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A New Prediction Method of Rain Attenuation along Millimeter Wave Links Based on a Bivariate Model for the Effective Path Length and Weibull Distribution

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Abstract—Cellular technology is moving towards its 5th generation (5G) that will employ millimeter wave (mmWave) frequencies in the attempt to offer more spectrum and multi-Gigabit-per-second (Gbps) data rates to mobile devices. Various unfavorable propagation phenomena affect mmWave communications, rain attenuation being the most severe one. Various rain attenuation prediction models can be taken into account in the design of terrestrial links based either on cumbersome statistical regression, when sufficient local experimental data are available, or on analytical models where only local rain rate measurements are provided. In this paper, a new prediction method for the rain attenuation is proposed based on a bivariate model for the numerical estimation of the effective path length of a millimeter wave terrestrial link and on Weibull distribution for the representation of the point rainfall rate statistics. To validate the proposed prediction method, the actual data taken into account are extracted from experiments included in the databank of ITU-R SG3. The numerical results obtained show a significant improvement of the prediction accuracy compared to existing prediction models.

1. INTRODUCTION

To address the increasing demand per subscriber in wireless data traffic, wireless networks migrate to the 5th generation (5G) that will use millimeter wave (mmWave) frequencies since they offer channel bandwidths more than ten times wider than the 20 MHz channels offered by the 4th generation Long-Term-Evolution (4G LTE) [1]. In service-driven wireless networks, the operators aim at efficiently and flexibly providing diversified services such as enhanced mobile broadband, ultra-reliable and low-latency communications and massive machine type communications [2]. Since, compared to 4G microwave frequencies currently in use by LTE, at mmWave the wavelengths shrink by an order of magnitude, atmospheric precipitation, diffraction, and material penetration will incur greater attenuation. Hence, determining the effect of atmospheric phenomena in the design of new mmWave protocols becomes critical and using more accurate propagation models becomes imperative. Over the past few years, measurements and models concerning a plethora of scenarios have been presented by many companies and research groups [3–6]. However, due to the fundamental differences between mmWave communications and the existing communication systems operating in the microwaves band, such as in the 2.4 GHz and 5 GHz bands, many propagation problems related to mmWave communications affecting the physical (PHY) layer and thereafter the medium access control (MAC) and subsequent layers are expected to have a severe impact on 5G wireless networks.

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Attenuation due to rain imposes important constraints on frequency bands above 10 GHz as it restricts the range of radio communication systems. Rainfall is a complex meteorological phenomenon due to its stochastic behavior with regard to duration, location and occurrence frequency. Since for every location on Earth the statistical distribution of rain attenuation is obtained from local data concerning the rainfall rate distribution, the accuracy of the rainfall rate measurements affects the accuracy of the rain attenuation estimation.

The usual procedure for the estimation of rain attenuation statistics for a given terrestrial or Earth-space link is to insert local or regional precipitation statistics into a propagation model. In other words, to successfully estimate the rain fade margin of a future link, annual point rainfall rate statistics must be available in the location where the radio communication link is planned. Rain rate data within an average year allow the estimation of the percentage of time during which attenuation due to rain is significant. Unfortunately, long-term rainfall rate data covering the whole distribution are not always available everywhere. Therefore, there is a need for an accurate mathematical model for predicting rain rate cumulative distribution within an average year.Several propagation models have been proposed so far and their main differences depend either on the assumption regarding the probability distribution that represents the rainfall rate statistics properly or on how they take into account the rainfall structure across the propagation path [7–10]. The most usually employed distributions for the representation of the rainfall rate in analytical models are (i) Lognormal [7], (ii) Gamma [8] and (iii) Weibull [11]. The Lognormal model gives a good approximation for many climatic zones only in the region of low rainfall rates, whereas the Gamma model is quite accurate in the region of high rainfall rates. Livieratos et al. [11] proposed Weibull distribution for the representation of the point rainfall rate statistics which showed an almost excellent fitting to rainfall rate data distributions over all climatic zones. The superiority of the approach employing Weibull distribution to represent the rain rate statistical behavior is due to the fact that it is simple and it works globally and more accurately than other distributions [11]. Moreover, it would be expected that if rain rate is Weibull distributed the assumption that the specific rain attenuation and/or rain attenuation is also Weibull distributed can lead to more accurate predictions retaining the mathematical validity of the prediction model. As far as the rainfall structure across the radio link is concerned, the existing propagation models are also differentiated to each other with respect to how the effective path length of the radio propagation path is numerically estimated. The effective path length aims at replacing the actual path length with a hypothetical one that is equivalently affected by uniform point rainfall along its span.

The existing ITU-R prediction model for rain induced attenuation over terrestrial line-of-sight (LOS) links does not perform accurately on a global level [12]. Although it is widely used by engineers and researchers in the field of radio propagation, it is theoretically hampered by the fact that it is rather a regression model with parameters being expressed by formulas which cannot be easily interpreted with a physical meaning. This constitutes the main motivation to formulate alternative prediction models which, by employing appropriate attributes, apply better to specific locations or climatic zones and ideally could be expressed in a simple and probabilistically solid manner. In any case, it is not an easy task to employ a complex prediction model for rain attenuation that applies to any location and climatic region. In the present paper a new prediction method is presented that is based on two new approaches. A new bivariate model for the numerical estimation of the effective path length is introduced along with the employment of Weibull distribution not only for the description of the rainfall rate statistics but also for the specific rain attenuation. The proposed method accomplishes an improved accuracy of the rain attenuation prediction over terrestrial LOS links.

The rest of the paper is organized as follows. Section 2 presents the most important prediction models of rain attenuation over terrestrial LOS links that are currently in use. Section 3 presents the basic mathematical background referring to Weibull distribution for both rainfall rate and specific rain attenuation and how they are related to each other algebraically. In Section 4, the proposed method is analyzed. Also, its prediction accuracy is validated by performing performance comparison with relevant prediction models, taking into account real data for rainfall rate and rain attenuation extracted from the experimental databank of ITU-R. Finally, Section 5 concludes the paper and presents fields for further study and application of the proposed method.

2. EXISTING RAIN ATTENUATION PREDICTION MODELS

Rain induced attenuation may severely degrade radio wave propagation. Due to absorption and scattering at frequencies above 10 GHz, liquid rain drops become a serious contribution to transmission losses [13]. In terrestrial LOS links or Earth-space links operating above 10 GHz, the occurrence of rain along the transmission path constitutes the most important impairment factor for microwave system degradation [14]. The rain attenuation over a terrestrial path is determined as the product of the specific rain attenuation γ_R (dB/km) and the effective propagation path length d_{eff} (km) [13]. The specific rain attenuation, which is the main parameter characterizing the rain attenuation on a local basis, depends on frequency, polarization, and latitude [11]. The ITU-R recommendation P.838-3 [15] establishes the procedure relating the specific rain attenuation γ_R to the rain intensity R (mm/h). γ_R is obtained from R exceeded at p_{exc} percent of the time on a yearly basis $R_{p_{exc}}$ (mm/h) using the power law relationship

$$\gamma_R = k(R_{p_{exc}})^{\alpha} \tag{1}$$

where k and a depend on the frequency and polarization of the electromagnetic wave. These parameters appear in recommendation tables in [15] and can also be obtained by interpolation on a logarithmic scale for k and linear for a. The rain attenuation A (dB) exceeded for p_{exc} percent of the time on a yearly basis is calculated from

$$A = \gamma_R d_{\mathit{eff}} \tag{2}$$

Since rainfall is not uniform along the actual propagation path, the effective propagation path length d_{eff} depends drastically on the actual path length. Four of the most frequently encountered rain attenuation prediction models are following.

2.1. ITU-R P.530-16

Based on the previous approach for the calculation of the specific attenuation the ITU-R Recommendation 530-16 [16] determines the path attenuation exceeded for 0.01% of the time. Employing an empirical formula, the results obtained are scaled by to percentages of time ranging from 1% to 0.001%. This method is advised to be used in all parts of the world which stated that the rain attenuation needs to be considered for any operating frequency beyond 5 GHz and for frequencies up to 100 GHz with path lengths up to 60 km. The prediction calculations rely on the following steps:

i. Determine the effective propagation path length of the link, d_{eff}, by multiplying the actual path length L (km) by the path reduction factor r, which can be expressed as:

$$r = \frac{1}{0.477 L^{0.633} R_{0.01\%}^{0.073a} f^{0.123} - 10.579\left(1 - \exp\left(-0.024L\right)\right)} \tag{3}$$

where f (GHz) is the frequency, and a is the exponent of the specific rain attenuation model. Maximum recommended r is 2.5, so if the denominator of Eq. (3) is less than 0.4 use $r = 2.5$. $R_{0.01\%}$ is the rain rate exceeded for 0.01% of the time in a year. If this information is not locally available, an estimate can be obtained from the information given in Recommendation ITU-R P.837-7 [17].

ii. The rain attenuation exceeded for 0.01% of the yearly time is calculated as

$$A_{0.01\%} = \gamma_R d_{\mathit{eff}} = \gamma_R L r \tag{4}$$

The purpose of introducing the path reduction factor is to replace the actual path length with a hypothetical path length equivalently affected by uniform point rainfall.

iii. The prediction of the rain attenuation for other exceeded time percentages P, ranging from 0.001% to 1%, is given by

$$A\left(P\right) = A_{0.01\%} C_1 P^{-\left(C_2 + C_3 \log_{10} P\right)} \tag{5}$$

with C_1, C_2, and C_3 being empirical coefficients depending on frequency f [16].

2.2. Da Silva Mello Model

This model uses the numerical coefficients derived for effective rain rate and equivalent rain cell diameter obtained by multiple non-linear regressions, using the measured data available in the ITU-R databank. In the model proposed by Da Silva Mello et al. in [18], the effective path length d_{eff} is calculated as:

$$d_{\mathit{eff}} = \frac{1}{1+\frac{L}{d_0}} L \tag{6}$$

where

$$d_0 = 119 R(P)^{-0.244} \tag{7}$$

The prediction of the rain attenuation exceeded for P percent of the time is achieved as:

$$A\left(P\right) = k R_{\mathit{eff}}^{a} d_{\mathit{eff}} \tag{8}$$

where R_{eff}, i.e., the effective rain rate, is:

$$R_{\mathit{eff}} = 1.763 R(P)^{0.753+0.197/L} \tag{9}$$

2.3. Moupfouma's Model

This model uses only the parameter $R_{0.01\%}$ (mm/h) which, in the area of interest, represents the rainfall rate value exceeded for 0.01% per year. This model does not need rainfall rate numerical values for all time percentages. Similarly to the ITU-R model, the prediction method proposed by Moupfouma in [19] receives $R_{0.01\%}$ as the input to predict A as:

$$A\left(P\right) = k R_{0.01\%}^{a} L_{eq}(PL) \tag{10}$$

L_{eq} is the equivalent path length denoting what exactly d_{eff} expresses. It is calculated as:

$$L_{eq}\left(P, L\right) = L \exp\left(-\frac{R(P)}{1+\zeta(L) R(P)}\right) \tag{11}$$

where

$$\zeta\left(L\right) = \begin{cases} -100, & L \le 7\,(\mathrm{km}) \\ \left[\dfrac{44.2}{L}\right]^{0.78}, & L > 7\,(\mathrm{km}) \end{cases} \tag{12}$$

2.4. Lin Model

The methodology proposed by Lin in [20] takes advantage of the concept of the path reduction factor to estimate rain attenuation statistics on terrestrial links. The method accounts for partially correlated rainfall rate variations along the propagation path length. According to this model,the rain attenuation exceeded for a percentage P of the yearly time can be calculated as:

$$A\left(P\right) = k R(P)^{a} d_{\mathit{eff}} = k R(P)^{a} L r \tag{13}$$

$R(P)$ is the rainfall rate exceeded for the same percentage P of the time. The factor r takes the following simple expression:

$$r = \frac{1}{1+\dfrac{L}{L(R)}} \tag{14}$$

where

$$L\left(R\right) = \frac{2623}{R\left(P\right) - 6.2} \tag{15}$$

3. SPECIFIC RAIN ATTENUATION AS A WEIBULL DISTRIBUTED RANDOM VARIABLE

The fundamental assumption of the present paper is that the rainfall rate is a Weibull distributed random variable (rv) on a global level as presented and verified in [11]. For simplicity reasons let us consider the rv X denoting the rainfall rate. The probability density function (pdf) of X is

$$f_X(x) = \frac{b}{\lambda}\left(\frac{x}{\lambda}\right)^{b-1} e^{-\left(\frac{x}{\lambda}\right)^b} \tag{16}$$

where $b > 0$ is the shape parameter, and $\lambda > 0$ is the scale parameter of the distribution. The corresponding cumulative distribution function (cdf) is

$$F_X(x) = \text{Prob}\left(X < x\right) = 1 - e^{-\left(\frac{x}{\lambda}\right)^b} \tag{17}$$

Further, let us consider the rv Y denoting the specific rain attenuation. Taking into account Eq. (1) we have

$$Y = g(X) = kX^a \tag{18}$$

where parameters k and a have been mentioned in Section 2. Moreover, g is a differentiable and monotonically increasing function. Given that the distribution of X is Weibull, we are coming up with a direct formula to find the distribution of Y, which is a function of X, in the cases where the function g is differentiable and monotonic, as follows:

$$F_Y(y) = \text{Prob}\left(Y \leq y\right) = \text{Prob}\left(g(X) \leq y\right) = \text{Prob}\left(X \leq g^{-1}(y)\right) = F_X\left(g^{-1}(y)\right) \tag{19}$$

where F_Y is the cdf of Y, and g^{-1} is the inverse function of g. Taking the derivative of Eq. (19) we obtain

$$f_Y(y) = F_Y'(y) = \frac{\partial}{\partial y}\left(F_X\left(g^{-1}(y)\right)\right) = f_X\left(g^{-1}(y)\right)\frac{\partial}{\partial y}\left(g^{-1}(y)\right) \tag{20}$$

Solving the equation expressed in Eq. (18) with respect to X, the root is

$$X = \left(\frac{Y}{k}\right)^{\frac{1}{a}} \tag{21}$$

Inserting (21) in (20) the pdf of Y can be obtained

$$f_Y(y) = f_X\left(\left(\frac{y}{k}\right)^{\frac{1}{a}}\right)\frac{\partial}{\partial y}\left(\left(\frac{y}{k}\right)^{\frac{1}{a}}\right) \tag{22}$$

which after some algebraic calculations becomes

$$f_Y(y) = \frac{\frac{b}{a}}{k\lambda^{\alpha}}\left(\frac{y}{k\lambda^{\alpha}}\right)^{\frac{b}{a}-1} e^{-\left(\frac{y}{k\lambda^{\alpha}}\right)^{\frac{b}{a}}} \tag{23}$$

By the above we conclude that if the rainfall rate is a Weibull distributed rv with shape parameter b and scale parameter λ, then the corresponding specific rain attenuation is also a Weibull distributed rv with shape parameter $\frac{b}{a}$ and scale parameter $k\lambda^{\alpha}$ where k and a depend on the frequency and polarization of the electromagnetic wave and are calculated employing [15].

4. THE PROPOSED PREDICTION METHOD AND NUMERICAL RESULTS

ITU-R has proposed a model for the prediction of rain attenuation on terrestrial radio links [16]. However, the ITU-R model does not perform satisfactorily in all climatic zones [12]. In general, it is not possible to apply a prediction model to any location or any climatic region. As discussed so far, Weibull distribution is modeling the rainfall rate better than the usually employed Lognormal and Gamma distributions [11] on a global scale. In the previous Section 3, it was verified that if the rain rate is a Weibull distributed rv the specific rain attenuation is also a Weibull distributed rv. Analytical models that employ Weibull representation of the rainfall rate and specific rain attenuation, being more accurate are expected to perform better in general in the prediction of rain attenuation provided that all other assumptions remain the same as those of the models that employ Lognormal and Gamma distributions [11]. As it is shown in Section 2, the existing rain attenuation prediction models, amongst others, are differentiated one another mainly with respect to how the effective propagation path length d_{eff} is modelled and numerically calculated. So, it seems that a more efficient model for d_{eff} can facilitate a more accurate prediction of the rain attenuation. In general, the inputs required by most prediction models for rain induced attenuation over terrestrial links are the rainfall rate exceeded for a specific time percentage, the propagation path length, the operation frequency, and the wave polarization.

The terrestrial LOS links included in the ITU-R databank provide experimental data for the location and climatic zone such as local rainfall rate, path length, operation frequency, wave polarization and the corresponding rain induced attenuation for specific exceedance probability levels, namely 0.001%, 0.002%, 0.003%, 0.006%, 0.01%, 0.02%, 0.03%, 0.06% and 0.1% referred to on one-year period. Hereafter, these levels will be called reference exceedance probability levels (REPLs). $A(p\%)$ is the rain attenuation in dB exceeded for the REPL that is equal to $p\%$ on a per year basis. In general, this is expressed as

$$\mathrm{Prob}\left(A > A\left(p\%\right)\right) = p\% \tag{24}$$

Taking into account (2), (24) becomes

$$\mathrm{Prob}\left(\gamma_R d_{\mathit{eff}} > A\left(p\%\right)\right) = p\% \tag{25}$$

and equivalently

$$\mathrm{Prob}\left(d_{\mathit{eff}} > \frac{A\left(p\%\right)}{k(R(p\%))^{\alpha}}\right) = p\% \tag{26}$$

From Eq. (26) it is obvious that d_{eff} can take different values for the various REPLs for the same terrestrial radio link.

The availability of data regarding rainfall rate and rain induced attenuation referred to real terrestrial LOS links of the ITU-R databank that operate in various locations encourages the development of an alternative bivariate regression model for calculating the effective propagation path length. The proposed method represents the effective propagation path length, denoted as $d_{\mathit{eff},\,biv}$, as a bivariate function of the effective path length determined in the frame of the ITU-R rain attenuation model, denoted as $d_{\mathit{eff},\,\mathrm{ITU}}$, and of the actual path length L:

$$d_{\mathit{eff},\,biv}\left(p\%\right) = f_{\mathrm{regression}}\left(d_{\mathit{eff},\,\mathrm{ITU}},\, L\right) \tag{27}$$

The actual values of $d_{\mathit{eff},\,biv}$ can be calculated as

$$d_{\mathit{eff},\,biv}\left(p\%\right) = \frac{A\left(p\%\right)}{k(R(p\%))^{\alpha}} \tag{28}$$

where

$A(p\%)$ is the rain induced attenuation in dB exceeded for each REPL that is equal to $p\%$,

$R(p\%)$ is the rainfall rate exceeded for each REPL that is equal to $p\%$, and k and a depend on the frequency and polarization of the electromagnetic wave which range from 7 GHz to 137 GHz and from 0 degrees to 90 degrees respectively.

The algebraic expression proposed for the regression function is

$$d_{\mathit{eff},\,biv}\left(p\%\right) = c_1\left(p\%\right) d_{\mathit{eff},\,\mathrm{ITU}}^{c_2(p\%)} L^{c_3(p\%)} \tag{29}$$

where

$d_{\mathit{eff},\,biv}(p\%)$ is the effective propagation path length corresponding to a REPL equal to $p\%$,

$d_{\mathit{eff},\,\mathrm{ITU}}(p\%)$ is the effective propagation path length as calculated for the ITU-R model corresponding to a REPL equal to $p\%$,

L is the length of the real propagation path and ranges from 0.5 km to 58 km, and $c_1(p\%)$, $c_2(p\%)$, $c_3(p\%)$ are coefficients to be determined as a result of the regression process.

Table 1. Coefficients of the proposed model for the effective propagation path length obtained from bivariate regression.

REPL	c_1	c_2	c_3	R^2
0.001%	1.381	0.5136	0.15230	88.9%
0.002%	1.154	0.7017	0.10540	90.3%
0.003%	1.139	0.7823	0.06675	90.6%
0.006%	1.165	0.7949	0.08788	92.0%
0.01%	1.176	0.9280	0.00720	89.3%
0.02%	1.206	0.8342	0.11490	88.3%
0.03%	1.081	1.0080	0.04603	86.1%
0.06%	1.296	0.8431	0.17980	82.4%
0.1%	1.531	0.6182	0.34860	79.4%

The numerical results obtained from the regression analysis that employed all the experiments of the ITU-R databank are tabulated in Table 1.

The effectiveness of the regression is expressed employing the R^2 metric that ranges from 0 to 1. R^2 is a statistical measure called coefficient of determination and provides a measure of how well the observed outputs are replicated by the regression function. R^2 essentially expresses the proportion of the variance in the dependent response $d_{\mathit{eff},biv}$ that is predictable from the independent variables $d_{\mathit{eff},\mathrm{ITU}}$ and L. Indicatively, the interpretation of $R^2 = 90\%$ is that 90% of the variance in the response can be attributed to the employed independent variables whereas the remaining 10% can be attributed to unknown, lurking variables or inherent variability. The proposed bivariate regression model can be applied independently from the distribution employed to describe the rainfall rate statistics. The proposed bivariate model for d_{eff} is essentially a synthetic variable that takes into account the real length of the radio path and the estimation of the ITU-R model aiming at correcting the former estimation and bringing it closer to its actual values as calculated using Eq. (26). Employing synthetic variables is usually followed in the field of data analytics as a facilitator of accurate regression and/or classification modeling so that they can successfully replace original variables [21].

If the rain attenuation A, expressed in dB, exceeds $A(p\%)$ for $p\%$ on a per year basis, as provided in the ITU-R databank, the following equation is satisfied

$$\mathrm{Prob}\left(A > A\left(p\%\right)\right) = p\% \tag{30}$$

Given Eq. (2), Eq. (30) becomes

$$\mathrm{Prob}\left(\gamma_R d_{\mathit{eff}}\left(p\%\right) > A\left(p\%\right)\right) = p\% \tag{31}$$

which equivalently becomes

$$\mathrm{Prob}\left(\gamma_R > \frac{A\left(p\%\right)}{d_{\mathit{eff}}\left(p\%\right)}\right) = p\% \tag{32}$$

Taking into account that the specific rain attenuation γ_R is a Weibull distributed rv with parameters b_{γ_R} and λ_{γ_R}, Eqs. (17), (32) can take the following algebraic form

$$e^{-\left(\frac{\frac{A(p\%)}{d_{\mathit{eff}}(p\%)}}{\lambda_{\gamma_R}}\right)^{b_{\gamma_R}}=p\%} \tag{33}$$

In Section 3 it was proved that if the rainfall rate R is a Weibull distributed rv with shape parameter b_R and scale parameter λ_R, then the specific rain attenuation γ_R is also a Weibull distributed rv with shape parameter

$$b_{\gamma_R} = \frac{b_R}{a} \tag{34}$$

and scale parameter

$$\lambda_{\gamma_R} = k\lambda_R^a \tag{35}$$

where k and a depend on the frequency and polarization of the electromagnetic wave and are calculated employing [15]. Hence, if the rainfall rate data of each experiment is used to determine via regression the exact Weibull distribution that describes the local rainfall rate behavior, the parameters of the distribution of the corresponding specific rain attenuation can be determined too by using Eqs. (34) and (35). If in Eq. (33) $d_{\mathit{eff}}(p\%)$ is replaced with $d_{\mathit{eff},\,biv}(p\%)$, then applying the logarithmic function in Eq. (33) it is obtained

$$A_W\left(p\%\right) = d_{\mathit{eff},\,biv}\left(p\%\right)\left(-\ln p\%\right)^{\frac{1}{b_{\gamma_R}}}\lambda_{\gamma_R} \tag{36}$$

where

b_{γ_R} is the shape parameter of the specific rain attenuation γ_R given in Eq. (34),

λ_{γ_R} is the scale parameter of the specific rain attenuation γ_R given in Eq. (35),

$d_{\mathit{eff},\,biv}(p\%)$ is the effective propagation path length expressed in Eq. (29) along with the parameters c_1, c_2 and c_3 which are tabulated in Table 1, and $A_W(p\%)$ is the rain induced attenuation in dB exceeded for each REPL that is equal to $p\%$ if rainfall rate is taken as a Weibull distributed rv with shape parameter b_R and scale parameter λ_R.

It is important to consider how the proposed method performs in comparison to the existing prediction models for rain attenuation that were presented in Section 2. Numerical calculations were performed using rainfall rate data extracted from the ITU-R experiment databank. First, a regression analysis was conducted regarding the local rainfall data for each experiment included in the databank employing Weibull distribution so that the parameters b_R and λ_R were determined. By applying the calculation process recommended in [15] the parameters of specific rain attenuation, which is also a Weibull distributed rv, were also determined for each experiment. Using Eq. (36) the rain induced attenuation exceeded for each REPL can be calculated provided that $d_{\mathit{eff},\,biv}$ has been calculated using Eq. (29) and Table 1. To become comparable, the respective results have been normalized adopting the test variable ρ_V proposed by ITU-R in Recommendation P.311-13 [22]. According to [22], for each time percentage examined and each radio link of the ITU-R databank considered, say the ith link, the ratio of the predicted rain attenuation, A_p (dB), to the measured rain attenuation, A_m (dB), is calculated from

$$S_i = \frac{A_{p,\,i}}{A_{m,\,i}} \tag{37}$$

Next, the variable V_i is calculated from

$$V_i = \begin{cases} \left(\dfrac{A_{m,\,i}}{10}\right)^{0.2} \ln S_i, & \text{for } A_{m,\,i} < 10\,\text{dB} \\ \ln S_i, & \text{for } A_{m,\,i} \geq 10\,\text{dB} \end{cases} \tag{38}$$

Then, the mean μ_V and standard deviation σ_V of the V_i values for each time percentage are calculated. Finally, the test variable is determined as the RMS (root mean square) value

$$\rho_V = \sqrt{\mu_V^2 + \sigma_V^2} \tag{39}$$

In comparing the various prediction methods, it should be noted that the lower the test variable ρ_V is, the better the prediction method is. The numerical results obtained following the above comparison procedure are depicted in Fig. 1. As readily observed from Fig. 1, the proposed prediction method based on Weibull distribution performs significantly better than the four prediction models under comparison whereas its accuracy is similar to or slightly less than that of the ITU-R model for exceeded times lower than 0.003% on a yearly basis. Obviously the prediction models mentioned in Section 2, other than the ITU-Rone, are simulation models that have lower accuracy and are differentiated as far as d_{eff} is concerned. On the other hand, the proposed model, which employs the Weibull distribution, perfectly fits the rain rate data for all climatic zones and has a solid probabilistic foundation. In addition, it is proved that the specific rain attenuation is also a Weibull distributed rv, with parameters that can be calculated based on the corresponding ones of rain rate rv. Hence, it'd be expected and it is verified that the proposed model performs better than the existing ones for all REPLs whereas it is shown that it performs better than the ITU-R model too, for REPLs higher than 0.003%.

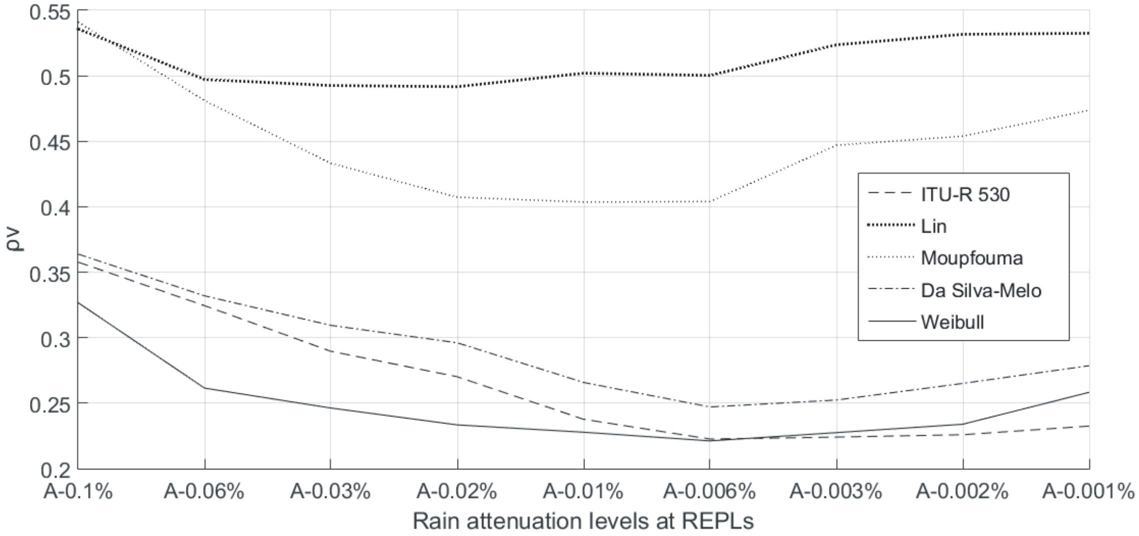

Figure 1. Regression effectiveness expressed in the form of the test variable ρ_V employing various rain attenuation prediction models and the proposed prediction method employing a bivariate function for the effective radio propagation path as well as Weibull distribution for the statistical representation of point rainfall rate and specific rain attenuation.

Although the effectiveness of the proposed model is illustrated in Fig. 1 in terms of the test variable ρ_V [22], a similar check can be made considering the following Table 2, in which the RMS value of the regression error is tabulated for the various REPLs. In particular, the RMS error is defined as

$$\varepsilon_{\mathrm{RMS}}\,(\mathrm{dB}) = \sqrt{\varepsilon_{\mathrm{mean}}^2\,(\mathrm{dB}) + \varepsilon_{\mathrm{std}}^2\,(\mathrm{dB})} \tag{40}$$

where $\varepsilon_{\mathrm{mean}}$ is the mean value of the regression error, $A_m - A_p$, and is calculated for each REPL, and $\varepsilon_{\mathrm{std}}$ is the standard deviation of the regression error, $A_m - A_p$, and is calculated for each REPL.

Table 2. RMS error of the proposed and the existing rain attenuation prediction models.

REPL	$\varepsilon_{\mathrm{RMS,\,Weibull}}$ (dB)	$\varepsilon_{\mathrm{RMS,\,ITU\text{-}R.530}}$ (dB)	$\varepsilon_{\mathrm{RMS,\,Da\ Silva\ Mello}}$ (dB)	$\varepsilon_{\mathrm{RMS,\,Moupfouma}}$ (dB)	$\varepsilon_{\mathrm{RMS,\,Lin}}$ (dB)
0.001%	7.46	7.35	9.37	13.79	18.02
0.002%	7.49	7.08	8.15	11.87	15.88
0.003%	6.69	6.81	7.11	10.90	14.09
0.006%	5.73	5.65	6.68	8.84	13.58
0.01%	5.70	6.03	6.99	8.87	13.54
0.02%	5.17	5.43	6.52	6.46	11.10
0.03%	4.69	5.13	6.07	6.03	10.90
0.06%	3.68	4.15	4.33	4.39	8.10
0.1%	3.10	3.50	3.57	3.76	5.77

The mean, standard deviation, and RMS values of the regression error have been calculated taking into account all the LOS links included in the ITU-R databank which covers all the climatic zones, path lengths that range from 0.5 km to 58 km, operating frequencies that range from 7 GHz to 137 GHz and electromagnetic wave polarization that ranges from 0 degrees to 90 degrees.

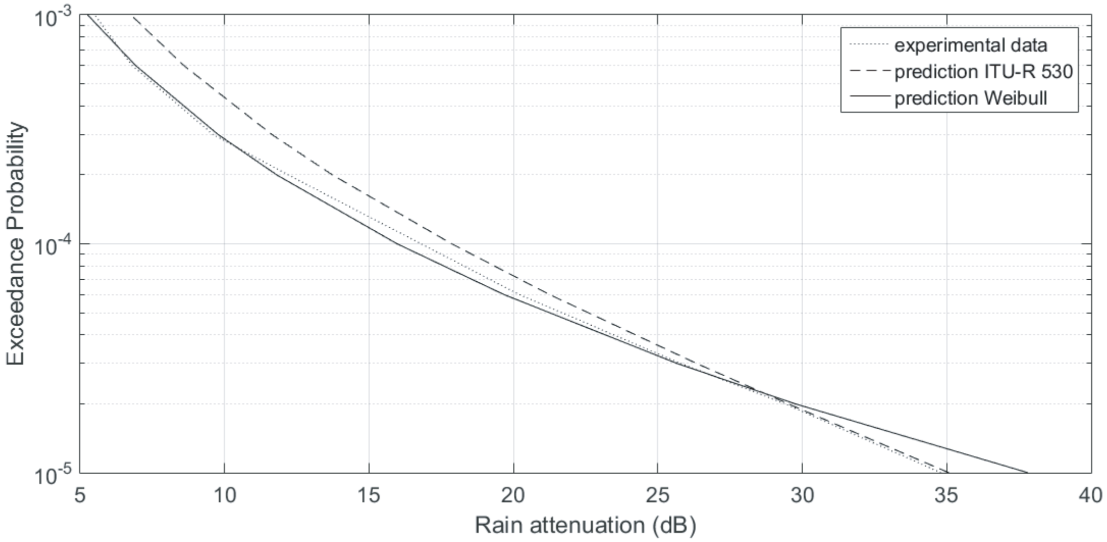

Figure 2. Exceedance probability, Mendlesham, UK, frequency 20.1 GHz, path length 16.6 km and vertical polarization.

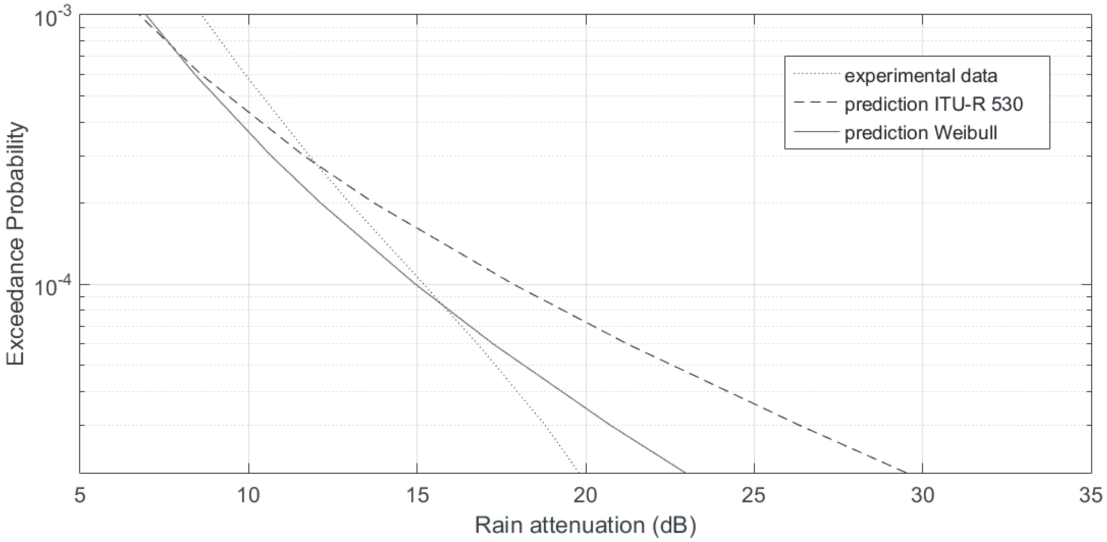

Figure 3. Exceedance probability, Chilbolton, UK, frequency 137 GHz, path length 0.5 km and vertical polarization.

At this point, it is worthwhile to investigate how the proposed method performs if it is applied to particular experimental configurations. In Fig. 2 the exceedance probability that the rain induced attenuation will be higher than a certain level in dB, which is mathematically expressed as Prob(Rain attenuation > level), for a terrestrial link in Mendlesham, UK, which is included in the ITU-R databank, is illustrated. Apart from the rainfall rate and the rain attenuation actual data of the aforementioned link that are available, its other operational characteristics are $f = 20.1\,\text{GHz}$, path length $L = 16.6\,\text{km}$ and the used polarization is vertical. It is obvious that the proposed method performs better than the ITU-R model for exceedance probabilities higher than $2\ 10^{-5}$ as shown by the Weibull prediction curve which almost coincides with the experimental data curve. The corresponding time percentage is 0.002% on a per year basis, which refers to approximately 10 min per year. The interpretation of the above is that the probability that the rain attenuation will be higher than 30 dB is approximately $2\ 10^{-5}$ or equivalently the rain attenuation will be higher than 30 dB for 0.002% of the year, i.e., 10 min provided that one year has 525,600 minutes.

It is of high importance to see how the proposed method performs for higher frequencies closer to mmWave wavelengths. In Figs. 3, 4, and 5 the exceedance probability that the rain induced attenuation will be higher than a certain level in dB is illustrated for the same terrestrial link in Chilbolton, UK, which is included in the ITU-R databank. Apart from the rainfall rate and the rain attenuation actual data of the aforementioned link that are available, its other operational characteristics are $f = 137\,\text{GHz}$, 97 GHz and 57 GHz respectively whereas path length $L = 0.5\,\text{km}$ and the used polarization is vertical. In Fig. 3, where the operating frequency is 137 GHz, the proposed method fits much better to the experimental data than the ITU-R model. Particularly for probabilities ranging from $2\ 10^{-5}$ to $2\ 10^{-4}$ the proposed method has a very slight deviation from the actual experimental data whereas the ITU-R model has a significantly lower accuracy. The aforementioned range of exceedance probabilities is of high importance in practice. It refers to exceeded time percentages ranging from 0.002% to 0.02% on

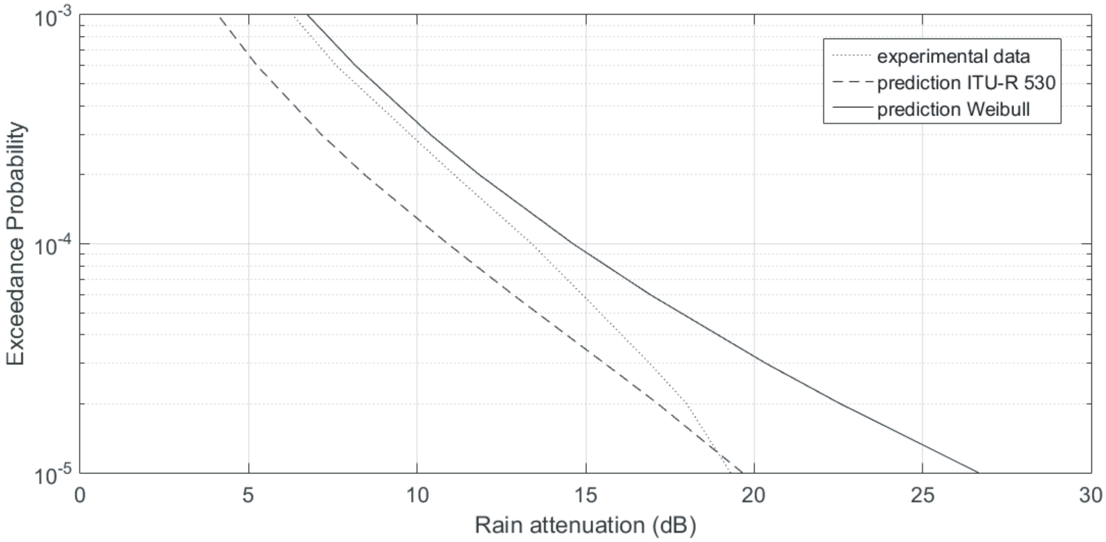

Figure 4. Exceedance probability, Chilbolton, UK, frequency 97 GHz, path length 0.5 km and vertical polarization.

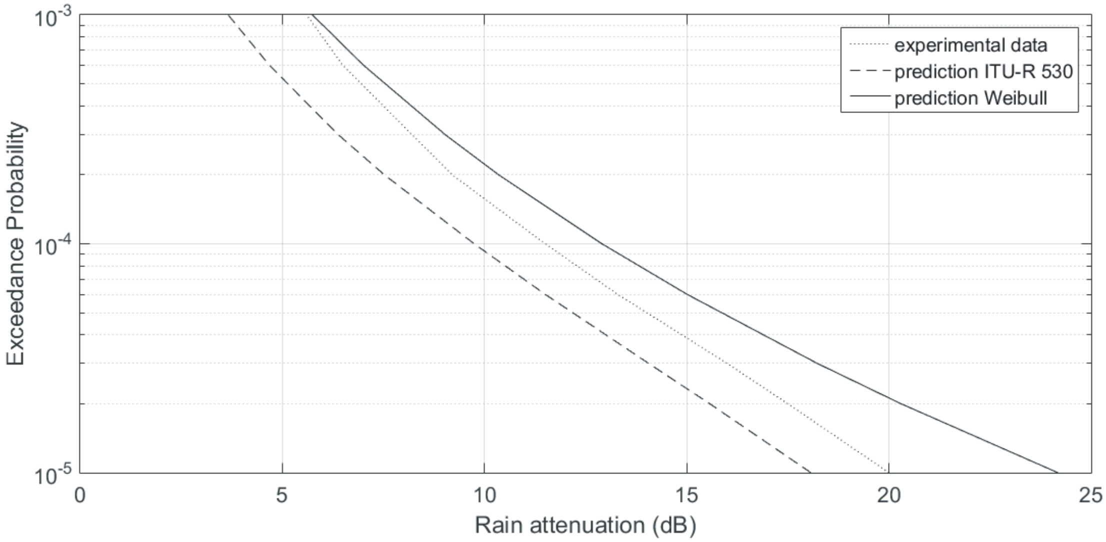

Figure 5. Exceedance probability, Chilbolton, UK, frequency 57 GHz, path length 0.5 km and vertical polarization.

a per year basis and in turn to cumulative time periods that rain attenuation is expected to exceed the respective levels of 20 dB and 13 dB which are 10 min and 105 min respectively. Such information can be very useful for the network planning as it facilitates the estimation of reliability and availability figures of the links under study.

In Fig. 4 the same experimental configuration is considered, but the operating frequency is 97 GHz. The proposed method fits much better to the experimental data than the ITU-R model especially for probabilities ranging from $5\ 10^{-5}$ to 10^{-3} which correspond to cumulative time periods that rain attenuation is expected to exceed the respective levels of 16 dB and 6 dB which are 26 min and 525 min respectively. Moreover, the proposed method slightly overestimates the exceedance probability providing an additional safety margin for planning purposes. Likewise, important conclusions can be obtained from Fig. 5 where the experimental configuration remains the same but the operating frequency is 57 GHz. The proposed method fits much better to the experimental data than the ITU-R model for the entire range of exceedance probabilities providing an additional safety margin as in the previous case.

5. CONCLUSIONS

In this paper, two new concepts were introduced. Firstly, a new bivariate regression for the estimation of the effective propagation path length was presented. This approach is differentiated from the existing methods intended to estimate the effective propagation path length, which essentially go along with and reflect the various rain attenuation prediction models. The independent variables are the real path length and the effective propagation path length as calculated from the ITU-R model. Further investigation may be conducted possibly aiming at a more efficient multivariate approach. Secondly, Weibull distribution for both rainfall rate and specific rain attenuation statistics was employed and the algebraic expressions of their interrelation were provided. The proposed method to predict rain attenuation is based on the combination of the two aforementioned concepts in a single prediction model as presented in Section 4. The proposed method has been validated taking into account experimental data from the ITU-R databank concerning LOS terrestrial links. The numerical results obtained showed a prediction accuracy improvement over the prediction accuracy offered by the existing prediction models currently in use, reaching very high accuracy levels. New propagation data can be collected if new experiments can be properly planned, so that to be employed towards improving the proposed method and dealing with various propagation phenomena.

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