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New TLM Formulation for Modeling Epstein Plasma

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Abstract—In plasma physics, the interaction with electromagnetic waves is related to the electrons contained in plasma. To analyze this interaction, the behaviour of electrons contained must be understood and modeled. In this paper, a new TLM formulation for dispersive media called the exponential time differencing (ETD) transmission line matrix (TLM) technique is introduced to model the interaction with dispersive media. To verify the high accuracy and efficiency of this method, the reflection and transmission coefficients of electromagnetic wave through a non-magnetized collisional plasma slab are computed and compared to the analytical solution. The electron density in plasma can be distributed as a Epstein formula, and its distribution as a function of the grads coefficient σ, and the effect of this parameter and the electron collision frequency υ_c on the reflection coefficient is calculated. The results show that with different values of σ and υ_c, the reflection coefficient is affected and can be reduced.

1. INTRODUCTION

The Transmission Line Matrix (TLM) method with symmetrical condensed node (SCN) [1, 2] remains a robust and efficient numerical tool to model electromagnetic waves interaction with dispersive media, in the time domain. Over the last decade, there have been numerous studies on TLM dispersive medium formulations. The methods employed include: the current density recursive convolution (CDRC) method [3], piecewise linear recursive convolution (PLRC) method [4], (CRC) with voltage and current sources [5], and JE convolution (JEC) with voltage sources method [6]. Another technique derived from the TLM is the PLCDRC-TLM method [7]. In this paper, we propose a new technique to simulate wave propagation in isotropic dispersive cold plasma, named ETD-TLM. The formulation of the above mentioned method is discussed in detail in the next section. In order to verify the validity and accuracy of the new method, the formulation is used to calculate the plasma reflectance with a constant electron density. The results are compared to the analytical solution. Furthermore, the plasma analyzed is modeled as a plasma of Epstein distribution, and the reflection coefficients are again calculated. This kind of analysis has been the subject of many articles [8], which illustrates the importance of the plasma physics. It can be applied in many fields including: astrophysics, industry, and also medicine, since the blood is composed of plasma and can be modeled using the Epstein distribution. The figures presented below show the relationship between different values of the distribution grads coefficient σ and the reflectance, also the relationship between different values of the electron collision frequency υ_c and the reflectance. The reasons for the relationships are discussed.

2. FORMULATIONS AND EQUATIONS

The formulation proposed using ETD approach can exceed the passage through the convolution used in PLCDRC [9]. This new formulation allows the obtaining of current density update equations directly using multiplication of the medium constitutive equation by an exponential term, followed by an

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integration of the obtained equation over an interval whose width is equal to the time step. This approach has been used for FDTD modeling in [10, 11].

For isotropic cold plasma with collisions, the Maxwell's equations and constitutive relation are given by:

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} \tag{1}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \tag{2}$$

$$\frac{d\mathbf{J}}{dt} = \varepsilon_0 \omega_p^2 \mathbf{E} - \upsilon_c \mathbf{J} \tag{3}$$

where $\mathbf{E}$ is the electric field, $\mathbf{H}$ the magnetic intensity, $\mathbf{J}$ the polarisation current density, ε_0 the permittivity of free space, μ_0 the permeability of free space, υ_c the electron collision frequency, and ω_p the plasma frequency.

The central difference approximation of Equation (1) allows us to express it in this form:

$$\mathbf{E}^{n+1} = \mathbf{E}^n + \frac{\Delta t}{\varepsilon_0} (\nabla \times \mathbf{H})^{n+\frac{1}{2}} - \frac{\Delta t}{\varepsilon_0} \mathbf{J}^{n+\frac{1}{2}} \tag{4}$$

$\mathbf{J}^{n+\frac{1}{2}}$ can be approximated by:

$$\mathbf{J}^{n+\frac{1}{2}} = \frac{1}{2} \left(\mathbf{J}^{n+1} + \mathbf{J}^{n-1} \right) \tag{5}$$

The multiplication of Equation (3) by $e^{\upsilon_c t}$ followed by an integration on the interval $[n - \frac{1}{2}, n + \frac{1}{2}]$ allows us to write:

$$\mathbf{J}^{n+\frac{1}{2}} = e^{-\upsilon_c t} \mathbf{J}^{n-\frac{1}{2}} + \varepsilon_0 \omega_p^2 e^{\upsilon_c t} \int_{\frac{-\Delta t}{2}}^{\frac{+\Delta t}{2}} e^{\upsilon_c \tau} \mathbf{E}(n\Delta t + \tau) d\tau \tag{6}$$

with $\mathbf{E}(n\Delta t + \tau) = \mathbf{E}(n\Delta t) + 0(\Delta t)$ so:

$$\mathbf{J}^{n+\frac{1}{2}} = e^{-\upsilon_c \Delta t} \mathbf{J}^{n-\frac{1}{2}} + \frac{1}{\upsilon_c} \left(1 - e^{\upsilon_c \Delta t} \right) \varepsilon_0 \omega_p^2 \mathbf{E}^n \tag{7}$$

Equation (7) gives us the expression of updating current density. On the other hand, Equations (4) and (5) provide the equation:

$${}_{n+1}V_u = {}_n V_u + \frac{\Delta t \Delta l}{\varepsilon_0} (\nabla \times \mathbf{H})_u^{n+\frac{1}{2}} - \frac{\Delta t \Delta l}{2\varepsilon_0} \left(\mathbf{J}_u^{n+1} + \mathbf{J}_u^n \right) \tag{8}$$

By following the same procedure written in the ADE-TLM [12] approach, we obtain a null value of the admittance of the stubs, which reduces the expressions of the total voltages and becomes:

$$\begin{pmatrix} {}_{n+1}V_x \\ {}_{n+1}V_y \\ {}_{n+1}V_z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} {}_{n+1}V_1^i + {}_{n+1} V_2^i + {}_{n+1} V_9^i + {}_{n+1} V_{12}^i \\ {}_{n+1}V_3^i + {}_{n+1} V_4^i + {}_{n+1} V_8^i + {}_{n+1} V_{11}^i \\ {}_{n+1}V_5^i + {}_{n+1} V_6^i + {}_{n+1} V_7^i + {}_{n+1} V_{10}^i \end{pmatrix} + \frac{1}{4} \begin{pmatrix} {}_{n+1}V_x^s \\ {}_{n+1}V_y^s \\ {}_{n+1}V_z^s \end{pmatrix} \tag{9}$$

with:

$$\begin{pmatrix} {}_{n+1}V_x^s \\ {}_{n+1}V_y^s \\ {}_{n+1}V_z^s \end{pmatrix} = \begin{pmatrix} -{}_nV_x^s - 4\frac{\Delta t \Delta l}{2\varepsilon_0} \left(\mathbf{J}_x^{n+1} + \mathbf{J}_x^n \right) \\ -{}_nV_y^s - 4\frac{\Delta t \Delta l}{2\varepsilon_0} \left(\mathbf{J}_y^{n+1} + \mathbf{J}_y^n \right) \\ -{}_nV_z^s - 4\frac{\Delta t \Delta l}{2\varepsilon_0} \left(\mathbf{J}_z^{n+1} + \mathbf{J}_z^n \right) \end{pmatrix} \tag{10}$$

3. VALIDITY OF ETD-TLM

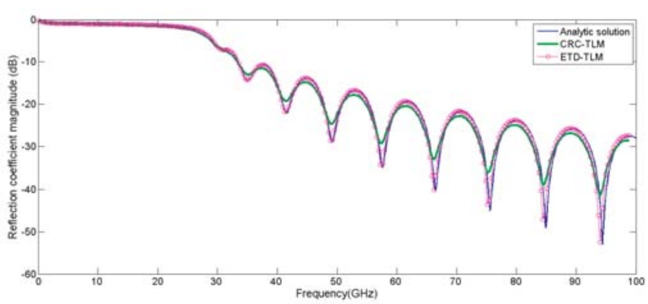
To verify the validity and accuracy of the ETD-TLM method, the reflection and transmission coefficients are calculated and compared to the analytical solution and to another numerical method which is CRC-TLM. Both coefficients are computed by using the same equations presented in the article [13] and calculated using the following parameters: plasma thickness of 1.5 cm, plasma pulsation of $\omega_p = 2\pi \times 28.7 \times 10^9$ rad/s, and an electron collision frequency of $\upsilon_c = 20 \times 10^9$ rad/s.

The incident wave used is a Gaussian pulsed plane wave of expression:

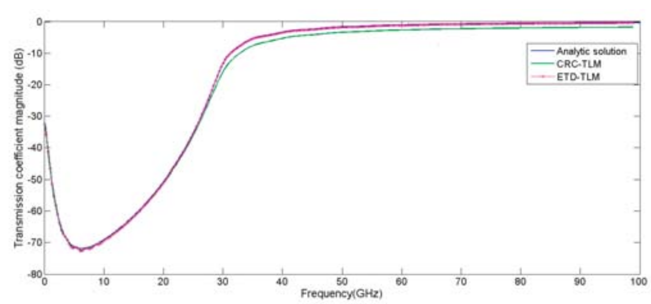
$$E_i(t) = \exp\left(-\frac{(t \times \Delta t - 10 \times T_0)^2}{\tau^2}\right) \tag{11}$$

where $\Delta t = \frac{\Delta l}{2C}$, $T_0 = \frac{1}{2F_0}$, $\tau = 4 \times T_0$ and $F_0 = 100 \times 10^9$ Hz.

The computational domain is subdivided into 800 cells. Cells from 301 to 500 are occupied by cold plasma, and the rest are occupied by the free space. The simulations are allowed to run 20000 time steps. Results are shown in Fig. 1. For the reflection coefficient shown in Fig. 1(a), the ETD-TLM curve follows the analytical curve closer than the CRC-TLM curve at all frequencies. For the transmission coefficient shown in Fig. 1(b), the ETD-TLM solution is also closer to the analytical solution than the CRC-TLM solution. As a conclusion, the ETD-TLM results are in excellent agreement with analytical solutions, which in turn proves the high accuracy of the proposed method.



(a)



(b)

Figure 1. Comparison between analytical, CRC-TLM and ETD-TLM results with plasma thickness of 1.5 cm.

4. EPSTEIN DISTRIBUTION

Mediums that contains electrons of high density can be modeled using the Epstein distribution. It is particularly useful in modeling the plasmas. Epstein (1930) discussed a general electron density distribution with three arbitrary constants and wave propagation in absorbing media. Years after, Vidmar (1990) adapted the Epstein distribution to one suitable also for modeling plasmas [14]. The Epstein distribution of the plasma electron density in the slab geometry utilized is:

$$\eta(z) = \frac{n_0}{1 + \exp\left(-\frac{z - 0.5z_0}{\sigma}\right)} \tag{12}$$

where n_0 is the peak value of the electron density at $z = +\infty$, z_0 the plasma thickness, and σ the grads coefficient.

The plasma slab is of Epstein distribution with a thickness of 1.5 cm, and the peak value of the electron density is $n_0 = 3.11 \times 10^{19} m^3$. The reflection coefficient is calculated for nine cases, three different values of the plasma collision frequency 20 GHz, 50 GHZ, and 100 GHz, and for each collision frequency, the reflection is calculated for σ values of 0.5 cm, 1.0 cm, and 5.0 cm. So the effects of the two parameters (υ_c, σ) on the plasma reflectance are compared.

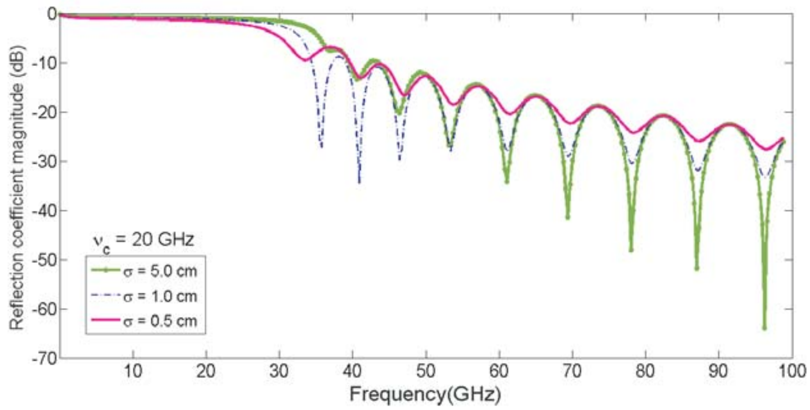


Figure 2. Effect of sigma in Epstein distribution on plasma reflectance at $\upsilon_c = 20\,\text{GHz}$.

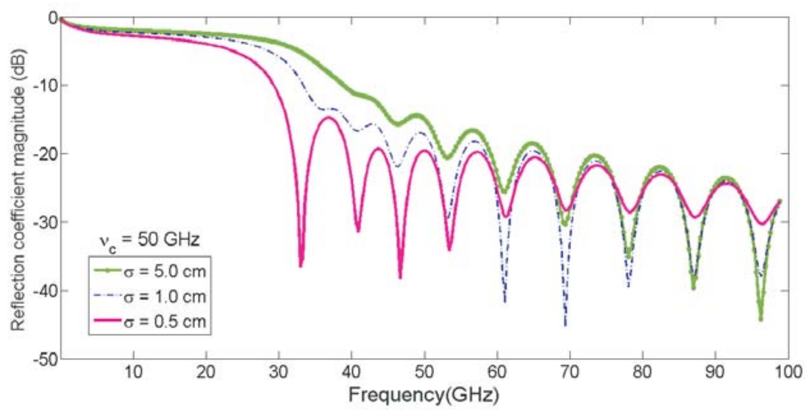


Figure 3. Effect of sigma in Epstein distribution on plasma reflectance at $\upsilon_c = 50\,\text{GHz}$.

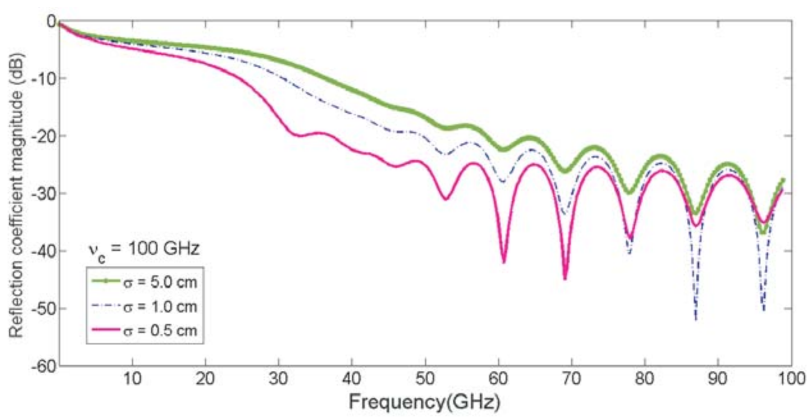


Figure 4. Effect of sigma in Epstein distribution on plasma reflectance at $\upsilon_c = 100\,\text{GHz}$.

As shown in Figs. 2, 3, and 4, with the increase of σ from 0.5 cm to 5 cm, going through 1.0 cm, the reflection coefficient changes consistently.

Figure 2 — with $\upsilon_c = 20\,\text{GHz}$ — shows the effects of different grads coefficients of 0.5 cm, 1.0 cm, and 5.0 cm on reflection coefficient as follows:

(i) There is no effect within the incidence frequency range of $0 \sim 25\,\text{GHz}$.

(ii) There is almost no effect of different σ on reflection coefficient within the incidence frequency range of $25 \sim 33$ GHz, and the three curves decrease consistently.

(iii) Within the incidence frequency range of $33 \sim 53$ GHz, the different values of σ affect the reflection coefficient significantly. With σ of 0.5 cm and 5.0 cm, the reflectance is not affected enormously, and the curves swing smoothly. But with $\sigma = 1.0$ cm, the reflectance is affected clearly, and the curves swing sharply. Also the values of reflection coefficient and minimum are both small with certain variation.

(iv) Within the incidence frequency range of $53 \sim 100$ GHz, for $\sigma = 0.5$ cm, there is no significant effects, and the curve continues swinging smoothly. But with σ of 1.0 cm and 5.0 cm, the reflectance is affected, and both the curves have the same form, but with different minimum values. The minimum value is greater with σ of 1.0 cm and smallest with σ of 5.0 cm.

For Fig. 3 with $\upsilon_c = 50$ GHz, the effects of different grads coefficients of 0.5 cm, 1.0 cm, and 5.0 cm on reflection coefficient can be interpreted as follows:

(i) There is no effect within the incidence frequency range of $0 \sim 26$ GHz.

(ii) Within the incidence frequency range of $26 \sim 51$ GHz, for $\sigma = 0.5$ cm, the reflectance is affected clearly, and the curves swing sharply. But with σ of 1.0 cm and 5.0 cm, the reflection coefficient changes consistently. As a result, when $\sigma > 0.5$ cm, there is almost no effect on the reflection coefficient.

(iii) Within the incidence frequency range of $51 \sim 76$ GHz, the reflectance is affected, and the three curves have the same form, but with different minimum values.

(iv) Within the incidence frequency range of $76 \sim 100$ GHz, for $\sigma = 0.5$ cm, the curve swings smoothly with no effect. But with σ of 1.0 cm and 5.0 cm, the reflectance is affected clearly.

(v) With σ of 0.5 cm, 1.0 cm and 5.0 cm, the optimal regions obtained are as follows: $31 \sim 54$ GHz, $34 \sim 69$ GHz and $77 \sim 97$ GHz correspondingly.

For Fig. 4 with $\upsilon_c = 100$ GHz:

(i) There are almost no effects of different σ on the reflection coefficient within the incidence frequency range of $0 \sim 58$ GHz, and the three curves decrease consistently.

(ii) Within the incidence frequency range of $58 \sim 100$ GHz, different values of σ do not cause reflection coefficient variation of incidence frequency position, except for minimum.

As a result of the comparison between different figures, the electron collision frequency and grads coefficient are two parameters that play an important role in reducing the reflectance.

5. CONCLUSION

In this paper, a new TLM method for modeling dispersive media is derived. This technique is based on the symmetrical condensed TLM method, the relation between the density current J and electric field E, and the exponential time differencing method. The accuracy of this method is verified by comparing it to the analytical and numerical solutions. Results demonstrate excellent agreement between the proposed and analytical models, which prove that ETD-TLM is a powerful tool that can also be applied to model complex structures in two and three dimensions. As the electron density of the plasma can be distributed as the Epstein formula, and the distribution depends on the grads coefficient, the effect of this parameter and electron collision frequency on the reflection is studied, and the results affirm that the reflectance can be controlled by changing the two parameters. This is a study of the interaction with cold plasma, but it has to be mentioned that interaction of electromagnetic waves with nanoparticles in charged plasma should also have other interesting consequences, as shown in [15], which could be an idea for a future work.

REFERENCES

1. Johns, P. B., "A symmetrical condensed node for the TLM method," *IEEE Trans. Microwave Theory Tech.*, Vol. 35, No. 4, 370–377, 1987.

2. Hoefer, J. R., "The transmission line matrix method, theory and applications," *IEEE Trans. Microwave Theory Tech.*, Vol. 33, No. 10, 882–893, 1985.
3. Adraoui, E. S., K. mounirh, A. Zugari, M. Iben Yaich, and M. Khalladi, "Novel CDRC-TLM algorithm for the analysis of magnetized plasma," *Optik — International Journal for Light and Electron Optics*, Vol. 125, No. 1, 276–279, 2014.
4. Adraoui, E. S., A. Zugari, M. Bassouh, MI. Yaich, and M. Khalladi, "Novel PLRC-TLM algorithm implementation for modeling electromagnetic wave propagation in gyromagnetic media," *I. J. Adv. Sci. Technol.*, Vol. 6, No. 1, 26–32, 2013.
5. Yaich, I. M., M. Khalladi, I. Zekik, and J. A Morente, "Modeling of frequency-dependent magnetized plasma in hybrid symmetrical condensed TLM method," *IEEE Microwave and Wireless Components Letters*, Vol. 12, No. 8, 293–295, 2002.
6. Abrini, R., M. Iben Yaich, and M. Khalladi, "Efficient modeling of isotropic cold plasma media using JE-TLM method," *IEICE Electron.*, Vol. 4, No. 15, 492–49, 2007.
7. Mounirh, K., S. El Adraoui, M. Charif, M. Khalladi, and M. Iben Yaich, "Modeling of anisotropic magnetized plasma media using PLCDRC-TLM method," *Optik — International Journal for Light and Electron Optics*, Vol. 126, No. 1, 1479–1482, 2015.
8. Yang, H., Y. Zhou, Y. Zan, and R. Chen, "SO-FDTD analysis of the plasma reflectance of Epstein distribution," *Plasma Science and Technology*, Vol. 8, No. 6, 2013.
9. Ekdiha, Y., K. Mounirh, S. El Adraoui, M. Khalladi, and M. Iben Yaich, "Analysis of Epstein distribution effect on the plasma reflectance," *Proceedings of the 1st International Conference on Electronic Engineering and Renewable Energy*, Springer, Saidia, 2018.
10. Huang, Sh. J. and Y. Zhon, "Exponential time differencing FTDT formulation for plasma," *Microwave and Optical Technology Letters*, Vol. 49, No. 6, 1393–1364, 2007.
11. Huang, Sh. J. and F. Li, "FTDT implementation for magnetoplasma medium using exponential time differencing," *IEEE Microwave and Wireless Components Letters*, Vol. 15, No. 3, 183–185, 2005.
12. Mounirh, K., S. El Adraoui, Y. Ekdiha, M. I. Yaich, and M. Khalladi, "Modeling of dispersive chiral media using the ADE-TLM method," *Progress In Electromagnetics Research*, Vol. 64, No. 10, 157–166, 2018.
13. Yaich, M. I., M. Khalladi, I. Zekik, and J. A. Morente, "Modeling of frequency-dependent magnetized plasma in hybrid symmetrical condensed TLM method," *IEEE Microwave and Wireless Components Letters*, Vol. 12, No. 8, 293–295, 2013.
14. Becker, K. H., U. Kogelschatz, K. H. Schoenbach, and R. J. Barker, *Non-equilibrium Air Plasmas at Atmospheric Pressure, Series in Plasma Physics*, Nov. 2004.
15. Kocifaj, M., J. Klaka, F. Kundracik, and G. Videen, "Charge-induced electromagnetic resonances in nanoparticles," *Annalen der Physik*, Vol. 527, No. 11, 765–769, 2015.