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Spreading of Four-Petal Lorentz-Gauss Beams Propagating through Atmospheric Turbulence

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Abstract—The analytical propagation equation of a four-petal Lorentz-Gauss (FPLG) beam propagating through atmospheric turbulence is derived, and the spreading of average intensity is analyzed by using numerical examples. It is found that the FPLG beam propagating through atmospheric turbulence will evolve into Gaussian beam due to the influences of atmospheric turbulence, and the atmospheric turbulence will accelerate the spreading of FPLG beam as the propagation distance increases. It is also found that the FPLG beam with different N or Lorentz widths propagating through atmospheric turbulence will have the same beam spot when the FPLG beam evolves into the Gaussian beam at the same propagation distance.

1. INTRODUCTION

Recently, the free space optical communication has become a topic of laser applications in high rate communication under atmospheric turbulence. Therefore, the evolution properties of laser beam propagating in atmospheric turbulence have been widely studied, such as two Gaussian laser beams [1], apertured partially coherent beam [2], superimposed partially coherent Hermite-Gaussian beams[3], pseudo-Bessel correlated beams [4], Lorentz-Gauss beam [5], radial phased-locked partially coherent anomalous hollow beam array [6], Gaussian array beams [7], four-petal Gaussian vortex beam [8], Airy beam [9], multi-cosine-Laguerre-Gaussian correlated Schell-model beam [10], cusped random beam [11], partially coherent anomalous elliptical hollow Gaussian beam [12], pseudo-Bessel-Gaussian Schell-mode beam [13], partially coherent beam [14], partially coherent crescent-like optical beam [15], partially coherent Lorentz-Gauss beam [16], partially coherent flat-topped vortex hollow beam [17], pulsed Laguerrian beam [18], radial phased-locked partially coherent Lorentz-Gauss array beam [19], phase-locked partially coherent flat-topped array laser beam [20], partially coherent Lorentz-Gauss vortex beam [21], partially coherent four-petal Gaussian vortex beams [22], and ring Airy Gaussian beams with optical vortices [23]. With the development of laser optics, a new beam called four-petal Lorentz-Gauss (FPLG) beam has been introduced and studied [24, 25]. However to our knowledge, the evolution properties of FPLG beam propagating through atmospheric turbulence have not been given. In this paper, we derive the analytical propagation equations of FPLG beam propagating through atmospheric turbulence based on the extended Huygens-Fresnel integral, and the average intensity of FPLG beam propagating through atmospheric turbulence is analyzed by using numerical examples.

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2. PHYSICAL MODEL

In a Cartesian coordinate system, within the validity of the paraxial propagation, propagation of a laser beam through atmospheric turbulence from the source plane $z = 0$ to the space z can be described by the extended Huygens-Fresnel principle [1–9]:

$$\begin{aligned}\langle I(\mathbf{r}, z)\rangle &= \frac{k^2}{4\pi^2 z^2}\int d\mathbf{r}_{10} d\mathbf{r}_{20} E(\mathbf{r}_{10}, 0) E^*(\mathbf{r}_{20}, 0)\\ &\quad\times\exp\left[-\frac{ik}{2z}(\mathbf{r}-\mathbf{r}_{10})^2 + \frac{ik}{2z}(\mathbf{r}-\mathbf{r}_{20})^2\right]\langle\exp[\psi(\mathbf{r}, \mathbf{r}_{10}) + \psi^*(\mathbf{r}, \mathbf{r}_{10})]\rangle \end{aligned}\tag{1}$$

where $\mathbf{r}_{10} = (x_{10}, y_{10})$ and $\mathbf{r}_{10} = (x_{20}, y_{20})$ are the position vectors in the plane $z = 0$; $\mathbf{r} = (x, y)$ is the position vector in the plane z; $k = 2\pi/\lambda$ is the wave number with λ is the wavelength; $E(*, 0)$ is the electric field of laser beam at the plane $z = 0$; $\psi(\mathbf{r}, \mathbf{r}_{10})$ is the random complex phase perturbation caused by the atmospheric turbulence; and the asterisk denotes the complex conjugation. In Equation (1), $\langle\exp[\psi(\mathbf{r}, \mathbf{r}_{10}) + \psi^*(\mathbf{r}, \mathbf{r}_{20})]\rangle$ can be written as

$$\langle\exp[\psi(\mathbf{r}, \mathbf{r}_{10}) + \psi^*(\mathbf{r}, \mathbf{r}_{20})]\rangle = \exp\left[-\frac{(\mathbf{r}_{10}-\mathbf{r}_{20})^2}{\rho_0^2}\right]\tag{2}$$

where $\rho_0 = \left(0.545C_n^2 k^2 z\right)^{-3/5}$ is the spatial coherence length of a spherical wave propagating through the atmospheric turbulence, and C_n^2 is the constant of refractive index structure constant of atmospheric turbulence.

Recently, the Lorentz beam has been given to describe the output of diode laser [26], and a new array beam called the four-petal Lorentz-Gauss (FPLG) beam has also been introduced, which is composed by four Lorentz-Gauss beams and takes the form as [25]:

$$E(\mathbf{r}_0, 0) = \frac{1}{w_{0x}w_{0y}\left[1+\left(\frac{x_0}{w_{0x}}\right)^2\right]\left[1+\left(\frac{y_0}{w_{0y}}\right)^2\right]}\left(\frac{x_0 y_0}{w_0^2}\right)^{2N}\exp\left(-\frac{x_{10}^2+y_{10}^2}{w_0^2}\right)\tag{3}$$

where $\mathbf{r}_0 = (x_0 y_0)$ is the position vector in the plane $z = 0$; w_{0x} and w_{0y} are the beam widths of Lorentz beam in the x-axis and y-axis, respectively. w_0 is the beam width of the Gaussian beam; N is the order of the FPLG beam.

Substituting Equation (3) into Equation (1), the analytical expressions of FPLG beam propagating through atmospheric turbulence can be derived as

$$I(\mathbf{r}, z) = \frac{k^2}{4\pi^2 z^2}\left(\frac{1}{2w_{0x}^2 w_{0y}^2}\right)^2\left(\frac{1}{w_0^2}\right)^{4N}\sum_{m1=0}^{M}\sum_{n1=0}^{M}\sigma_{2m1}\sigma_{2n1}\sum_{m2=0}^{M}\sum_{n2=0}^{M}\sigma_{2m2}\sigma_{2n2} I(x, z) I(y, z)\tag{4}$$

where

$$\begin{aligned}I(x, z) &= \exp\left(-\frac{k^2}{4a_x z^2}x^2\right)\frac{\pi}{\sqrt{a_x b_x}}\sum_{t=0}^{m1}\frac{(-1)^t(2m1)!}{t!(2m1-2t)!}\left(\frac{2}{w_{0x}}\right)^{2m1-2t}(2N+2m1-2t)!\\ &\quad\times\left(\frac{1}{a_x}\right)^{(2N+2m1-2t)}\sum_{s=0}^{\left[\frac{2N+2m1-2t}{2}\right]}\frac{1}{s!(2N+2m1-2t-2s)!}\left(\frac{a_x}{4}\right)^s\\ &\quad\times\sum_{e=0}^{2N+2m1-2t-2s}\frac{(2N+2m1-2t-2s)!}{e!(2N+2m1-2t-2s-e)!}\left(\frac{ik}{2z}x\right)^{2N+2m1-2t-2s-e}\left(\frac{1}{\rho_0^2}\right)^e\\ &\quad\times\sum_{h=0}^{m2}\frac{(-1)^h(2m2)!}{h!(2m2-2h)!}\left(\frac{2}{w_{0x}}\right)^{2m2-2h}2^{-(2N+e+2m2-2h)}i^{2N+e+2m2-2h}\exp\left(\frac{c_x^2}{b_x}\right)\\ &\quad\times\left(\frac{1}{\sqrt{b_x}}\right)^{2N+e+2m2-2h}H_{2N+e+2m2-2h}\left(-\frac{ic_x}{\sqrt{b_x}}\right)\end{aligned}\tag{5}$$

$$
\begin{aligned}
I\left(y,z\right)] \;=\; & \exp\left(-\frac{k^2}{4a_y z^2}y^2\right)\frac{\pi}{\sqrt{a_y b_y}}\sum_{t=0}^{n1}\frac{\left(-1\right)^t\left(2n1\right)!}{t!\left(2n1-2t\right)!}\left(\frac{2}{w_{0y}}\right)^{2n1-2t}\left(2N+2n1-2t\right)! \\
& \times\left(\frac{1}{a_y}\right)^{(2N+2m1-2t)}\sum_{s=0}^{\left[\frac{2N+2n1-2t}{2}\right]}\frac{1}{s!\left(2N+2n1-2t-2s\right)!}\left(\frac{a_y}{4}\right)^s \\
& \times\sum_{e=0}^{2N+2n1-2t-2s}\frac{\left(2N+2n1-2t-2s\right)!}{e!\left(2N+2n1-2t-2s-e\right)!}\left(\frac{ik}{2z}y\right)^{2N+2n1-2t-2s-e}\left(\frac{1}{\rho_0^2}\right)^e \\
& \times\sum_{h=0}^{n2}\frac{\left(-1\right)^h\left(2n2\right)!}{h!\left(2n2-2h\right)!}\left(\frac{2}{w_{0y}}\right)^{2n2-2h}2^{-(2N+e+2n2-2h)}i^{2N+e+2n2-2h}\exp\left(\frac{c_y^2}{b_y}\right) \\
& \times\left(\frac{1}{\sqrt{b_y}}\right)^{2N+e+2n2-2h}H_{2N+e+2n2-2h}\left(-\frac{ic_y}{\sqrt{b_y}}\right)
\end{aligned}
\tag{6}
$$

$$a_j = \frac{1}{2w_{0j}^2}+\frac{1}{w_0^2}+\frac{1}{\rho_0^2}+\frac{ik}{2z}\,(j{=}x,y) \tag{7}$$

$$b_j = \frac{1}{2w_{0j}^2}+\frac{1}{w_0^2}+\frac{1}{\rho_0^2}-\frac{ik}{2z}-\frac{1}{a_j}\left(\frac{1}{\rho_0^2}\right)^2 \tag{8}$$

$$c_j = \left(\frac{1}{a_j\rho_0^2}-1\right)\frac{ik}{2z}j \tag{9}$$

In the derivation Equation (4), the following equations have been used [27, 28]

$$\int_{-\infty}^{+\infty}x^n\exp\left(-px^2+2qx\right)dx = n!\exp\left(\frac{q^2}{p}\right)\left(\frac{q}{p}\right)^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{1}{k!\left(n-2k\right)!}\left(\frac{p}{4q^2}\right)^k \tag{10}$$

$$\frac{1}{\left(x^2+w_{0x}^2\right)\left(y^2+w_{0y}^2\right)} = \frac{\pi}{2w_{0x}^2w_{0y}^2}\sum_{m=0}^{M}\sum_{n=0}^{M}\sigma_{2m}\sigma_{2n}H_{2m}\left(\frac{x}{w_{0x}}\right)H_{2n}\left(\frac{y}{w_{0y}}\right)\times\exp\left(-\frac{x^2}{2w_{0x}^2}-\frac{y^2}{2w_{0y}^2}\right) \tag{11}$$

where M is the number of the expansions for Lorentz function. σ_{2m} and σ_{2n} are the expanded coefficients and given in [27], and M is chosen as $M = 5$. The Hermite polynomial $H_{2m}(x)$ can be written as [28]

$$H_{2m}\left(x\right) = \sum_{l=0}^{m}\frac{\left(-1\right)^l\left(2m\right)!}{l!\left(2m-2l\right)!}\left(2x\right)^{2m-2l} \tag{12}$$

3. NUMERICAL RESULTS AND ANALYSES

Here, we carry out the average intensity of an FPLG beam propagating through atmospheric turbulence. The parameters of FPLG beam are chosen as $w_{0x} = w_{0y} = 1\,\mathrm{cm}$, $w_0 = 2\,\mathrm{cm}$, and $N = 1$ in the numerical calculations.

To investigate the influences of the atmospheric turbulence on the evolution of FPLG beam, 3D normalized FPLG beam propagating through atmospheric turbulence and free space are shown in Figures 1 and 2, respectively. The refractive index structure constant of atmospheric turbulence C_n^2 is set as $C_n^2 = 10^{-14}\,\mathrm{m}^{-2/3}$ in the numerical analysis. The FPLG beam propagating through atmospheric turbulence will gradually evolve from four-petal pattern into the Gaussian beam as the propagation distance increases. To compare with Figure 1, Figure 2 gives the 3D normalized of a FPLG beam propagating through free space ($C_n^2 = 0$). One can find that the FPLG beam propagating through free space will also lose the four-petal pattern, but the FPLG beam propagating through free space will

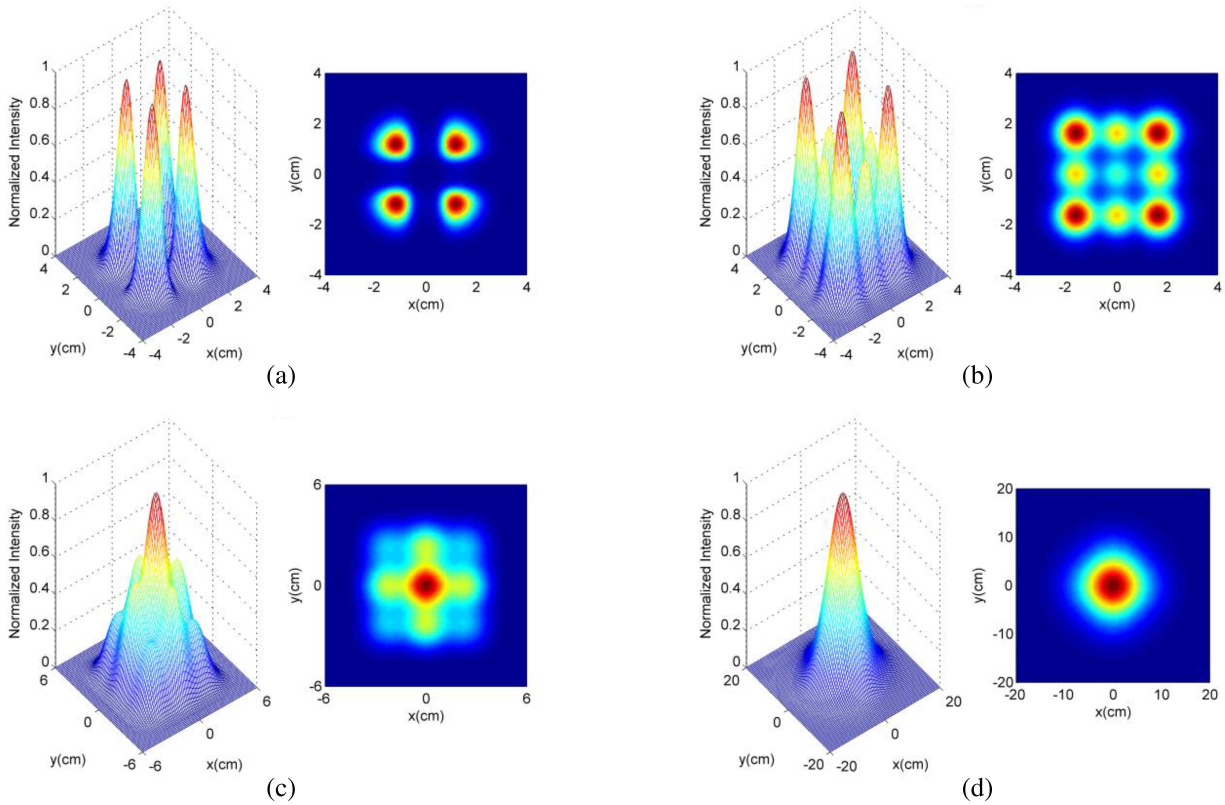


Figure 1. The 3D average intensity and contour graphs of a FPLG beam propagating through atmospheric turbulence. (a) $z = 100\,\mathrm{m}$, (b) $z = 500m$, (c) $z = 1000\,\mathrm{m}$, (d) $z = 3000\,\mathrm{m}$.

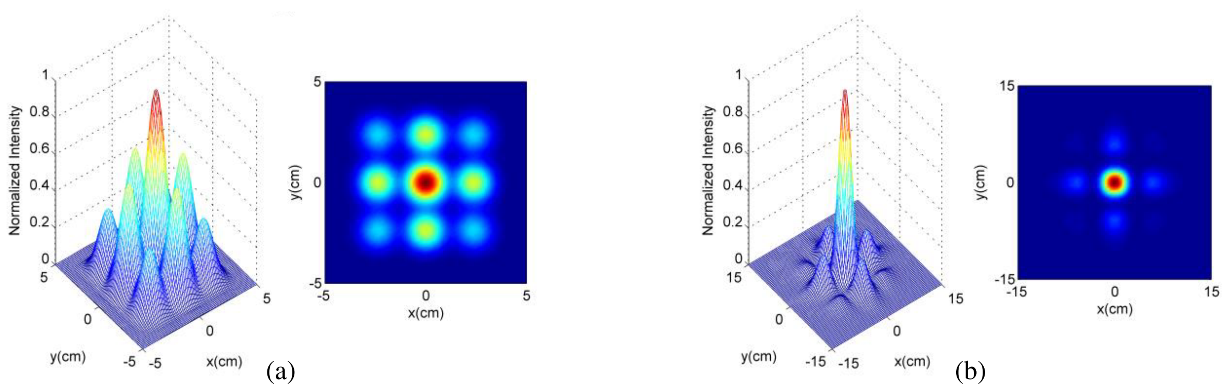


Figure 2. The 3D average intensity and contour graphs of a FPLG beam propagating through free space. (a) $z = 1000\,\mathrm{m}$, (b) $z = 3000\,\mathrm{m}$.

evolve into a Gaussian-like beam around by the smaller beams which is accordance with the previous reports [24]. By comparing Figure 1(d) with Figure 2(b), we can conclude that the FPLG beam propagating through atmospheric turbulence evolving into Gaussian beam is caused by the atmospheric turbulence.

To carry out the effects of Lorentz widths $w_{0x} = w_{0y}$ on the spreading of FPLG beam, the cross lines of an FPLG beam propagating through atmospheric turbulence with $C_n^2 = 10^{-14}\,\mathrm{m}^{-2/3}$ for the different $w_{0x} = w_{0y}$ is plotted in Figure 3. From Figure 3(a), it is shown that the FPLG beam with smaller

$w_{0x} = w_{0y}$ will have a smaller beam spot as the shorten propagation distance. As the propagation distance increases into a longer distance, the FPLG beam with the different $w_{0x} = w_{0y}$ will evolve into a Gaussian beam with the same beam profile due to the influences of atmospheric turbulence (Figure 3(b)).

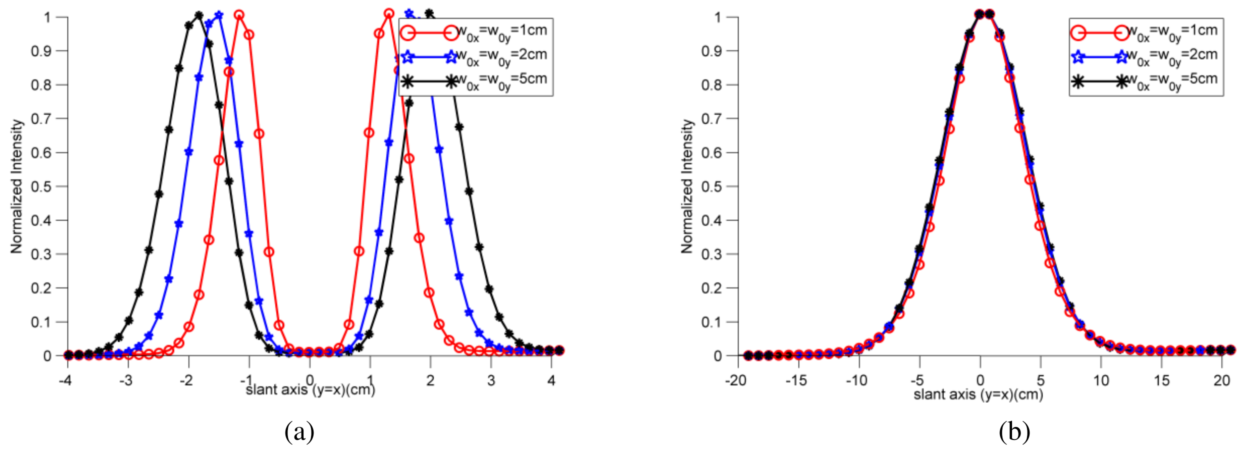


Figure 3. Cross lines of a FPLG beam propagating through atmospheric turbulence along the slanted axis ($y = x$) for the different $w_{0x} = w_{0y}$. (a) $z = 100\,\text{m}$, (b) $z = 3000\,\text{m}$.

The effects of N on the spreading of an FPLG beam propagating through atmospheric turbulence with $C_n^2 = 10^{-15}\,\text{m}^{-2/d3}$ is given in Figure 4. One can find that the FPLG beam with smaller N will first have a smaller beam spot, and the beam with the different N will evolve into a similar beam spot with the Gaussian distribution as the propagation distance increases.

To study the influences of refractive index structure constant of atmospheric turbulence C_n^2 on the evolution properties of FPLG beam, the cross lines of an FPLG beam propagating through atmospheric turbulence and free space for the different C_n^2 are illustrated in Figure 5. It is found that FPLG beam propagating through stronger atmospheric turbulence (larger C_n^2) will evolve into the Gaussian-like beam faster (Figure 4(a)), and the beam propagating through stronger atmospheric turbulence will have a larger beam spot at the longer propagation distance. Thus, the atmospheric turbulence will accelerate the spreading of FPLG beam.

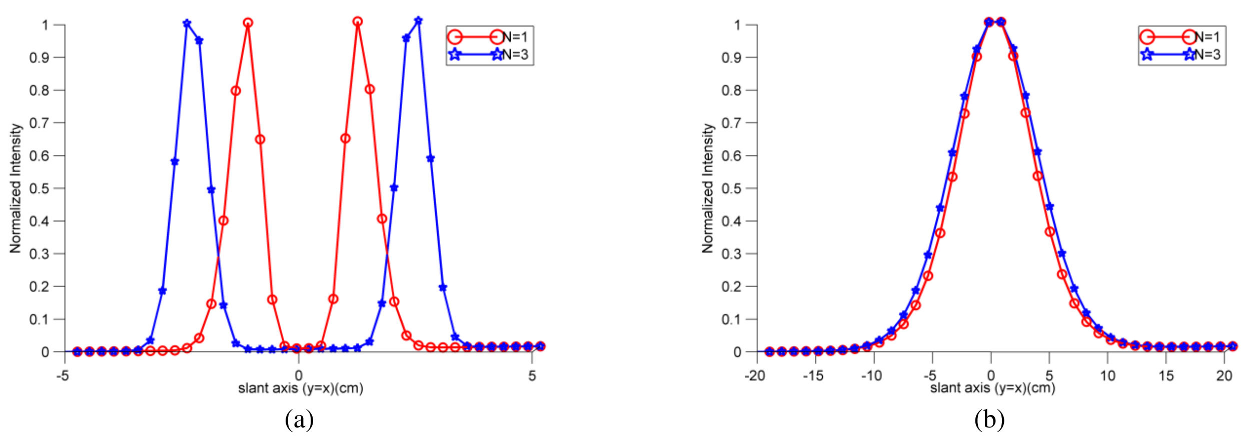
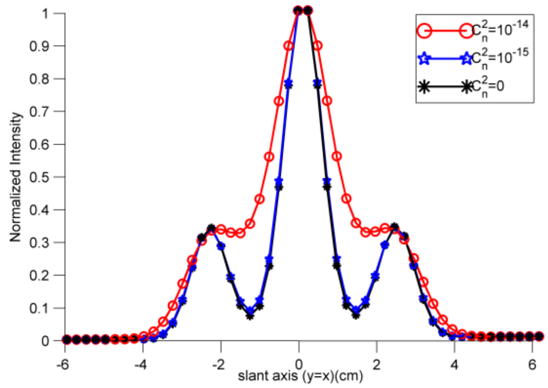
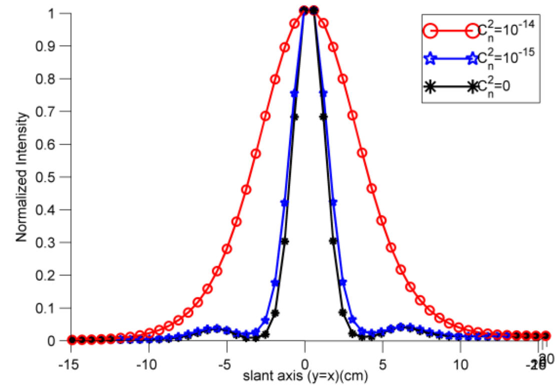


Figure 4. Cross lines of a FPLG beam propagating through atmospheric turbulence along the slanted axis ($y = x$) for the different N. (a) $z = 100\,\text{m}$, (b) $z = 3000\,\text{m}$.



(a)



(b)

Figure 5. Cross lines of a FPLG beam propagating through atmospheric turbulence along the slanted axis ($y = x$) for the different C_n^2. (a) $z = 1000\,\text{m}$, (b) $z = 3000\,\text{m}$.

4. CONCLUSIONS

Based on the extended Huygens-Fresnel principle, the average intensity equation of FPLG beam propagating through atmospheric turbulence is derived, and the effects of beam parameters and atmospheric turbulence on spreading of FPLG beam are investigated by using derived equations. It is found that the atmospheric turbulence will accelerate the spreading of FPLG beam, and the FPLG beam propagating through atmospheric turbulence will evolve gradually from four-petal pattern into Gaussian beam due to the influences of atmospheric turbulence.

REFERENCES

1. Peleg, A. and J. V. Moloney, "Scintillation index for two Gaussian laser beams with different wavelengths in weak atmospheric turbulence," *J. Opt. Soc. Am. A Opt. Image. Sci. Vis.*, Vol. 23, 3114–3122, 2006.
2. Ji, X. and G. Ji, "Spatial correlation properties of apertured partially coherent beams propagating through atmospheric turbulence," *Applied Physics B*, Vol. 92, 111–118, 2008.
3. Li, X., X. Chen, and X. Ji, "Influence of atmospheric turbulence on the propagation of superimposed partially coherent Hermite-Gaussian beams," *Opt. Commun.*, Vol. 282, 7–13, 2009.
4. Gu, Y. L. and G. Gbur, "Scintillation of pseudo-Bessel correlated beams in atmospheric turbulence," *J. Opt. Soc. Am. A*, Vol. 27, 2621–2629, 2010.
5. Zhou, G. Q. and X. X. Chu, "Average intensity and spreading of a Lorentz-Gauss beam in turbulent atmosphere," *Opt. Express*, Vol. 18, 726–731, 2010.
6. Wang, K. L. and C. H. Zhao, "Propagation properties of a radial phased-locked partially coherent anomalous hollow beam array in turbulent atmosphere," *Opt. Laser Technol.*, Vol. 57, 44–51, 2014.
7. Tang, M. M. and D. M. Zhao, "Regions of spreading of Gaussian array beams propagating through oceanic turbulence," *Appl. Optics*, Vol. 54, 3407–3411, 2015.
8. Liu, D. J., L. Chen, Y. C. Wang, and H. M. Yin, "Intensity properties of four-petal Gaussian vortex beams propagating through atmospheric turbulence," *Optik*, Vol. 127, 3905–3911, 2016.
9. Yin, X. and L. C. Zhang, "Quantum polarization fluctuations of an Airy beam in turbulent atmosphere in a slant path," *J. Opt. Soc. Am. A*, Vol. 33, 1348–1352, 2016.
10. Zhu, J., X. Li, H. Tang, and K. Zhu, "Propagation of multi-cosine-Laguerre-Gaussian correlated Schell-model beams in free space and atmospheric turbulence," *Opt. Express*, Vol. 25, 20071–20086, 2017.

11. Wang, F. and O. Korotkova, "Circularly symmetric cusped random beams in free space and atmospheric turbulence," *Opt Express*, Vol. 25, 5057–5067, 2017.
12. Tian, H. H., Y. G. Xu, T. Yang, Z. R. Ma, S. J. Wang, and Y. Q. Dan, "Propagation characteristics of partially coherent anomalous elliptical hollow Gaussian beam propagating through atmospheric turbulence along a slant path," *J. Mod. Optic*, Vol. 64, 422–429, 2017.
13. Zheng, G., J. Wang, L. Wang, M. Zhou, Y. Chen, and M. Song, "Scintillation analysis of pseudo-Bessel-Gaussian Schell-mode beams propagating through atmospheric turbulence with wave optics simulation," *Optics* & *Laser Technology*, Vol. 100, 139–144, 2018.
14. Wang, F., X. L. Liu, and Y. J. Cai, "Propagation of partially coherent beam in turbulent atmosphere: A review," *Progress In Electromagnetics Research*, Vol. 150, 123–143, 2015.
15. Wang, F., J. Li, G. Martinez-Piedra, and O. Korotkova, "Propagation dynamics of partially coherent crescent-like optical beams in free space and turbulent atmosphere," *Opt. Express*, Vol. 25, 26055–26066, 2017.
16. Liu, D., G. Wang, and Y. Wang, "Average intensity and coherence properties of a partially coherent Lorentz-Gauss beam propagating through oceanic turbulence," *Optics* & *Laser Technology*, Vol. 98, 309–317, 2018.
17. Liu, D. J., X. X. Luo, H. M. Yin, G. Q. Wang, and Y. C. Wang, "Effect of optical system and turbulent atmosphere on the average intensity of partially coherent flat-topped vortex hollow beam," *Optik*, Vol. 130, 227–236, 2017.
18. Banakh, V. A. and L. O. Gerasimova, "Strong scintillations of pulsed Laguerrian beams in a turbulent atmosphere," *Opt. Express*, Vol. 24, 19264–19277, 2016.
19. Liu, D. and Y. Wang, "Evolution properties of a radial phased-locked partially coherent Lorentz-Gauss array beam in oceanic turbulence," *Optics* & *Laser Technology*, Vol. 103, 33–41, 2018.
20. Golmohammady, S. and B. Ghafary, "Stokes parameters of phase-locked partially coherent flat-topped array laser beams propagating through turbulent atmosphere," *Laser Phys.*, Vol. 26, 2016.
21. Liu, D., H. Yin, G. Wang, and Y. Wang, "Propagation of partially coherent Lorentz-Gauss vortex beam through oceanic turbulence," *Appl. Optics*, Vol. 56, 8785–8792, 2017.
22. Liu, D. J., Y. C. Wang, and H. M. Yin, "Propagation properties of partially coherent four-petal Gaussian vortex beams in turbulent atmosphere," *Opt. Laser Technol.*, Vol. 78, 95–100, 2016.
23. Zhi, D., R. M. Tao, P. Zhou, Y. X. Ma, W. M. Wu, X. L. Wang, and L. Si, "Propagation of ring Airy Gaussian beams with optical vortices through anisotropic non-Kolmogorov turbulence," *Opt. Commun.*, Vol. 387, 157–165, 2017.
24. Liu, D., H. Zhong, G. Wang, and Y. Wang, "Model of a four-petal Lorentz-Gauss beam and its paraxial propagation," *Optik*, Vol. 179, 492–498, 2019.
25. Liu, D., G. Wang, H. Zhong, H. Yin, A. Dong, and Y. Wang, "Properties of a four-petal Lorentz-Gauss beam propagating in uniaxial crystal orthogonal to the optical axis," *Optik*, Vol. 183, 257–265, 2019.
26. El Gawhary, O. and S. Severini, "Lorentz beams and symmetry properties in paraxial optics," *J. Opt. A: Pure Appl. Opt.*, Vol. 8, 409–414, 2006.
27. Schmidt, P., "A method for the convolution of lineshapes which involve the Lorentz distribution," *Journal of Physics B*, Vol. 9, 2331–2339, 1976.
28. Jeffrey, H. D. A., *Handbook of Mathematical Formulas and Integrals*, 4th Edition Academic Press Inc, 2008.