes the study.

2. FORMULATION OF THE PROBLEM

The general governing equation of motion in the presence of a magnetic field force in a homogenous elastic media is considered to be [11, 12, 34–36]

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \vec{F}_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad i = j = 1, 2, ..., \tag{1}$$

where ρ is the density, and σ_{ij} is the stress-strain relation given by

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}, \tag{2}$$

with λ and μ being the Lame's elastic constants, δ_{ij} the Kronecker delta, and ε_{ij} the strain-displacement relation given by

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

Furthermore, $\vec{F}_i$ is the magnetic force obtainable from the linearized Maxwell equations for electromagnetic field of a conducting medium given by

$$Curl\vec{H} = \vec{J} \times \epsilon_0 \frac{\partial \vec{E}}{\partial t},$$

$$Curl\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t},$$

$$Div\vec{H} = 0, \quad Div\vec{E} = 0,$$

$$\vec{E} = \mu_0 \left(\frac{\partial \vec{u}}{\partial t} \times \vec{H} \right),$$

$$\vec{h} = Curl(\vec{u} \times \vec{H}),$$

as follows [34–36]

$$\vec{F}_i = \mu_0 H_0^2 \left(\frac{\partial e}{\partial x_i} - \epsilon_0 \mu_0 \frac{\partial^2 u_i}{\partial t^2} \right), \tag{3}$$

where $\vec{H} = H_0 + \vec{h}$, $\vec{h}$ is the induced magnetic field, ϵ_0 the electric field permeability, μ_0 the magnetic permeability, and $e = \Delta \vec{u}$, Δ the Laplacian operator. Additionally, in the rotating frame of reference, the acceleration on the right hand side of Eq. (1) takes the following form [36]

$$\frac{\partial^2 u_i}{\partial t^2} \rightarrow \frac{\partial^2 u_i}{\partial t^2} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2\vec{\Omega} \times \frac{\partial \vec{u}}{\partial t}, \tag{4}$$

where Ω is the angular velocity of the medium, and $\vec{\Omega} \times (\vec{\Omega} \times \vec{u})$ and $2\vec{\Omega} \times \frac{\partial \vec{u}}{\partial t}$ are the centripetal and Coriolis accelerations, respectively.
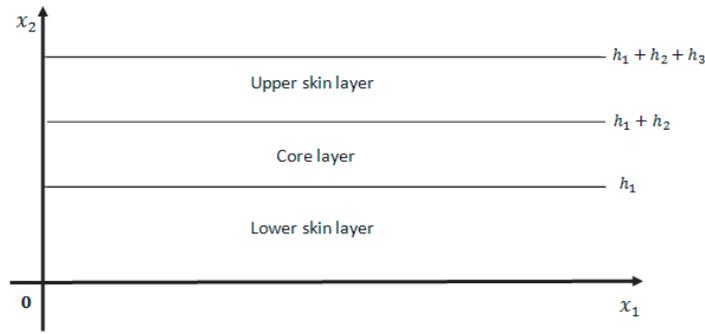
Figure 1. Asymmetric three-layered plate.

However, the anti-plane shear motion [22] of an asymmetric isotropic three-layered plate as shown in Figure 1 is considered; consisting of the lower skin layer of thickness h_1, the core layer of thickness h_2, and the upper skin layer of thickness h_3 (the skin layers are considered to be of the same material). Also, the skin layers are subjected to an external force under the influence of the magnetic field $\vec{F}$, while the core layer is assumed in a rotational frame of reference.

Let (x_1, x_2, x_3) be a Cartesian coordinates system. Then, the anti-plane motion [22] in (x_1, x_2) plane with displacements (u_1, u_2, u_3) such that

$$u_1(x_1, x_2, t) = 0, \qquad u_2(x_1, x_2, t) = 0, \qquad u_3(x_1, x_2, t) = u, \tag{5}$$

is considered, where t is the time variable.

The equations of motions in the skin layers under the influence of the magnetic force from Eqs. (1)–(3) are given by

$$\frac{\partial \sigma_{13}^s}{\partial x_1} + \frac{\partial \sigma_{23}^s}{\partial x_2} = (\rho_s + \epsilon_0 \mu_0^2 H_0^2)\frac{\partial^2 u^s}{\partial t^2}, \quad s = l, u, \tag{6}$$

for $s = l, u$ standing for the lower and upper skin layers, respectively, and $\rho_l = \rho_u$.

Moreover, the equation of motion in the core layer with rotational effect from Eqs. (1), (2), and (4) is given by

$$\frac{\partial \sigma_{13}^c}{\partial x_1} + \frac{\partial \sigma_{23}^c}{\partial x_2} = \rho_c \left(\frac{\partial^2 u^c}{\partial t^2} - \Omega^2 u^c \right), \tag{7}$$

where $\vec{\Omega} = \Omega(0, 0, 1)$, that is, the core layer medium is assumed to rotate along x_3 axis; the superscript c denotes the core layer.

The shear stresses σ_{j3}^r, in Eqs. (6), (7) are obtained from Eq. (2) to be

$$\sigma_{j3}^r = \mu_r \frac{\partial u^r}{\partial x_j}, \quad j = 1, 2, \quad r = c, l, u, \tag{8}$$

where $\mu_l = \mu_u$.

The continuity conditions between the layers are prescribed as follows

$$\text{(i) } u^l(x_1, x_2, t) = u^c(x_1, x_2, t), \text{ at } x_2 = h_1,$$

$$\text{(ii) } \sigma_{23}^l(x_1, x_2, t) = \sigma_{23}^c(x_1, x_2, t), \text{ at } x_2 = h_1,$$

$$\text{(iii) } u^c(x_1, x_2, t) = u^u(x_1, x_2, t), \text{ at } x_2 = h_1 + h_2,$$

$$\text{(iv) } \sigma_{23}^c(x_1, x_2, t) = \sigma_{23}^u(x_1, x_2, t), \text{ at } x_2 = h_1 + h_2,$$

$$\tag{9}$$

and the traction-free boundary conditions on the outer faces of the skin layers as

$$\text{(v) } \sigma_{23}^l(x_1, x_2, t) = 0, \text{ at } x_2 = 0,$$

$$\text{(vi) } \sigma_{23}^u(x_1, x_2, t) = 0, \text{ at } x_2 = h_1 + h_2 + h_3.$$

$$\tag{10}$$

3. SOLUTION OF THE PROBLEM

In this section, the formulated problem will be solved by first reducing the given partial differential equations to suitable ordinary differential equations via the use of the harmonic wave solution assumptions. The related displacements and shear stresses in the respective layers will also be determined using the prescribed continuity and traction-free boundary conditions.

3.1. The Dynamics of the Skin Layers with Magnetic Effects

Substituting the stress equation given in Eq. (8) into Eq. (8), the following wave equation is obtained

$$\frac{\partial^2 u^s}{\partial x_1^2} + \frac{\partial^2 u^s}{\partial x_2^2} = \left(\frac{1}{c_s^2} + \frac{\epsilon_0 \mu_0^2 H_0^2}{\mu_s} \right) \frac{\partial^2 u^s}{\partial t^2}, \quad s = l, u, \tag{11}$$

where $c_s = \sqrt{\frac{\mu_s}{\rho_s}}$ is the transverse shear velocity for the skin layers.

Furthermore, a harmonic wave solution along x_1 direction of the following form is assumed

$$u^s(x_1, x_2, t) = v^s(x_2) e^{ik(x_1 - ct)}, \quad s = l, u, \tag{12}$$

where $i = \sqrt{-1}$, k is the wave number, and c is the phase speed velocity. Thus, Eq. (11) is reduced to

$$\frac{d^2 v^s}{dx_2^2} + k^2 \left(\frac{c^2}{c_s^2} + R \right) v^s = 0, \quad s = l, u, \tag{13}$$

where

$$R = \frac{c^2 \epsilon_0 \mu_0^2 H_0^2}{\mu_s} - 1.$$

The solutions to Eq. (13) in the lower and upper skin layers are respectively given as

$$v^l(x_2) = A_1 \cos\left(kx_2 \sqrt{\frac{c^2}{c_s^2} + R} \right) + A_2 \sin\left(kx_2 \sqrt{\frac{c^2}{c_s^2} + R} \right),$$

$$\tag{14}$$

$$v^u(x_2) = B_1 \cos\left(kx_2 \sqrt{\frac{c^2}{c_s^2} + R} \right) + B_2 \sin\left(kx_2 \sqrt{\frac{c^2}{c_s^2} + R} \right).$$

where A_n, B_n, $(n = 1, 2)$ are constants to be determined from the given conditions above.

3.2. The Dynamics of the Core Layer with Rotational Effects

Also, from Eqs. (7) and (8), the following wave-like equation is obtained

$$\frac{\partial^2 u^c}{\partial x_1^2} + \frac{\partial^2 u^c}{\partial x_2^2} = \frac{1}{c_c^2} \left(\frac{\partial^2 u^c}{\partial t^2} - \Omega^2 u^c \right), \tag{15}$$

where $c_c = \sqrt{\frac{\mu_c}{\rho_c}}$ is the transverse shear velocity for the core layer. Again, assuming the harmonic solution along x_1 direction in Eq. (15) of the form

$$u^c(x_1, x_2, t) = v^c(x_2) e^{ik(x_1 - ct)}, \tag{16}$$

then Eq. (15) is reduced to

$$\frac{d^2 v^c}{dx_2^2} + k^2 \left(\frac{c^2}{c_c^2} + N \right) v^c = 0, \tag{17}$$

where

$$N = \frac{\Omega^2}{k^2 c_c^2} - 1.$$

Therefore, the solution of Eq. (17) of the core layer is given as

$$v^c(x_2) = C_1 \cos\left(kx_2\sqrt{\frac{c^2}{c_c^2} + N}\right) + C_2 \sin\left(kx_2\sqrt{\frac{c^2}{c_c^2} + N}\right), \tag{18}$$

where C_1 and C_2 are constants to be determined from the given conditions above.

3.3. The Related Exact Displacements and Stresses

The related exact dimensional displacements and stresses in the lower skin layer, upper skin layer, and core layer are respectively given as follows

$$u^l = \cos(kM_s\eta_2),$$
$$\sigma_{13}^l = i\mu_s k \cos(kM_s\eta_2), \tag{19}$$
$$\sigma_{23}^l = -\mu_s kM_s \sin(kM_s\eta_2),$$

$$u^u = \chi \sec(h_3 kM_s)\cos(k(\xi_2 - (h_1 + h_2 + h_3))M_s),$$
$$\sigma_{13}^u = i\mu_s k\chi \sec(h_3 kM_s)\cos(k(\xi_2 - (h_1 + h_2 + h_3))M_s), \tag{20}$$
$$\sigma_{23}^u = -\mu_s kM_s\chi \sec(h_3 kM_s)\cos(k(\xi_2 - (h_1 + h_2 + h_3))M_s),$$

and

$$u^c = \cos(h_1 kM_s)\sin(kM_c(h_1 + \zeta_2)) - G\sin(h_1 kM_s)\cos(kM_c(h_1 + \zeta_2)),$$
$$\sigma_{13}^c = i\mu_c k(\cos(h_1 kM_s)\sin(kM_c(h_1 + \zeta_2)) - G\sin(h_1 kM_s)\cos(kM_c(h_1 + \zeta_2))), \tag{21}$$
$$\sigma_{23}^c = \mu_c kM_c(\cos(h_1 kM_s)\cos(kM_c(h_1 + \zeta_2)) + G\sin(h_1 kM_s)\sin(kM_c(h_1 + \zeta_2))),$$

where

$$\chi = \cos(h_2 kM_c)\cos(h_1 kM_s) - G\sin(h_2 kM_c)\sin(h_1 kM_s), \tag{22}$$

over the interval,

$$0 \leq \eta_2 \leq h_1,$$
$$h_1 \leq \xi_2 \leq h_1 + h_2, \tag{23}$$
$$h_1 + h_2 \leq \zeta_2 \leq h_1 + h_2 + h_3.$$

Note that the exponential multiple $e^{ik(x_1-ct)}$ is omitted in Eqs. (19)-(21), while M_c, M_s and G are given in the next section.

4. THE RAYLEIGH-LAMB DISPERSION RELATION

In this section, the Rayleigh-Lamb dispersion relation or rather the dispersion relation of the formulated problem is derived and analysed for certain cases of interests.

4.1. General Dispersion Relation

To derive the general dispersion relation for the formulated problem, the continuity and traction-free conditions prescribed in Eqs. (9), (10) coupled to the obtained solutions found in Eqs. (12) and (16) via Eqs. (14) and (18) are considered. Therefore, the following system of homogeneous equations is obtained:

$$A\mathbf{x} = 0, \tag{24}$$

where $\mathbf{x} = (A_1, B_1, B_2, C_1, C_2)^T$, and

$$A = \begin{pmatrix} 0 & \cos(klM_s) & \sin(klM_s) & -\cos(klM_c) & -\sin(klM_c) \\ -\cos(kh_1M_s) & 0 & 0 & \cos(kh_1M_c) & \sin(kh_1M_c) \\ 0 & G\sin(klM_s) & -G\cos(klM_s) & -\sin(klM_c) & \cos(klM_c) \\ G\sin(kh_1M_s) & 0 & 0 & -\sin(kh_1M_c) & \cos(kh_1M_c) \\ 0 & -\sin(khM_s) & \cos(khM_s) & 0 & 0 \end{pmatrix}, \tag{25}$$

where

$$
\begin{cases}
l = h_1 + h_2, \quad h = h_1 + h_2 + h_3, \\[2mm]
M_s = \sqrt{\dfrac{c^2}{c_s^2} + R}, \quad M_c = \sqrt{\dfrac{c^2}{c_c^2} + N}, \\[2mm]
G = \dfrac{M_s \mu_s}{M_c \mu_c}.
\end{cases}
\tag{26}
$$

Therefore, from the 5×5 matrix given in Eq. (25), the general dispersion relation is obtained as follows:

$$
\tan\left(h_1 k M_s\right) \tan\left(h_3 k M_s\right) - \frac{M_c \mu_c}{M_s \mu_s} \cot\left(h_2 k M_c\right)\left(\tan\left(h_1 k M_s\right) + \tan\left(h_3 k M_s\right)\right) = \left(\frac{M_c \mu_c}{M_s \mu_s}\right)^2.
\tag{27}
$$

The general dispersion relation in Eq. (27) is the needed dispersion relation posed by the formulated problem under consideration for the propagation of Rayleigh waves in an asymmetric three-layered inhomogeneous structure in the presence of some effects as described earlier. However, the general dispersion relation given in Eq. (27) is analyzed in the subsequent subsection.

4.2. Some Cases of the Dispersion Relation

Here, the obtained general dispersion relation is analyzed by studying both the symmetric and antisymmetric structure cases. Also, the situations with and without magnetic and rotational effects will be investigated.

4.2.1. Symmetric Structure

Consider a symmetric structure, which can be seen from Figure 1 when the skin layers are assumed to be of the same length. Thus, $h_1 = h_3$ is set in Eq. (27) to obtain the corresponding dispersion relation for the symmetric structure as follows:

$$
\tan^2\left(h_1 k M_s\right) - 2\left(\frac{M_c \mu_c}{M_s \mu_s}\right) \cot\left(h_2 k M_c\right) \tan\left(h_1 k M_s\right) = \left(\frac{M_c \mu_c}{M_s \mu_s}\right)^2.
\tag{28}
$$

Case one:

If $R \to 0$ and $N \to 0$, then the symmetric dispersion relation given in Eq. (28) is reduced to:

$$
\tan^2\left(k\frac{c}{c_s}h_1\right) - 2\left(\frac{c_s \mu_c}{c_c \mu_s}\right) \cot\left(k\frac{c}{c_c}h_2\right) \tan\left(k\frac{c}{c_s}h_1\right) = \left(\frac{c_s \mu_c}{c_c \mu_s}\right)^2.
\tag{29}
$$

Case two:

If $H_0 \to 0$, $\Omega \to 0$, $\mu_s \to \mu_c \to \mu$, and $\rho_s \to \rho_c \to \rho$, that is, in the absence of magnetic field and rotation, and also assuming the layers to be of the same materials, the symmetric dispersion relation given in Eq. (28) is reduced to:

$$
\tan^2\left(h_1 k \sqrt{\frac{c^2}{c_1^2} - 1}\right) - 2\cot\left(h_2 k \sqrt{\frac{c^2}{c_1^2} - 1}\right) \tan\left(h_1 k \sqrt{\frac{c^2}{c_1^2} - 1}\right) = 1,
\tag{30}
$$

where $c_1 = \sqrt{\frac{\mu}{\rho}}$.

Case three:

If $H_0 \to 0$ and $\Omega \to 0$, that is, in the absence of magnetic field in the skin layers and rotation in the core layer, the symmetric dispersion relation given in Eq. (28) is reduced to:

$$
\tan^2\left(h_1 k \sqrt{\frac{c^2}{c_s^2} - 1}\right) - 2\left(\frac{\mu_c c_s}{\mu_s c_c}\right) \sqrt{\frac{c^2 - c_c^2}{c^2 - c_s^2}} \cot\left(h_2 k \sqrt{\frac{c^2}{c_c^2} - 1}\right) \tan\left(h_1 k \sqrt{\frac{c^2}{c_s^2} - 1}\right) = \frac{c_s^2\left(c^2 - c_c^2\right)\mu_c^2}{c_c^2\left(c^2 - c_s^2\right)\mu_s^2}.
\tag{31}
$$

4.2.2. Antisymmetric Structure

We analyse the obtained antisymmetric dispersion relation given in Eq. (27) for the following special cases:

Case one:

If $R \to 0$ and $N \to 0$, then the antisymmetric dispersion relation given in Eq. (27) is reduced to:

$$\tan\left(h_1\frac{c}{c_s}k\right)\tan\left(h_3\frac{c}{c_s}k\right) - \frac{c_s\mu_c}{c_c\mu_s}\cot\left(h_2\frac{c}{c_c}k\right)\left(\tan\left(h_1\frac{c}{c_s}k\right) + \tan\left(h_3\frac{c}{c_s}k\right)\right) = \left(\frac{c_s\mu_c}{c_c\mu_s}\right)^2. \quad (32)$$

Case two:

If $H_0 \to 0$, $\Omega \to 0$, $\mu_s \to \mu_c \to \mu$, and $\rho_s \to \rho_c \to \rho$, that is, in the absence of magnetic field and rotation, and also assuming the layers to be of the same materials, the antisymmetric dispersion relation given in Eq. (27) is reduced to:

$$\cot\left(h_2k\sqrt{\frac{c^2}{c_1^2}-1}\right)\left(\tan\left(h_1k\sqrt{\frac{c^2}{c_1^2}-1}\right) + \tan\left(h_3k\sqrt{\frac{c^2}{c_1^2}-1}\right)\right)$$
$$= \tan\left(h_1k\sqrt{\frac{c^2}{c_1^2}-1}\right)\tan\left(h_3k\sqrt{\frac{c^2}{c_1^2}-1}\right) - 1, \quad (33)$$

where $c_1 = \sqrt{\frac{\mu}{\rho}}$.

Case three:

If $H_0 \to 0$ and $\Omega \to 0$, that is, in the absence of magnetic field in the skin layers and rotation in the core layer, the antisymmetric dispersion relation given in Eq. (27) is reduced to:

$$\frac{\mu_c c_s}{\mu_s c_c}\sqrt{\frac{c^2-c_c^2}{c^2-c_s^2}}\cot\left(\sqrt{\frac{c^2}{c_c^2}-1}h_2k\right)\left(\tan\left(h_1k\sqrt{\frac{c^2}{c_s^2}-1}\right) + \tan\left(h_3k\sqrt{\frac{c^2}{c_s^2}-1}\right)\right)$$
$$= \tan\left(h_1k\sqrt{\frac{c^2}{c_s^2}-1}\right)\tan\left(h_3k\sqrt{\frac{c^2}{c_s^2}-1}\right) - \frac{c_s^2\left(c^2-c_c^2\right)\mu_c^2}{c_c^2\left(c^2-c_s^2\right)\mu_s^2}. \quad (34)$$

5. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results and their interpretations are demonstrated. These involve the analysis of the obtained displacements and shear stresses of the formulated problem given in Eqs. (19)–(21) and the general dispersion relation given in Eq. (27). Mathematica software is adopted for the present numerical simulations and also makes use of c_q as μ_q/ρ_q for our convenience. In this regard, the inhomogeneous plate is considered to have the following thicknesses in the respective layers: the lower skin layer $h_1 = 6\,\mathrm{m}$, the core layer $h_2 = 10\,\mathrm{m}$, and the upper skin layer $h_3 = 8\,\mathrm{m}$, and also fixed $k = 0.01$. Further, the following values for c (speed of light is considered), μ_0 (magnetic permeability), and ϵ_0 (electric field permeability) are considered as follows [13]

$$c = 2.998 \times 10^8\,\mathrm{ms}^{-1}, \quad \mu_0 = 4\pi \times 10^{-7}, \quad \epsilon_0 = 8.85 \times 10^{-12}. \quad (35)$$

Moreover, the upper and lower skin layers are assumed to be of copper materials with the following material properties [34]

$$\rho_s = 8.954 \times 10^3\,\mathrm{Kgm}^{-3}, \quad \mu_s = 3.86 \times 10^{10}\,\mathrm{Pa}, \quad (36)$$

while the core layer is of aluminum material with the following material parameters [35]:

$$\rho_c = 2.66 \times 10^3\,\mathrm{Kgm}^{-3}, \quad \mu_c = 2.46 \times 10^{10}\,\mathrm{Pa}. \quad (37)$$
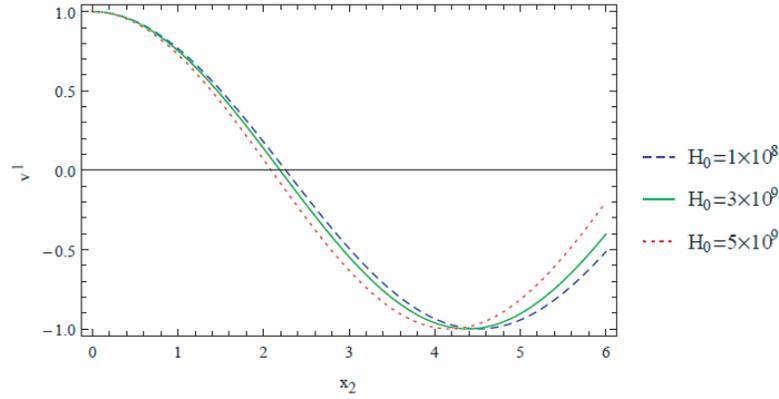
Figure 2. Variation of the displacement v^l in the lower skin layer given in Eq. (19) with variation of magnetic field.
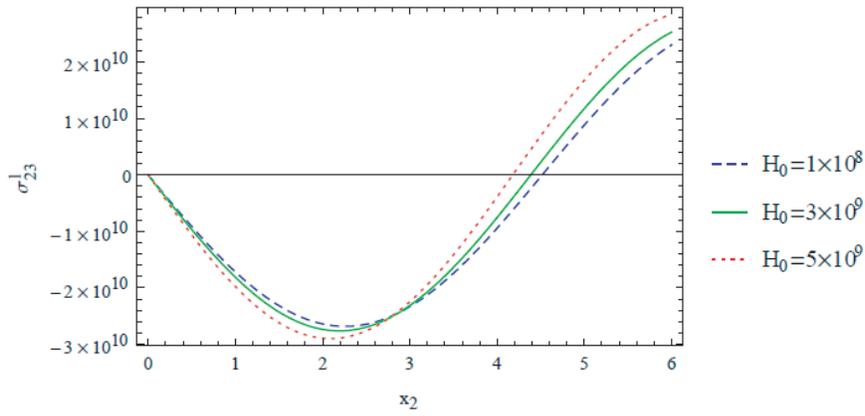

Figure 3. Variation of the shear stress σ_{23}^l in the lower skin layer given in Eq. (19) with variation of magnetic field.

Figures 2 and 3 give the variations of the displacements and shear stresses in the lower skin layer with respect to the space variable x_2, in the presence of magnetic field H_0. The displacements start off from 1, while the shear stresses start from 0 and gradually oscillate periodically with uniform wavelengths and amplitudes; however, these can be seen clearly on a broader interval of x_2. Further, it is observed that both the displacement and shear stress decrease with respect to the space variable x_2 with an increase in magnetic field, and at the side time a significant increase in the respective amplitudes is noted.

In Figures 4 and 5, the variations of the displacements and shear stresses in the upper skin layer with respect to the space variable x_2 in the presence of variation of magnetic field H_0 with fixed rotation $\Omega = 1 \times 10^6$ are depicted. The wave displacements and shear stresses move periodically with uniform wavelengths and amplitudes. Also, it is observed that both the displacement and shear stress increase with respect to the space variable x_2 with an increase in magnetic field and at the same time with a decrease in the respective wavelengths.

In Figures 6 and 7, the variations of the displacements and shear stresses in the core layer with respect to the space variable x_2 in the presence rotation Ω with fixed value of magnetic field, i.e., $H_0 = 1 \times 10^6$, are shown. It is observed that both the displacement and shear stress decrease periodically with respect to the space variable x_2 with an increase in rotation.

Figure 8 displays the variation of the dimensional dispersion relation given in Eq. (27) with respect to the rotation with variation in magnetic field. It is noted that an increase in magnetic field results in a decrease of the dimensional dispersion relation curves with respect to the rotation. In Figure 9,
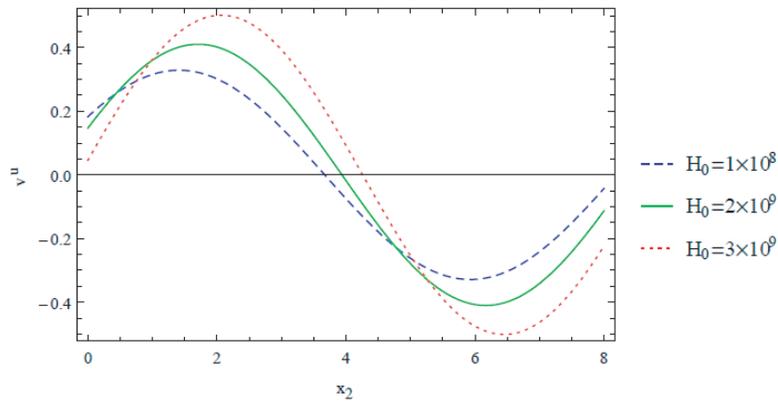
Figure 4. Variation of the displacement v^u in the upper skin layer given in Eq. (20) with variation of magnetic field.
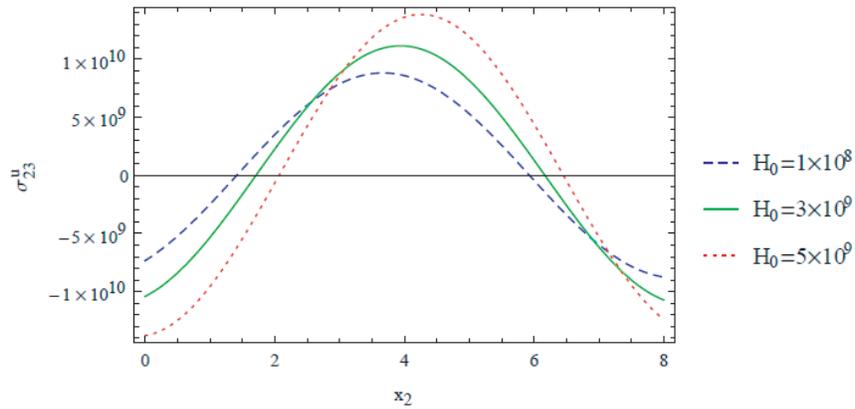

Figure 5. Variation of the shear stress σ_{23}^u in the upper skin layer given in Eq. (20) with variation of magnetic field.
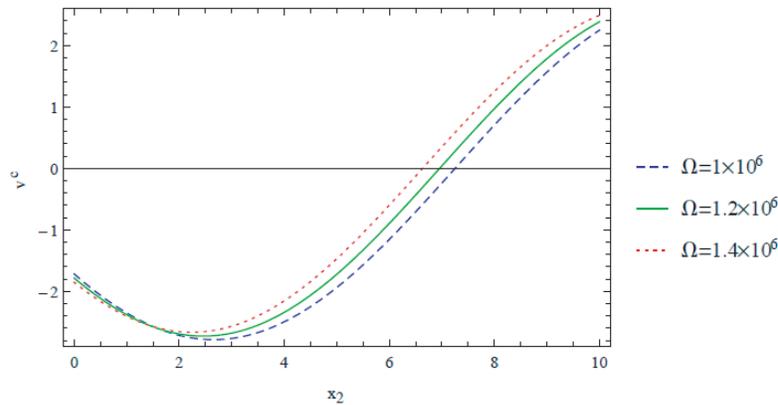

Figure 6. Variation of the displacement v^c in core layer given in Eq. (21) with variation of rotation.

increase in rotation produces an increase in the dispersion relation curves with respect to the magnetic field. It is also noted here that the dispersion relation curves decrease continuously with an increase in magnetic field.

In Figure 10, the variation of the general dispersion relation given in Eq. (27) with respect to the magnetic field with variation in electric field with $\Omega = 1 \times 10^6$ is shown. Figure 11 depicts the
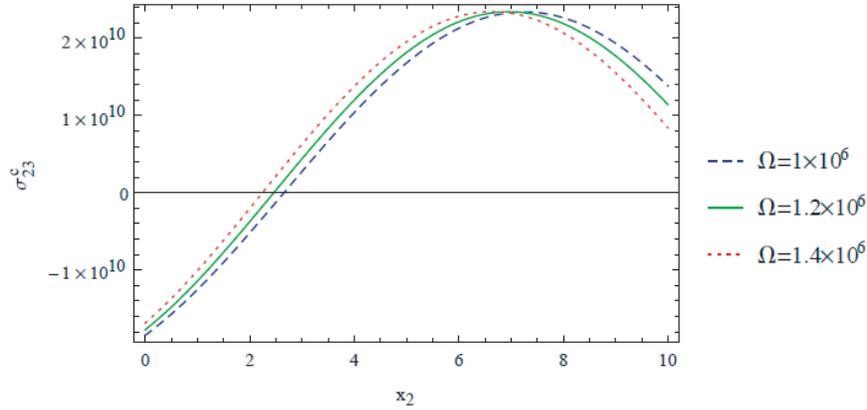
Figure 7. Variation of the shear stress σ_{23}^c in the core layer given in Eq. (21) with variation of rotation.
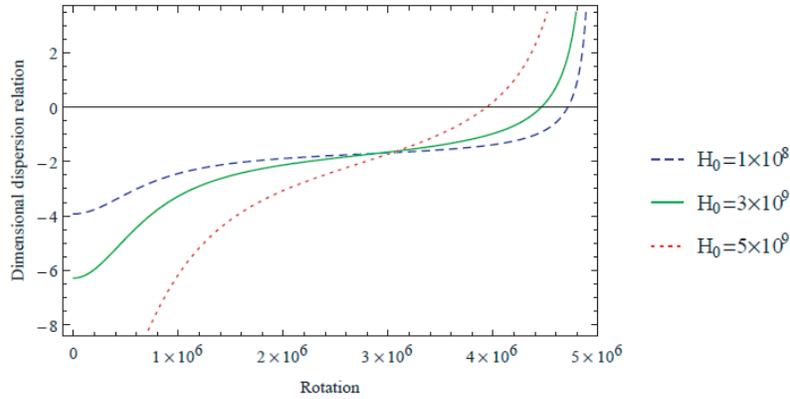

Figure 8. Variation of the dispersion relation given in Eq. (27) with respect to the rotation with variation in magnetic field.
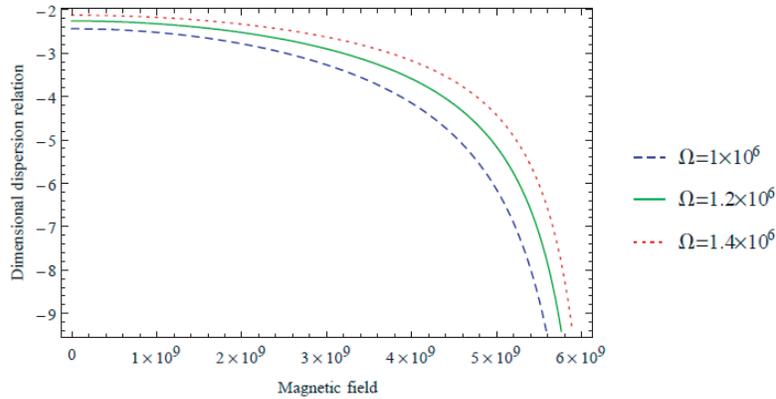

Figure 9. Variation of the general dispersion relation given in Eq. (27) with respect to the magnetic field with variation in rotation.

variation of the same dispersion relation with respect to the rotation with variation in electric field at $H_0 = 1 \times 10^9$. In the plots in Figures 10 and 11, the dispersion relation is noted to decrease with an increase in electric field. However, dispersion relation curves are seen decreasing continuously with an increase in magnetic field in Figure 10, while opposite trend is observed in Figure 11 with an increase in rotation.
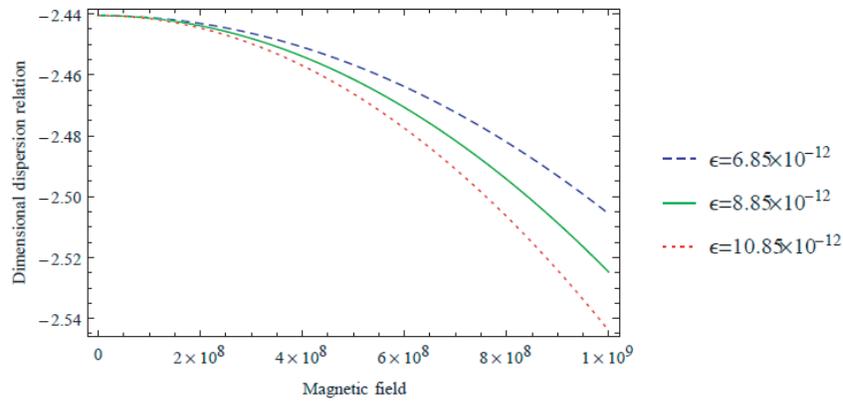
Figure 10. Variation of the general dispersion relation given in Eq. (27) with respect to the magnetic field with variation in electric field.
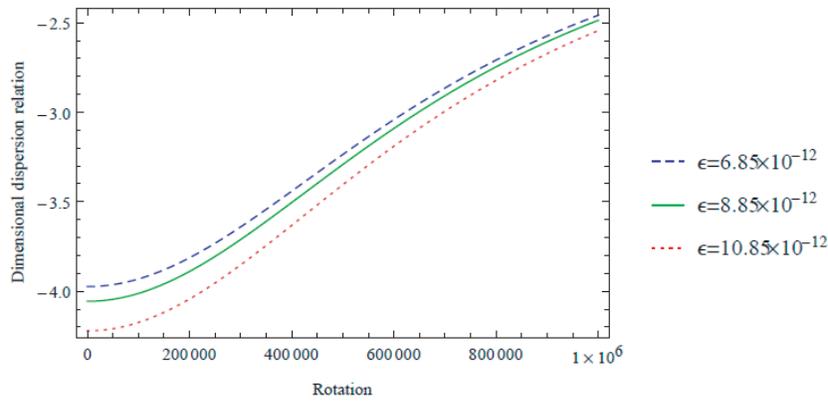

Figure 11. Variation of the general dispersion relation given in Eq. (27) with respect to the rotation with variation in electric field.

6. CONCLUSION

In conclusion, the present paper investigates the propagation and dispersion of elastic surface waves in an asymmetric inhomogeneous isotropic three-layered plate in the presence of magnetic field and rotation. The model is governed by an anti-plane equation of motion with prescribed continuity (interlayer) and free-ends conditions within and outside the plate, respectively. The skin layers are exposed to magnetic field force while the core layer is considered to be in a rotational frame of reference. The respective displacements, shear stresses, and general dispersion relation have been determined and analyzed. Some physical cases of interest in regards to the general dispersion relation are also investigated. Further, the obtained quantities alongside the general dispersion relation are examined numerically; the fundamental mode of the general dispersion relation is considered for the examination. Lastly, the presence of a magnetic field in the skin layers and rotation in the core layer are noted to greatly affect the dispersion of elastic waves in the plate. However, it is worth mentioning here that variations in the displacements and shear stresses (excluding the lower skin layer) and on the other hand in the general dispersion relation are only observed for higher values of rotation $\Omega \geq 1 \times 10^6$ unlike what is obtainable in most of the literature; this could be because the rotation is only assumed in the core layer.

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