# Analysis of "False Non Reciprocity" in 2-Port VNA Measurements of Reciprocal Devices

**Leonardo Zappelli**[*]

**Abstract**—The effect of measurement errors in the $S$-matrix of a reciprocal 2-port device is recognized in the (usually low) difference between $S_{12}$ and $S_{21}$, as the device is nonreciprocal. This "false non-reciprocity" is analyzed in this paper, and it is verified that, for low loss device, the difference acts principally on the phases of $S_{12}$ and $S_{21}$. This anomaly can be removed if a numerical correction is applied to the experimental $S$-matrix. In doing so, it is proved that the residual measurement errors have comparable amplitudes on all scattering parameters.

## 1. INTRODUCTION

Reciprocal devices are characterized by symmetric matrices representing relationships between electrical quantities at input and output sections. When a reciprocal device is analyzed by means of a theoretical approach or with the simulation performed by numerical approaches or commercial softwares, the reciprocity of such a device is always reflected in the symmetry of its matrices $S/Z/Y$. Unfortunately, the symmetry is not completely ensured when experimental measurements are performed on the same reciprocal device that has been previously analyzed with a theoretical approach or with a software. In fact, in this case, several scenarios can alter the measurements and produce a low "false non-reciprocity" even if the measured device is reciprocal. Typical measurement errors are related to the bandwidth used during the experimental test, to the matched/short loads used during calibration and to the evaluation of the calibration factors that can alter the measurement of the off diagonal values of the $S$-matrix, producing different values for $S_{12}$ and $S_{21}$. The difference $S_{12} - S_{21}$ can be maintained lower than one part per thousands but could increase up to one part per hundreds if the VNA IF bandwidth is not properly chosen. This difference can be neglected if we need a comparison between theoretical and experimental results. In fact, in this case, it does not alter the judgment about the correspondence between theoretical and experimental results. On the contrary, if we need to identify an equivalent circuit for the real reciprocal device, the difference between $S_{12}$ and $S_{21}$ introduces unwanted gyrators in the circuit that should not appear in the correct version. Another case where high measurement precision is required on $S_{ij}$ is the use of 2-port measurements to evaluate equivalent circuits for lossy 2-port reciprocal devices [1] or the scattering parameters of $n$-port reciprocal devices. In this case, the measure of the overall $S$-matrix can be obtained by means of $S$-matrix reconstruction transforms based on 2-port measurements [2] or with the use of equivalent circuits [3].

If a symmetric $S$-matrix is required, the usual choice of a researcher to correct the low discrepancy between $S_{12}$ and $S_{21}$ is one of the following options: (a) $S_{12} = S_{21}$, (b) $S_{21} = S_{12}$, or (c) define new off diagonal terms $\bar{S}_{12} = \bar{S}_{21} = \frac{S_{12}+S_{21}}{2}$. Hence, at this stage, the question is to choose the best solution.

* Corresponding author: Leonardo Zappelli (l.zappelli@univpm.it).
The author is with the Dipartimento di Ingegneria dell'Informazione, Università Politecnica delle Marche, Via Brecce Bianche, Ancona 60131, Italy.

Moreover, another problem should be discussed: if "false non-reciprocity" is present in the experimental values of $S_{12}, S_{21}$, how are $S_{11}$ and $S_{22}$ affected by errors similar to those occurring in $S_{12}, S_{21}$? Hence, should we also correct the values of $S_{11}, S_{22}$?

In this paper, the best choice to correct the values of $S_{12}, S_{21}$ is discussed together with the effect of that correction on $S_{11}, S_{22}$. As we could expect from the experimental practice, the best choice will be to define new off diagonal terms $\bar{S}_{12} = \bar{S}_{21} = \frac{S_{12}+S_{21}}{2}$, in order to obtain the same uncertainty affecting the measurement of $S_{11}, S_{22}$.

## 2. THEORY

The starting point of the analysis of the addressed problem is represented by the fundamental papers that Carlin and Youla wrote in the '50s–'60s [4–10]. They discussed and proved that a nonreciprocal lossy device can be identified with a proper network made by linear components as capacitors, inductors, real/complex transformers, positive/negative resistors, and gyrators. The key point is that a lossy nonreciprocal device can be represented by a nonsymmetric normalized impedance matrix $\zeta$ that can be decomposed in two parts connected in series, the first being relative to the lossy nonreciprocal part and the second to the lossless nonreciprocal part

$$\zeta = \zeta^{\text{lossy}} + \zeta^{\text{lossless}} \tag{1}$$

with († represents transpose and conjugate)

$$\zeta^{\text{lossy}} = \frac{\zeta + \zeta^{\dagger}}{2} \qquad \zeta^{\text{lossless}} = \frac{\zeta - \zeta^{\dagger}}{2} \tag{2}$$

For a 2-port device, it can be shown that Eq. (2) can be written as

$$\zeta^{\text{lossy}} = \left[ \begin{array}{cc} r_{11} & r_{12} + j\xi \\ r_{12} - j\xi & r_{22} \end{array} \right] \tag{3}$$

$$\zeta^{\text{lossless}} = \left[ \begin{array}{cc} jx_{11} & jx_{12} + \chi \\ jx_{12} - \chi & jx_{22} \end{array} \right] \tag{4}$$

The matrix in Eq. (3) is an "Hermitian" matrix relative to the lossy part of the device because its elements are purely real except for the off diagonal terms containing also two imaginary and opposite values representing the non-reciprocity, via a gyrator with amplitude $j\xi$.

The matrix in Eq. (4) is a "skew-Hermitian" matrix relative to the lossless part of the device because its elements are purely imaginary except for the off diagonal terms that contain also two real and opposite values representing the non-reciprocity, via a gyrator with amplitude $\chi$. Carlin and Youla proved that any 2-port device can be reduced to the series of Eqs. (3) and (4) and that a physical realization of Eqs. (3)–(4) can be obtained, even if the procedure to synthesize the "Hermitian" part in Eq. (3) is more involved than the "skew-Hermitian" part in Eq. (4). A recent synthesis procedure has been analyzed in [11], but it is not reported here, because the goal of the paper is to evaluate how to correct the "false non-reciprocity" due to experimental measurements.

In order to solve our problem, we transform Eqs. (3)–(4) in terms of the scattering matrix of the device, obtaining:

$$S = \left[ \begin{array}{cc} S_{11} & S_{12} \\ S_{21} & S_{22} \end{array} \right] = \left[ \begin{array}{cc} \dfrac{N_{11}}{D} & \dfrac{N_{12}}{D} \\ \dfrac{N_{21}}{D} & \dfrac{N_{22}}{D} \end{array} \right] \tag{5}$$

with

$$\begin{aligned} N_{11} &= -1 - r_{12}^2 - r_{22} + r_{11}(1 + r_{22}) - x_{11}x_{22} + x_{12}^2 + j\left[x_{11}(1 + r_{22}) - 2r_{12}x_{12} + x_{22}(r_{11} - 1)\right] \\ &\quad + (\chi + j\xi)^2 = N_{11}^{\text{rec}} + (\chi + j\xi)^2 \tag{6} \\ N_{12} &= 2\left[r_{12} + \xi + j(x_{12} + \chi)\right] = N_{12}^{\text{rec}} + 2(\chi + j\xi) \tag{7} \\ N_{21} &= 2\left[r_{12} - \xi + j(x_{12} - \chi)\right] = N_{12}^{\text{rec}} - 2(\chi + j\xi) \tag{8} \\ N_{22} &= -1 - r_{12}^2 - r_{11} + r_{22}(1 + r_{11}) - x_{11}x_{22} + x_{12}^2 + j\left[x_{22}(1 + r_{11}) - 2r_{12}x_{12} + x_{11}(r_{22} - 1)\right] \\ &\quad + (\chi + j\xi)^2 = N_{22}^{\text{rec}} + (\chi + j\xi)^2 \tag{9} \end{aligned}$$

$$D = 1 - r_{12}^2 + r_{22} + r_{11}(1 + r_{22}) + x_{12}^2 - x_{11}x_{22} + j\left[x_{22}(1 + r_{11}) + x_{11}(1 + r_{22}) - 2r_{12}x_{12}\right]$$
$$+(\chi + j\xi)^2 = D^{\text{rec}} + (\chi + j\xi)^2 \tag{10}$$

The main remark on Eqs. (5)–(10) is that any term can be written as the sum of the corresponding reciprocal term ($N_{ij}^{\text{rec}}, D^{\text{rec}}$) plus nonreciprocal terms related to the gyrators amplitude, $\chi, \xi$, and that the non-reciprocity acts with different effects on the numerators $N_{ij}$ and denominator $D$ of $S_{ij}$. In fact, $D, N_{11}$ and $N_{22}$ contain the gyrators amplitude, $\chi, \xi$, as square of their sum $(\chi + j\xi)^2$, while $N_{12}$ and $N_{21}$ contain the sum $\chi + j\xi$ at the first degree.

If we expand Eq. (5) in series with respect to the second order of $\chi, \xi$, which are usually small quantities for low loss devices, we obtain

$$S \approx S^{\text{rec}} + \delta S \tag{11}$$

$$S^{\text{rec}} = \begin{bmatrix} \dfrac{N_{11}^{\text{rec}}}{D^{\text{rec}}} & \dfrac{N_{12}^{\text{rec}}}{D^{\text{rec}}} \\ \dfrac{N_{12}^{\text{rec}}}{D^{\text{rec}}} & \dfrac{N_{22}^{\text{rec}}}{D^{\text{rec}}} \end{bmatrix} = \begin{bmatrix} S_{11}^{\text{rec}} & S_{12}^{\text{rec}} \\ S_{12}^{\text{rec}} & S_{22}^{\text{rec}} \end{bmatrix} \tag{12}$$

$$\delta S = \begin{bmatrix} \dfrac{\left(1 - S_{11}^{\text{rec}}\right)(\chi + j\xi)^2}{D^{\text{rec}}} & \dfrac{2(\chi + j\xi) - S_{12}^{\text{rec}}(\chi + j\xi)^2}{D^{\text{rec}}} \\ \dfrac{-2(\chi + j\xi) - S_{12}^{\text{rec}}(\chi + j\xi)^2}{D^{\text{rec}}} & \dfrac{\left(1 - S_{22}^{\text{rec}}\right)(\chi + j\xi)^2}{D^{\text{rec}}} \end{bmatrix} + o(\xi^3, \xi^2\chi, \xi\chi^2, \chi^3) \tag{13}$$

and we can observe that the errors in Eq. (13) are proportional to $(\chi + j\xi)^2$ for $S_{11}, S_{22}$ and to $\chi + j\xi$ for $S_{12}, S_{21}$, where the effect of $(\chi + j\xi)^2$ is lower than $\chi + j\xi$. Hence, the effect of the nonreciprocal gyrators is greater on $S_{12}, S_{21}$ than on $S_{11}, S_{22}$.

Another interesting information can be obtained if we evaluate the difference between the amplitudes and phases of the measured $S_{12}$ and $S_{21}$. In fact, from Eqs. (7), (8), (10), and (12), the series expansions in terms of $\chi, \xi$ at the first degree give:

$$|S_{12}| - |S_{21}| \approx 2|S_{12}^{\text{rec}}|\frac{r_{12}\xi + x_{12}\chi}{r_{12}^2 + x_{12}^2} + o(\xi^2, \xi\chi, \chi^2) \tag{14}$$

$$\Phi_{12} - \Phi_{21} \approx \frac{r_{12}\chi - x_{12}\xi}{r_{12}^2 + x_{12}^2} + o(\xi^2, \xi\chi, \chi^2) \tag{15}$$

where $\Phi_{ij}$ is the phase of $S_{ij}$. If the device is lossless, $\zeta^{\text{lossy}}$ in Eq. (2) is zero, with $r_{12} = 0$ and $\chi = 0$, and from Eq. (14), $|S_{12}| - |S_{21}| \approx 0$ and the error due to the "false non reciprocity" affects only the phase difference in Eq. (15). Similarly, for a low lossy device we can expect that the error on $|S_{12}| - |S_{21}|$ is lower than the error on $\Phi_{12} - \Phi_{21}$.

From Eqs. (7), (8), and (11)–(13), it is clear that the best choice to reduce the "false non-reciprocity" due to measurement errors is to evaluate a new symmetric value of $S_{12}$, i.e.,

$$\bar{S}_{12} = \bar{S}_{21} = \frac{S_{12} + S_{21}}{2} = \frac{2(r_{12} + jx_{12})}{D} = \frac{N_{12}^{\text{rec}}}{D} \approx S_{12}^{\text{rec}}\left[1 - \frac{(\chi + j\xi)^2}{D^{\text{rec}}}\right] + o(\xi^3, \xi^2\chi, \xi\chi^2, \chi^3) \tag{16}$$

The new definition of $\bar{S}_{12}$ has an error $S_{12}^{\text{rec}}\frac{(\chi + j\xi)^2}{D^{\text{rec}}}$ that is about the same amplitude of the error affecting $S_{11}, S_{22}$ in Eq. (13). Hence, even if measurement errors are yet contained in $S_{11}, S_{22}, \bar{S}_{12}$, we have obtained a reciprocal $S$-matrix by means of Eq. (16).

The other choices, i.e., to set $S_{21} = S_{12}$ or $S_{12} = S_{21}$, are not recommended because the error due to the "false non-reciprocity" is still proportional to $\chi + j\xi$.

Now we could discuss the effect of the gyrators $\chi, \xi$ on the errors contained in $S_{11}, S_{22}, \bar{S}_{12}$. From Eqs. (11)–(13),

$$S_{12} - S_{21} \approx 2\frac{\chi + j\xi}{D^{\text{rec}}} \tag{17}$$

and we can expect that

$$10^{-2} \leq \text{Re}\left\{\frac{\chi + j\xi}{D^{\text{rec}}}\right\}, \quad \text{Im}\left\{\frac{\chi + j\xi}{D^{\text{rec}}}\right\} \leq 10^{-3} \tag{18}$$

because the "false non-reciprocity" of the measured $S$-matrix usually alters the second or third decimal digit of $S_{12}, S_{21}$. From Eqs. (13) and (16), it can be seen that the gyrators amplitudes act on $S_{11}, S_{22}, \bar{S}_{12}$ proportionally to $(\chi + j\xi)^2$. Hence, from Eq. (18) we can deduce that the error on $S_{11}, S_{22}, \bar{S}_{12}$ is in the following range

$$10^{-4} \leq \mathrm{Re}\left\{\frac{(\chi + j\xi)^2}{D^{\mathrm{rec}}}\right\}, \quad \mathrm{Im}\left\{\frac{(\chi + j\xi)^2}{D^{\mathrm{rec}}}\right\} \leq 10^{-6} \tag{19}$$

Therefore, we can state that the "false non-reciprocity" produces a very low error on $S_{11}$ and $S_{22}$, proportional to Eq. (19), which is of the same amplitude of the error affecting $\bar{S}_{12}$. Hence, we can use the measured $S_{11}$ and $S_{22}$, without further corrections, and $\bar{S}_{12}$ to obtain a scattering matrix with similar measurement errors on all its elements.

Obviously, we can completely delete the "false experimental non-reciprocity" effects evaluating the amplitudes of $\chi$ and $\xi$ by the measured $S$-matrix, performing the steps discussed in [4–11] that consist in evaluating the impedance matrices in Eq. (2) and from Eqs. (3) and (4), the actual values of $\chi, \xi, r_{ij}$, and $x_{ij}$. To delete the effect of the "false" experimental non-reciprocity, it is sufficient to reconstruct the reciprocal $S$-matrix from Eqs. (3), (4) with $\chi = \xi = 0$ and the actual values of $r_{ij}, x_{ij}$. Obviously, this procedure requires to implement a proper numerical code that must be evaluated for each frequency point of the measured $S$-matrix, and this implementation is surely more involved than the simple correction proposed in Eq. (16). Anyway, the values of $\chi$ and $\xi$ are very low and are related to the VNA bandwidth chosen during the measurements, as will be shown in the Results.

## 3. RESULTS

In order to verify the possibility to correct the "false non-reciprocity" of a reciprocal device due to experimental errors, a lossy device is arranged, as shown in Fig. 1. It is made by a thick capacitive window in aluminum WR90 (aperture $22.86 \times 5.26 \, \mathrm{mm}^2$, thickness $4.98 \, \mathrm{mm}$) sandwiched between two Maury WR90 waveguides ($L = 152.75 \, \mathrm{mm}$) and connected to aluminum coax-waveguide adapters. All components are reciprocal. The measurements have been made in coaxial cable to calibrate the VNA with 85052B Agilent coax calibration set. The scattering parameters of the lossy device have been measured with FieldFox N9928A Agilent VNA, with bandwidth set to $1000 \, \mathrm{Hz}$, and they are shown in Fig. 2(a). It can be verified that the difference $|S_{12}| - |S_{21}|$ is very low, between $-0.009$ and $0.004$, as shown in Fig. 2(b) (blue dotted line), while the difference $|S_{12} - S_{21}|$ is less than $0.045$ as shown in the same figure (black continuous line). This discrepancy between the values of $|S_{12} - S_{21}|$ and $|S_{12}| - |S_{21}|$ could cause perplexity, but this is related to the effects of the non-reciprocity induced on $S_{12}, S_{21}$ that act principally on their phases $\Phi_{12}, \Phi_{21}$ (red starred line in Fig. 2(b)) and, with less impact, on their amplitudes $|S_{12}|, |S_{21}|$, as reported in Eqs. (14), (15), and (17). In fact, $\Phi_{12} - \Phi_{21}$ lies in a range about $\pm 3°$. On the contrary, $|S_{12} - S_{21}|$ is related to the real and imaginary parts of $S_{12}, S_{21}$ that take into account the amplitudes and phases of $S_{12}, S_{21}$ at the same time, as shown in Fig. 2(c). The real and imaginary parts of the difference $S_{12} - S_{21}$ are in a range about $\pm 0.05$ and have similar behaviors.



**Figure 1.** The device under test.

Just for an example, at $f = 11.98 \, \mathrm{GHz}$, the values of $S_{12}, S_{21}, |S_{12}|, |S_{21}|, \Phi_{12}$ and $\Phi_{21}$ are
- $S_{12} = 0.387 - j0.811, S_{21} = 0.430 - j0.798$
- $|S_{12}| = 0.898, \Phi_{12} = -64.464°$
- $|S_{21}| = 0.907, \Phi_{21} = -61.679°$
- $|S_{12} - S_{21}| = |-0.043 - j0.013| = 0.045$

(a)



(b)



(c)
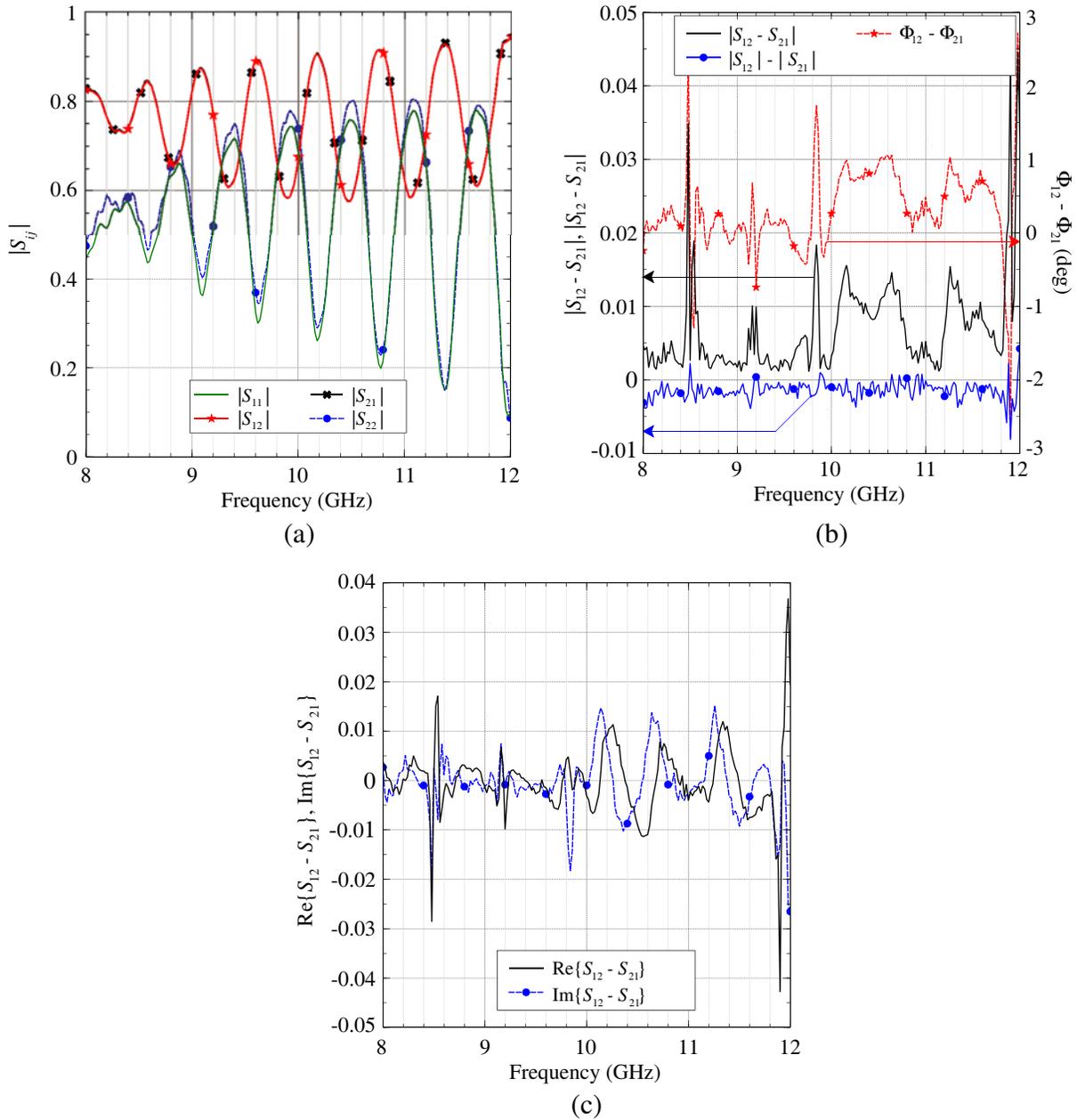
**Figure 2.** (a) The scattering parameters of the device under test and (b), (c) the difference between $S_{12}$ and $S_{21}$.

- $|S_{12}| - |S_{21}| = -0.009$
- $\Phi_{12} - \Phi_{21} = -2.78°$

As previously discussed, $|S_{12}|$ and $|S_{21}|$ are very similar while the difference between $\Phi_{12}$ and $\Phi_{21}$ is about 4.4%. This percentage difference can be recognized also in $\left|\frac{S_{12} - S_{21}}{S_{12}}\right| \approx 4.99\%$ and $\left|\frac{S_{12} - S_{21}}{S_{21}}\right| \approx 4.95\%$.

To delete the "false non-reciprocity" we can apply the definition in Eq. (16), obtaining $\bar{S}_{12} = \bar{S}_{21} = 0.408 - j0.804$, i.e., $|\bar{S}_{12}| = |\bar{S}_{21}| = 0.902, \arg(\bar{S}_{12}) = \arg(\bar{S}_{21}) = -63.065°$.

In doing so, the obtained $S$-matrix is symmetric, and $S_{11}, S_{12}, \bar{S}_{12}$ contain measurement errors of comparable amplitudes, as described in Eq. (19). To verify the entity of the actual errors, we can evaluate the amplitude of the gyrators amplitudes, shown in Figs. 3(a)–3(b) for different values of VNA
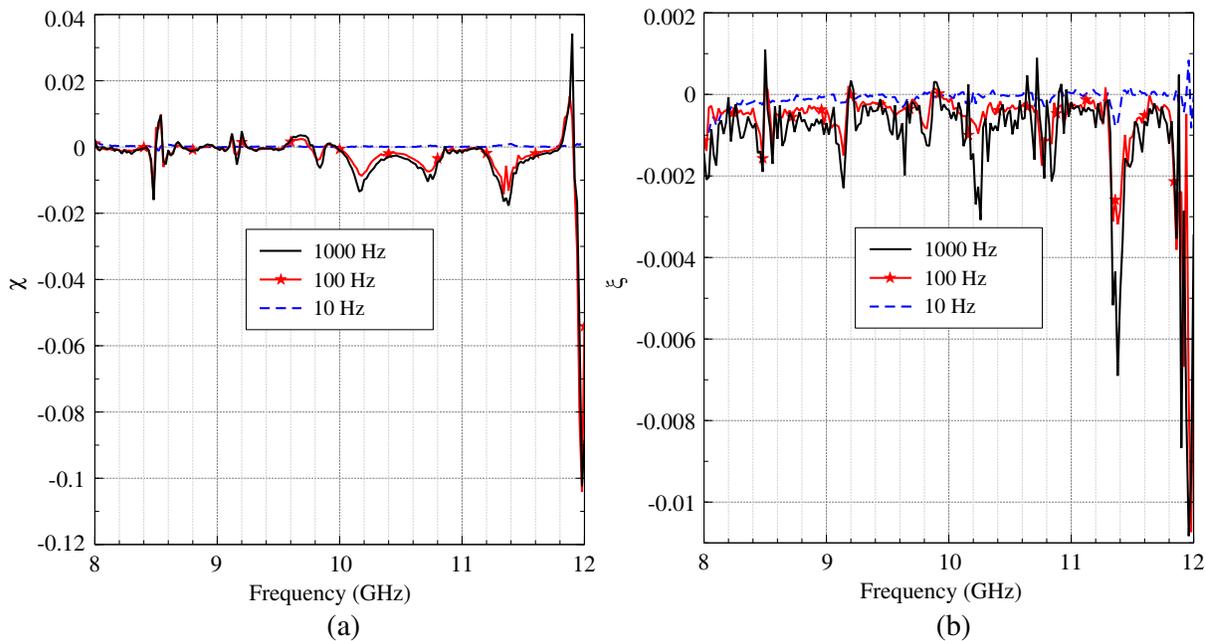
**Figure 3.** Amplitudes of the gyrators introduced by VNA measurement errors for different VNA IF bandwidth.

IF bandwidth [4–11]. The obtained values for $\chi, \xi$ confirm that the "false non-reciprocity" for the lossless part, $\chi$, is greater than that of the lossy part, $\xi$, because the losses of the analyzed device are low. Moreover, as expected, $\chi$ and $\xi$ decrease with VNA IF bandwidth.

We can also evaluate the actual values of $r_{ij}, x_{ij}$ of the lossy and lossless parts of the $\zeta$-matrix [11], here not shown for brevity, and reconstruct the "reciprocal" $S$-matrix in Eq. (12), $S^{\text{rec}}$, from Eqs. (2)–(4) with $\chi = \xi = 0$ and the obtained values of $r_{ij}, x_{ij}$. The differences between $S^{\text{rec}}$ and the measured $S_{11}, S_{12}, S_{22}$ and the approximated $\bar{S}_{12}$ are shown in Fig. 4. This figure confirms the previous discussion about the errors amplitude in Eq. (19). In fact, from Fig. 4 it is evident that the greatest error between the reconstructed reciprocal $S^{\text{rec}}$ and the experimental values occurs for $|S_{12} - S_{12}^{\text{rec}}|$ (red curve) and $|S_{21} - S_{21}^{\text{rec}}|$ (dashed blue with dots curve) that are almost the same, lying in the range $8 \cdot 10^{-4} \div 2 \cdot 10^{-2}$, and are proportional to $\chi + j\xi$.

The errors $|S_{11} - S_{11}^{\text{rec}}|$ (black curve) and $|S_{22} - S_{22}^{\text{rec}}|$ (dashed green with stars curve) are almost the same, lie in the range $10^{-7} \div 10^{-3}$, and are proportional to $(\chi + j\xi)^2$ as previously discussed. These errors are comparable with the VNA measurement errors in the X band: $\pm 0.009$ dB for $|S_{ij}|$ and $\pm 0.7$ degrees for $\Phi_{ij}$ [12]. Finally, the definition of $\bar{S}_{12} = \bar{S}_{21}$, Eq. (16), permits to obtain an error for $\left|\bar{S}_{12} - S_{12}^{\text{rec}}\right|$ lower than $|S_{12} - S_{12}^{\text{rec}}|$, as shown in Fig. 4 with the dashed blue curve. This error has an amplitude similar to the errors affecting $S_{11}$ and $S_{22}$, as previously discussed. Hence, the use of $\bar{S}_{12}$ evaluated with Eq. (16) permits to reproduce the reciprocal $S$ matrix of the device with an error of the same amplitude on all scattering parameters. Clearly, the precision of the experimental values of $S$ can be increased by decreasing VNA IF bandwidth, and consequently, the errors shown in Fig. 4 become even lower.

A second example is based on a standard waveguide of length $L_{wr} = 14.458 \pm 0.03$ mm, contained in a Flann WR90 calibration kit. The scattering matrix has been measured with the same VNA, and the symmetric $\bar{S}_{12}$ has been evaluated together with the reciprocal matrix, obtained with the procedure previously discussed. The differences between $S^{\text{rec}}$ and the measured $S_{12}, S_{21}$ and the approximated $\bar{S}_{12}$ are shown in Fig. 5. The lowest error is $\left|\bar{S}_{12} - S_{12}^{\text{rec}}\right|$, as expected by previous discussion. $S_{11}$ and $S_{22}$ are not reported because they are negligible.

As a final remark, it should be noted that the reconstructed reciprocal scattering matrix, $S^{\text{rec}}$, contains measurement errors that decrease with the VNA IF bandwidth.
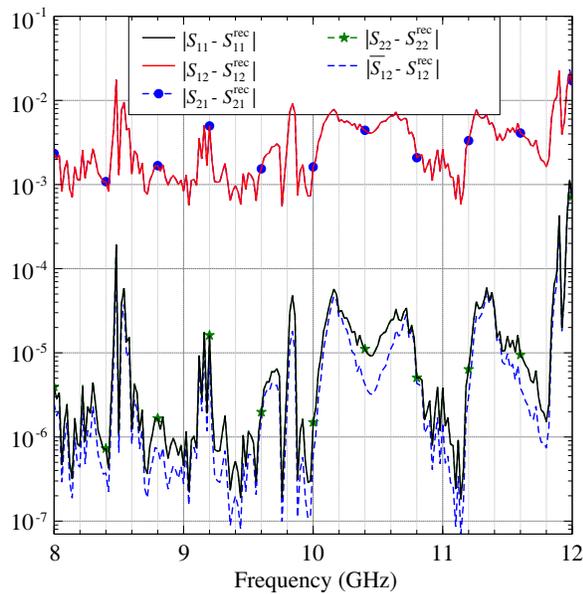
**Figure 4.** Amplitude of the difference between the measured and the reconstructed reciprocal scattering parameters $\left|S_{ij} - S_{ij}^{\text{rec}}\right|$ and $\left|\bar{S}_{12} - S_{12}^{\text{rec}}\right|$ for the device shown in Fig. 1.
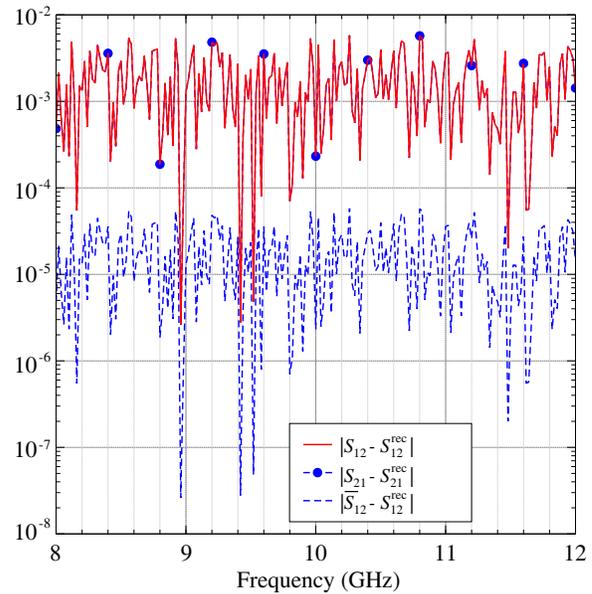
**Figure 5.** Amplitude of the difference between the measured and the reconstructed reciprocal scattering parameters $\left|S_{12} - S_{12}^{\text{rec}}\right|$, $\left|S_{21} - S_{21}^{\text{rec}}\right|$ and $\left|\bar{S}_{12} - S_{12}^{\text{rec}}\right|$ for a standard Flann WR90 waveguide.

## 4. CONCLUSION

A simplified model of the effects of the measurement errors in the scattering parameters of 2-port reciprocal lossy device have been discussed, and it has been proved that the experimental errors affect principally the phase of $S_{12}, S_{21}$ and, with less impact, their amplitude.

A simple correction on the off-diagonal terms $S_{12}, S_{21}$ has been analyzed to obtain scattering parameters that contain errors of comparable amplitude.

## REFERENCES

1. Zappelli, L., "Equivalent circuits of lossy two-port waveguide devices," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 67, No. 10, 4095–4106, Oct. 2019.

2. Chen, C. and T. Chu, "Accuracy criterion for *s*-matrix reconstruction transforms on multiport networks," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 59, No. 9, 2331–2339, Sep. 2011.

3. Zappelli, L., "Reconstruction of the *S*-matrix of n-port waveguide reciprocal devices from 2-port VNA measurements," *Progress In Electromagnetics Research B*, Vol. 72, 129–148, 2017.

4. Carlin, H., "On the physical realizability of linear non-reciprocal networks," *Proceedings of the IRE*, Vol. 43, No. 5, 608–616, May 1955.

5. Carlin, H., "The scattering matrix in network theory," *IRE Transactions on Circuit Theory*, Vol. 3, No. 2, 88–97, Jun. 1956.

6. Youla, D., L. Castriota, and H. Carlin, "Bounded real scattering matrices and the foundations of linear passive network theory," *IRE Transactions on Circuit Theory*, Vol. 6, No. 1, 102–124, Mar. 1959.

7. Youla, D., "Direct single frequency synthesis from a prescribed scattering matrix," *IRE Transactions on Circuit Theory*, Vol. 6, No. 4, 340–344, Dec. 1959.

8.  Carlin, H. and D. Youla, "Network synthesis with negative resistors," *Proceedings of the IRE*, Vol. 49, No. 5, 907–920, May 1961.

9.  Carlin, H. and D. Youla, "The realizability of the complex ideal transformer," *IRE Transactions on Circuit Theory*, Vol. 9, No. 4, 412–412, Dec. 1962.

10. Carlin, H., "On the existence of a scattering representation for passive networks," *IEEE Transactions on Circuit Theory*, Vol. 14, No. 4, 418–419, Dec. 1967.

11. Zappelli, L., "Identification of equivalent circuit based on polygon network for nonreciprocal lossy n-port device," *Progress In Electromagnetics Research B*, Vol. 77, 1–20, 2017.

12. https://www.keysight.com/us/en/assets/7018-03314/data-sheets/5990-9783.pdf.