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Physical Limits of Electromagnetic Responses of Layered Stacked Structures

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Abstract—In this paper, we theoretically investigate the electromagnetic response of the widely used layered stacked structure. For a causality and lossy system, relationships between maximum values of reflection and transmission coefficients are demonstrated, which are related with many parameters, such as absolute bandwidth, layers thicknesses and real parts of the static permittivity and static permeability. Different polarizations and incident conditions are discussed. The results can provide a criterion to judge different designs operating at different spectrum ranges with different thicknesses and materials by comparing them with achievable physical limits.

1. INTRODUCTION

Multilayered stacked structure (MSS) is the most widely used composition for devices such as field absorber, thin-film antireflection coatings, optical interference filter, dielectric mirrors, or broadband transparent diodes due to its simple fabrication and moderate performance [1, 2]. If the system only consists of linear, causal, and passive materials, with given thicknesses, the total electromagnetic response of the system can be derived based on transfer matrix method [3]. However, the criterion judging the physical limits of MSS with different thicknesses and different materials under different spectra was not studied carefully.

2. CAUSALITY AND ANALYTICITY

According to Nussenzveig [4], the response function $\widetilde{R}(\lambda)$ for a linear, lossy and casual system subject to a time-independent ($\sim e^{j\omega t}$) excitation has a regular analytic continuation in the upper half-plane of complex wavelengths $\lambda = \lambda' + j\lambda''$. As a consequence, $\widetilde{R}(\lambda)$ has no poles in the upper half-plane but may have nulls there. If the nulls are $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$, then an ancillary function $\widetilde{R}'(\lambda)$ can be defined as

$$\widetilde{R}'(\lambda) = \widetilde{R}(\lambda)\frac{(\lambda-\lambda_1^*)(\lambda-\lambda_2^*)\ldots(\lambda-\lambda_n^*)\ldots}{(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_n)\ldots}, \tag{1}$$

where $*$ represents the complex conjugation. Obviously, $\widetilde{R}'(\lambda)$ has neither poles nor nulls at the upper half-plane. ($\mathrm{Im}(\lambda) = \lambda'' > 0$) Hence the logarithm of $\widetilde{R}'(\lambda)$ is analytic function in the upper half-plane, and the Cauchy theorem can be applied, i.e., the integration of $\ln \widetilde{R}'(\lambda)$ along the real axis and the infinite semi-circle C_∞ in the upper half-plane is zero. Notice that $|\widetilde{R}'(\lambda)| = |\widetilde{R}(\lambda)|$ at real wavelengths

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and that the real part of $\ln \widetilde{R}(\lambda)$ is an even function of real λ. The real part of the Cauchy integral over the contour transforms to

$$\mathrm{Re}\int_C \ln \widetilde{R}' d\lambda = 2\int_0^\infty \ln\left|\widetilde{R}\right| d\lambda + \mathrm{Re}\int_{C_\infty} \ln \widetilde{R} d\lambda + \mathrm{Re}\int_{C_\infty} \ln\left(\frac{(\lambda-\lambda_1^*)(\lambda-\lambda_2^*)\ldots(\lambda-\lambda_n^*)\ldots}{(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_n)\ldots}\right) d\lambda = 0, \quad (2)$$

where the third term on the left hand side (LHS) can be calculated as,

$$\mathrm{Re}\int_{C_\infty} \ln\left(\frac{(\lambda-\lambda_1^*)(\lambda-\lambda_2^*)\ldots(\lambda-\lambda_n^*)\ldots}{(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_n)\ldots}\right) d\lambda = -4\pi \sum_n \mathrm{Im}\lambda_n. \quad (3)$$

For an electromagnetic field and matter interaction system, both the reflection and transmission coefficients are the response functions of the system, which satisfy Eq. (2).

3. REFLECTION BOUND OF METAL-BACKED SLABS

First, we consider a slab of thickness h, permittivity ε, and permeability μ, overlying a perfectly reflecting metal and illuminated at oblique incidence with angle θ_a by a monochromatic plane wave. Then the reflection coefficient $r(\omega)$ for TE polarized incident field can be obtained based on transfer matrix method:

$$r_{TE} = \frac{-\eta_{T,a}\cos(k_z h) + j\eta_{T,1}\sin(k_z h)}{\eta_{T,a}\cos(k_z h) + j\eta_{T,1}\sin(k_z h)} = \frac{-1 + j\beta_{TE}}{1 + j\beta_{TE}}, \quad (4)$$

where

$$\beta_{TE} = \frac{\mu_r \cos\theta_a}{\sqrt{\mu_r\varepsilon_r - \sin^2\theta_a}} \tan\left(k_0\sqrt{(\mu_r\varepsilon_r) - \sin^2\theta_a} h\right) \quad (5)$$

$k_z = k_0\sqrt{(\mu_r\varepsilon_r) - \sin^2\theta_a}$ is the z component of wave number in the slab, $\eta_{T,i} = \eta_i/\cos\theta_i = \sqrt{\mu_i/\varepsilon_i}/\cos\theta_i$ is the transverse characteristic impedance in slab i. ε_r and μ_r are the relative permittivity and permeability, respectively. Similarly, for TM polarized incident field, the reflection coefficient can be obtained as:

$$r_{TM} = \frac{-j\eta_{T,0}^{-1}\eta_{T,1}\sin(k_z h) + \cos(k_z h)}{j\eta_{T,0}^{-1}\eta_{T,1}\sin(k_z h) + \cos(k_z h)} = \frac{1 - j\beta_{TM}}{1 + j\beta_{TM}}, \quad (6)$$

where

$$\beta_{TM} = \frac{1}{\varepsilon_r}\frac{\sqrt{\mu_r\varepsilon_r - \sin^2\theta_a}}{\cos\theta_a} \tan\left(k_0\sqrt{(\mu_r\varepsilon_r) - \sin^2\theta_a} h\right), \quad (7)$$

and the transverse characteristic impedance for TM field is $\eta_{T,i} = \eta_i\cos\theta_i = \sqrt{\mu_i/\varepsilon_i}\cos\theta_i$. Substituting Eq. (4) and Eq. (6) into the second term on the LHS of Eq. (2) yields

$$\mathrm{Re}\int_{C_\infty} \ln r_{TE} d\lambda = 4\pi^2 \cos\theta_a \mu'_{sr} h, \quad (8)$$

for TE polarized field

$$\mathrm{Re}\int_{C_\infty} \ln r_{TM} d\lambda = \frac{4\pi^2}{\cos\theta_a}\left(\mu'_{sr} - \frac{\sin^2\theta_a}{\varepsilon'_{sr}}\right) h, \quad (9)$$

and for TM polarized field, where $\mu'_{sr} = \lim_{R\to\infty} \mathrm{Re}(\mu_r) = \lim_{R\to\infty} \mu'_r$ and $\varepsilon'_{sr} = \lim_{R\to\infty} \mathrm{Re}(\varepsilon_r) = \lim_{R\to\infty} \varepsilon'_r$ stands for the real part of the static relative permeability and permittivity, respectively. Therefore, Eq. (2) turns out to be

$$\int_0^\infty \ln|r_{TE}| d\lambda = -2\pi^2 \cos\theta_a \mu'_{sr} h + 2\pi \sum_n \mathrm{Im}\lambda_n, \quad (10)$$

for TE polarized field and

$$\int_0^\infty \ln|r_{TM}|\,d\lambda = -\frac{2\pi^2}{\cos\theta_a}\left(\mu'_{sr} - \frac{\sin^2\theta_a}{\varepsilon'_{sr}}\right)h + 2\pi\sum_n \mathrm{Im}\lambda_n, \tag{11}$$

for TM polarized field. The module of the reflection coefficient is smaller than unity, hence, the LHS of Eqs. (10) and (11) are negative. Besides, all of the nulls localized in the upper half plane, $\mathrm{Im}\lambda_n > 0$ for any n, indicate that the two terms on the right hand side (RHS) of Eqs. (10) and (11) are opposite in signs. Therefore,

$$\left|\int_0^\infty \ln|r_{TE}|\,d\lambda\right| \le 2\pi^2\cos\theta_a\mu'_{sr}h, \tag{12}$$

and

$$\left|\int_0^\infty \ln|r_{TM}|\,d\lambda\right| \le \frac{2\pi^2}{\cos\theta_a}\left(\mu'_{sr} - \frac{\sin^2\theta_a}{\varepsilon'_{sr}}\right)h. \tag{13}$$

The results in Eq. (12) and Eq. (13) can be readily extended to the multilayer slabs case. μ'_{sr} and ε'_{sr} are the real parts of the relative permeability and permittivity when wavelength tends to infinity. In such an approach, any finite thickness of material is very small compared to wavelength. Therefore, a multilayer stacked structure can be treated as an effective one layer structure with effective parameters defined as in Equations (14) and (15).

Considering both TE mode and TM mode for μ'_{sr}, the relationship is as shown in Eq. (14),

$$\mu'_{sr} = \frac{1}{h}\sum_{i=1}^{n}\mu'_{sr,i}h_i, \tag{14}$$

Considering TM mode for ε'_{sr}, the relationship is as shown in Eq. (15),

$$\frac{1}{\varepsilon'_{sr}} = \frac{1}{h}\sum_{i=1}^{n}\frac{1}{\varepsilon'_{sr,i}}h_i, \tag{15}$$

parameters with subscript i corresponding to that of layer i.

4. TRANSMISSION BOUND OF MULTI-LAYER STACKED SLABS

We consider a slab of thickness h, permittivity ε, and permeability μ, overlying a perfectly reflecting metal and illuminated at oblique incidence with angle θ_a by a monochromatic plane wave. Then the transmission coefficient $t(w)$ for TE polarized incident field can be obtained based on transfer matrix method

$$\begin{gathered}\mathrm{Re}\int_{C_\infty} \ln t_{TE}\,d\lambda = \pi^2 h_1\left(\varepsilon_{sr1}\sec\theta_a - \frac{1}{\mu_{sr1}}\sin\theta_a\tan\theta_a + \mu_{sr1}\cos\theta_a\right). \\ \varepsilon_{sr1} = \lim_{R\to\infty}\mathrm{Re}\left(\varepsilon_{r1}\right),\quad \mu_{sr1} = \lim_{R\to\infty}\mathrm{Re}\left(\mu_{r1}\right),\end{gathered} \tag{16}$$

Similarly for TM polarized incident field,

$$\begin{gathered}\mathrm{Re}\int_{C_\infty} \ln t_{TM}\,d\lambda = \pi^2 h_1\left(\mu_{sr1}\sec\theta_a - \frac{1}{\varepsilon_{sr1}}\sin\theta_a\tan\theta_a + \varepsilon_{sr1}\cos\theta_a\right) \\ \varepsilon_{sr1} = \lim_{R\to\infty}\mathrm{Re}\left(\varepsilon_{r1}\right),\quad \mu_{sr1} = \lim_{R\to\infty}\mathrm{Re}\left(\mu_{r1}\right)\end{gathered} \tag{17}$$

For single absorbing slab,

$$\int_0^\infty \ln|t|\,d\lambda = -\frac{1}{2}\mathrm{Re}\int_{C_\infty} \ln t d\lambda - \frac{1}{2}\mathrm{Re}\int_C \ln\left(\frac{(\lambda-\lambda_1^*)(\lambda-\lambda_2^*)\dots(\lambda-\lambda_n^*)\dots}{(\lambda-\lambda_1)(\lambda-\lambda_2)\dots(\lambda-\lambda_n)\dots}\right)d\lambda$$
$$= \begin{cases} -\frac{1}{2}\pi^2 h_1\left(\varepsilon_{sr1}\sec\theta_a - \frac{1}{\mu_{sr1}}\sin\theta_a\tan\theta_a + \mu_{sr1}\cos\theta_a\right) + 2\pi\sum_n \mathrm{Im}\lambda_n, & TE \\ -\frac{1}{2}\pi^2 h_1\left(\mu_{sr1}\sec\theta_a - \frac{1}{\varepsilon_{sr1}}\sin\theta_a\tan\theta_a + \varepsilon_{sr1}\cos\theta_a\right) + 2\pi\sum_n \mathrm{Im}\lambda_n, & TM \end{cases} \tag{18}$$

If we extend the results to multiple layers,

$$\int_0^\infty \ln|t|\,d\lambda = \begin{cases} -\frac{1}{2}\pi^2\sum_{i=1}^n\left(\varepsilon_{sri}h_i\sec\theta_a - \frac{1}{\mu_{sri}}h_i\sin\theta_a\tan\theta_a + \mu_{sri}h_i\cos\theta_a\right) + 2\pi\sum_n \mathrm{Im}\lambda_n, & TE \\ -\frac{1}{2}\pi^2\sum_{i=1}^n\left(\mu_{sri}h_i\sec\theta_a - \frac{1}{\varepsilon_{sri}}h_i\sin\theta_a\tan\theta_a + \varepsilon_{sri}h_i\cos\theta_a\right) + 2\pi\sum_n \mathrm{Im}\lambda_n, & TM \end{cases} \tag{19}$$

Since $|t| \leq 1$, the LHS is negative, and the second term on the RHS is negative. If the first term on the LHS is negative, we can achieve

$$\left|\int_0^\infty \ln|t|\,d\lambda\right| = \int_0^\infty |\ln|t||\,d\lambda \leq \begin{cases} \frac{1}{2}\pi^2\sum_{i=1}^n\left(\varepsilon_{sri}h_i\sec\theta_a - \frac{1}{\mu_{sri}}h_i\sin\theta_a\tan\theta_a + \mu_{sri}h_i\cos\theta_a\right), & TE \\ \frac{1}{2}\pi^2\sum_{i=1}^n\left(\mu_{sri}h_i\sec\theta_a - \frac{1}{\varepsilon_{sri}}h_i\sin\theta_a\tan\theta_a + \varepsilon_{sri}h_i\cos\theta_a\right), & TM \end{cases} \tag{20}$$

Further, if we restrict the finite frequency to range $[\lambda_{\min}, \lambda_{\max}]$ and assume that the transmission coefficient has the following relationship, we have

$$|\ln|t_0||(\lambda_{\max}-\lambda_{\min})$$
$$= |\ln|t_0||\,B_a \leq \left|\int_0^\infty \ln|t|\,d\lambda\right| \leq \begin{cases} \frac{1}{2}\pi^2\sum_{i=1}^n\left(\varepsilon_{sri}h_i\sec\theta_a - \frac{1}{\mu_{sri}}h_i\sin\theta_a\tan\theta_a + \mu_{sri}h_i\cos\theta_a\right), & TE \\ \frac{1}{2}\pi^2\sum_{i=1}^n\left(\mu_{sri}h_i\sec\theta_a - \frac{1}{\varepsilon_{sri}}h_i\sin\theta_a\tan\theta_a + \varepsilon_{sri}h_i\cos\theta_a\right), & TM \end{cases} \tag{21}$$

If the in and out materials are the same, we can simplify Eq. (21) and get the following relationship,

$$|T_{dB}| = 20\log_{10}|t| \leq \begin{cases} \frac{10\pi^2/\ln 10}{B_a}\sum_{i=1}^n\left(\varepsilon_{sri}h_i\sec\theta_a - \frac{1}{\mu_{sri}}h_i\sin\theta_a\tan\theta_a + \mu_{sri}h_i\cos\theta_a\right), & TE \\ \frac{10\pi^2/\ln 10}{B_a}\sum_{i=1}^n\left(\mu_{sri}h_i\sec\theta_a - \frac{1}{\varepsilon_{sri}}h_i\sin\theta_a\tan\theta_a + \varepsilon_{sri}h_i\cos\theta_a\right), & TM \end{cases} \tag{22}$$

5. CONCLUSION

The response function $R(\lambda)$ for a linear, lossy, and casual system has a continuation in the upper half-plane. Both the reflection and transmission coefficients can be achieved from the casualty and analytic system, and the judgment criterion can be achieved from the results

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