*Progress In Electromagnetics Research Letters, Vol. 82, 81–87, 2019*

# Impact of External DC Magnetic Bias Field and Frequency on the Bistability Features of a Nonlinear Microwave Meta-Atom

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**Abstract**—In this paper, we present our experimental study of the effect of the external DC magnetic bias field on the nonlinear properties of meta-atom loaded with ferrite elements of different shapes. It is demonstrated experimentally that the adjustment of the resonance frequency of the meta-atom loaded with the ferrite elements of different shapes is possible not only by the input microwave power but also by the external DC magnetic bias field. It is shown that as the external DC magnetic bias field is increased to a certain value, the resonance curve of the nonlinear meta-atom demonstrates bistability. In addition, we achieve significant enhancement of the meta-atom nonlinearity using the nonlinear properties of both ferrite and varactor diode.

## 1. INTRODUCTION

The nonlinearity of the dynamic processes of any oscillator in the vicinity of its eigen resonances is of great interest from the point of view of both fundamental and applied physics. In papers focused on optical bistability [1], the creation of bistable and multistable states in the Fabri-Perot resonators with nonlinear dialectic filling has been analyzed. Particularly, it was demonstrated that in such structures the intensity of the transmitted signal as a function intensity of the incident signal shows hysteresis.

The next step in this area was studying the nonlinearity of such resonators that included magnets [3, 10]. The nonlinearity of their behavior is caused by the nonlinear nature of the permeability in the vicinity of the well-known Electron Spin Resonance (ESR) phenomena observed in such magnets. Thus, the principal difference between the cases of optical bistability and "magnetic" bistability is that in the latter case we deal with the superposition of two types of oscillations — eigen oscillation of resonator and the ESR phenomenon. Thereby, the application of magnetic elements makes the nonlinear phenomena more pronounced. This simplifies the technological problems that occur during the design of nonlinear devices based on the above-mentioned phenomena.

The development of the physics of metamaterials has given a great boost to this field. Particularly, it was caused by the existence of many resonances in such artificial media with quite high frequency and spatial dispersion [2, 4].

It is known that a metamaterial can be represented as a system of elementary blocks of subwavelength size, called meta-atoms [5, 6]. To describe the features of nonlinear metamaterials, at first, it is necessary to study the nonlinear features of the meta-atoms. The essential step was made in [2] where the detailed study was presented for the nonlinear meta-atom made from a Split-Ring Resonator (SRR) with a varactor diode.

Note that the inclusion of the magnetoactive elements into metamaterials can amplify their nonlinear properties. The situation is similar to the above-mentioned resonators where the amplification

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*Received 27 November 2018, Accepted 15 February 2019, Scheduled 12 March 2019*

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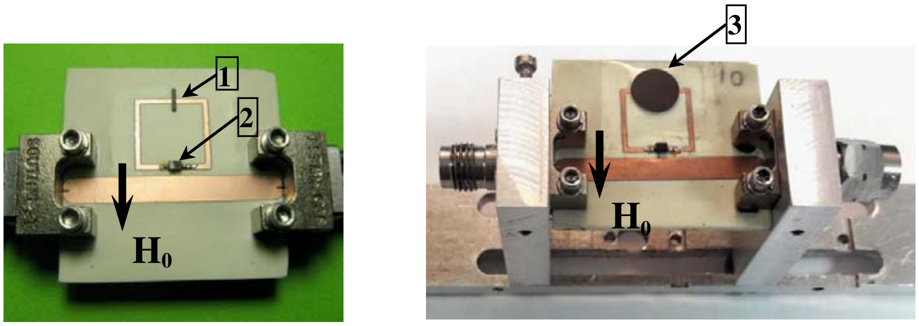
is caused by the superposition of the electrodynamical resonance and Electron Spin Resonance (ESR) (Ferromagnetic Resonance (FMR)) phenomena.

So, the aim of this paper is to present the experimental study of the effect of the ferrite frequency dispersion on the nonlinear properties of a meta-atom.

## 2. EXPERIMENT

In Figure 1, one can see the structure under study (meta-atom), which is formed by a rectangular Split-Ring Resonator (SRR) loaded with the first nonlinear element (varactor diode) and the second nonlinear element, ferrite element.



(a) (b)

**Figure 1.** (Color online) Structures under study (meta-atom with ferrite element in the form of a rectangular parallelepiped (a), meta-atom with ferrite element in the form of a disc (b)): 1 — the ferrite rectangular parallelepiped; 2 — the varactor diode; 3 — the ferrite disc.

The meta-atom is formed by a rectangular SRR with varactor diode 2 and loaded with a ferrite element 1, 3 (Fig. 1). SRR is $10 \times 10$ mm in size here. The microstrip line and SRR are conductively coupled through a copper strip (0.2 mm width). The thickness of the substrate (Rogers RO4450B ($\varepsilon = 3.3 + 0.003i$)) is $h = 1.524$ mm. The varactor diode (Infineon BB857) is placed in the SRR gap (0.5 mm). When the microwave radiation power changes, the diode capacitance varies as well. This results in a shift of the SRR resonance frequency. The magnetoactive properties of the meta-atom are provided by the ferrite rectangular parallelepiped 1 and/or ferrite disc 3. The ferrite rectangular parallelepiped (SN-475) is of dimensions $3.4 \times 0.5$ mm, the thickness 0.5 mm, and is located as shown in Fig. 1. The ferrite disc is 0.5 mm in thickness and 7 mm in diameter (Fig. 1). The external DC magnetic bias field (magnetic bias) $H_0$ is applied in the plane of the substrate.

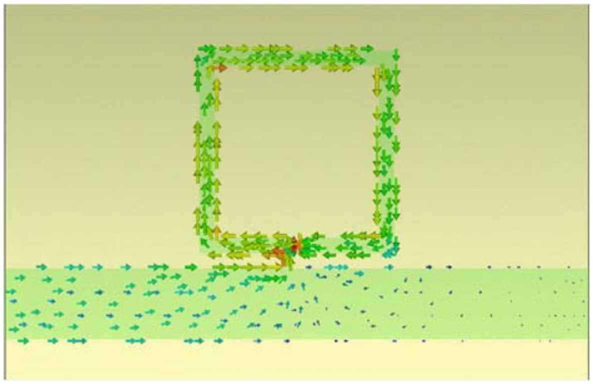


**Figure 2.** (Color online) Surface current distribution (numerical calculation) at frequency $f_{res} = 475$ MHz.

Figure 2 shows the numerical simulation for the distribution of currents at the resonance frequency SRR ($f_{res} = 475\,\mathrm{MHz}$) without nonlinearity. In this case, the varactor diode has been modeled as an ideal capacitance with $C = 6.5\,\mathrm{pF}$. It can be seen from Fig. 2 that the distribution of currents is circular. This corresponds to the magnetic mode in the SRR. It worth noticing that only in this mode the nonlinearity in the structure under study is observed experimentally.

Figure 3 shows the experimental dependence of the transmission on the frequency for different values of the input power at zero-field magnetic bias. It can be noticed that the resonance frequency decreases with the increase of the input power. Also, it is easy to see that the resonance curve demonstrates the behavior which is typical for a nonlinear oscillator [1–3, 10]. Namely, with the increase of the input power the resonance curve becomes nonsymmetrical, and for the sufficiently large nonlinearity the resonance curve becomes bistable [1]. This means that on the curve's shape appears the area, where the derivative of the transmission with respect to frequency tends to infinity.

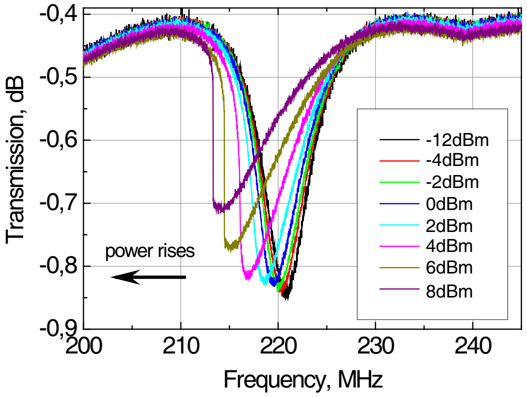


**Figure 3.** (Color online) Shift of transmission peak with the input power at zero-field magnetic bias (experiment).

This phenomenon is caused by the nonlinear properties of the varactor diode. As shown in [7], the differential equation for the nonlinear oscillator, which is the most suitable for the description of the varactor diode, has the form [7],

$$\frac{d^2q}{dt^2} + \gamma\frac{dq}{dt} + \omega_0^2 q + a\omega_0^2 q^2 + b\omega_0^2 q^3 = \omega_0^2 E\left(t\right), \tag{1}$$

where $q$ is the normalized charge; $\gamma$ is the linear damping constant, $\omega_0$ is the zero-bias resonance frequency; $E\left(t\right)$ is the amplitude of the driving voltage resulting from the electromagnetic field incident on the meta-atom. Solving this equation, one can express the third order nonlinear susceptibility as a function of frequency $\omega$ in the following form [7],

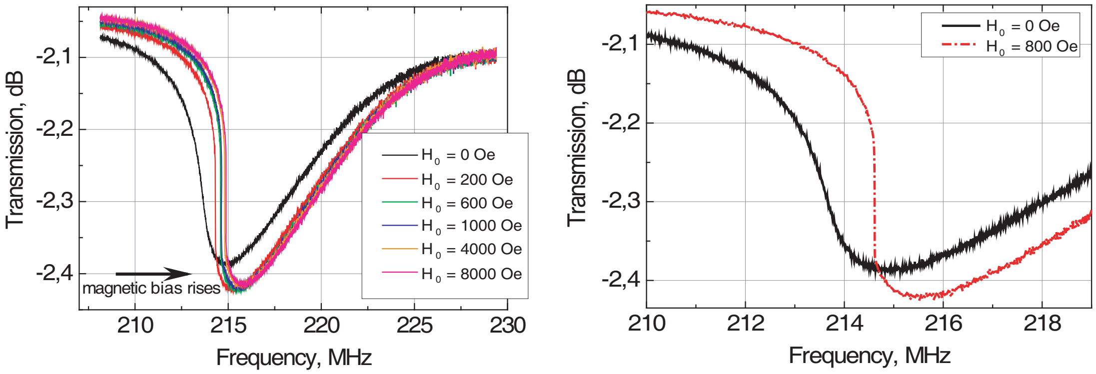
$$\chi^{(3)}_{yyyy}\left(\omega\right) = \frac{F\omega_0^6\omega^4 A^2\mu_0^2}{D^3\left(\omega\right)D\left(-\omega\right)}\left(\frac{4a^2\omega_0^2}{3D\left(0\right)} + \frac{2a^2\omega_0^2}{3D\left(2\omega\right)} - b\right) \tag{2}$$

where $\mu_0$ is the permeability of a vacuum; $F$ is the strength of the oscillator in the numerator of the linear SRR effective susceptibility [9]; $A$ is the effective area enclosed by the SRR structure; $D\left(\omega\right) \equiv \omega_0^2 - \omega^2 - i\gamma\omega$ is the resonant denominator. Then the expression for permeability, which depends on the microwave power, obtains the form [8],

$$\mu^{NL}_{eff}\left(\omega\right) = 1 + \frac{F\omega^2}{D\left(\omega\right)} + 3\chi^{(3)}_{yyyy}\left|H\right|^2, \tag{3}$$

where $H$ is the alternating magnetic field. From this expression, we can conclude that it is necessary to assign to the meta-atom under study the notion of the effective permeability. Such effective permeability is caused by the nonlinear properties of the meta-atom.

To understand the nonlinear properties of the meta-atom, we experimentally investigate the effect of the magnetic bias on the position and shape of the SRR resonance curve (Fig. 4). One can see that



(a) (b)

**Figure 4.** (Color online) (a) The dependence of the position of the resonance curve on the magnetic bias at the input power level 5 dBm; (b) enlarged Fig. 4(a) (experiment).

as the magnetic bias increases, the resonance frequency of this curve also increases. Such behavior of the resonance curve occurs because the frequency of the ferromagnetic resonance (FMR) in the ferrite increases with increase of the magnetic bias [3, 5, 10]. When the magnetic bias reaches a certain value, the frequency of the FMR attains the resonance frequency (eigen frequency) of the SRR. This, in turn, leads to a sharp change in the resonance frequency of the SRR. As a result, the amplitude of the signal varies drastically for these magnetic biases. In particular, Fig. 4 shows that as the magnetic bias increases, the nonlinearity of the meta-atom increases (Fig. 4(a)), and the resonance curve shows bistability at $H_0 > 800\,\mathrm{Oe}$ (Fig. 4(b)).

Now we analyze in details the nonlinear behavior of the meta-atoms for two shapes of the magnetic element.

### 2.1. Ferrite Element in the Form of a Thin Rectangular Parallelepiped

In order to study the influence of the frequency dispersion of the ferrite rectangular parallelepiped on the nonlinear properties of the meta-atom, the dependence of the resonance frequency of the SRR on the magnetic bias was measured for different values of the input power (Fig. 5). Note that the long edge of the ferrite rectangular parallelepiped is parallel to the vector of the magnetic bias ($H_0$). It can be seen that when the magnetic bias $H_0$ is increased, the resonance frequency $f_{res}$ increases too up to the value of the magnetic bias about 1000 Oe, then the curves reach plateau. This is because for the magnetic bias more than 1000 Oe, ferrite appears in a saturated state. It is also seen that with the increase of the input power, the resonance frequency (for each predetermined field value) decreases. This phenomenon is observed for all values of the input power. However, it should be noted that for greater field values, with a power change from $-10\,\mathrm{dBm}$ to $+7\,\mathrm{dBm}$, the difference between the resonance frequencies is significantly larger ($\delta f = 7\,\mathrm{MHz}$) than that for the smaller field values ($\delta f = 4\,\mathrm{MHz}$). This can be explained by the fact that the nonlinear properties of the meta-atom are caused by both the nonlinear properties of the ferrite and the nonlinearity of the varactor diode. As expected [3, 10], such a combined effect becomes more pronounced at greater fields.

In order to demonstrate the bistable state of a nonlinear meta-atom loaded with a ferrite rectangular parallelepiped, we plot the dependence of the output power (transmission) on the input power at the frequency $f_0 = 224\,\mathrm{MHz}$. Here $f_0$ is the SRR resonance frequency at the input power 6 dBm. We have analyzed this dependence in both the presence and absence of the magnetic bias.

As a result, we have experimentally obtained the bistability loops (Fig. 6) for the transmission through the meta-atom including the ferrite rectangular parallelepiped with a parallel orientation relative to magnetic biases ($H_0 = 0\,\mathrm{Oe}$ and $H_0 = 1000\,\mathrm{Oe}$).

It can be seen that with the increase (decrease) of the input power, the output power (transmission) demonstrates an abrupt jump-down (jump-up) behavior in the vicinity of the bistability area. Moreover,

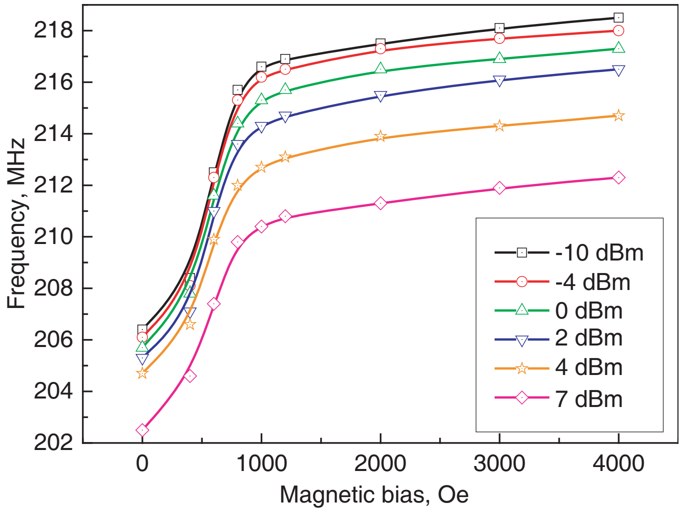


**Figure 5.** (Color online) Shift of the SRR resonance frequency with respect to the magnetic bias at various values of the input power (experiment).

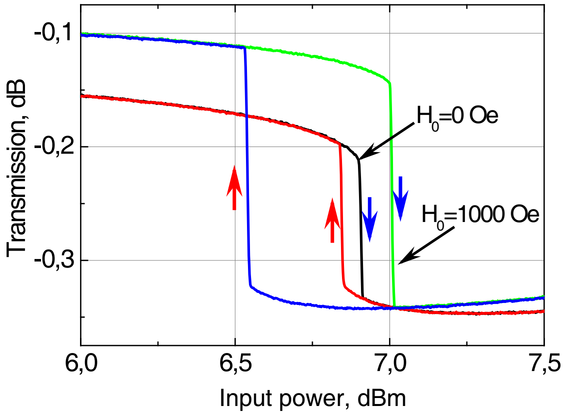


**Figure 6.** (Color online) Hysteresis loops for the ferrite rectangular parallelepiped $H_0 = 0\,\mathrm{Oe}$ and $H_0 = 1000\,\mathrm{Oe}$, $f_0 = 224\,\mathrm{MHz}$ (experiment).

as the magnetic bias increases, we observe the increase in the difference between the input power values at which the output power jumps down (or up). This indicates the increase in the degree of nonlinearity of the investigated meta-atom.

### 2.2. Ferrite Element in the Form of a Disc

In order to show the influence of the volume of the magnetic element on the nonlinear properties of the meta-atom, we performed the same experiment with the ferrite element in the shape of a disc (diameter — 7 mm; thickness — 0.5 mm, Fig. 1(b)). In this case, the ferrite element has been placed on the meta-atom in the vicinity of the electric field concentration. The dependence of the resonance line position on the magnetic bias for this case is shown in Fig. 7. It can be seen that as the magnetic bias $H_0$ increases, the resonance frequency $f_{res}$ increases too, and when the magnetic bias reaches about 800 Oe and increases further, the curves reach plateau. The explanation of this effect, in fact, is no different from the one given above (see Fig. 5).

As for the magnetic element in the form of a rectangular parallelepiped, we have investigated the effect of the frequency dispersion of the ferrite disk on the nonlinear properties of the meta-atom.

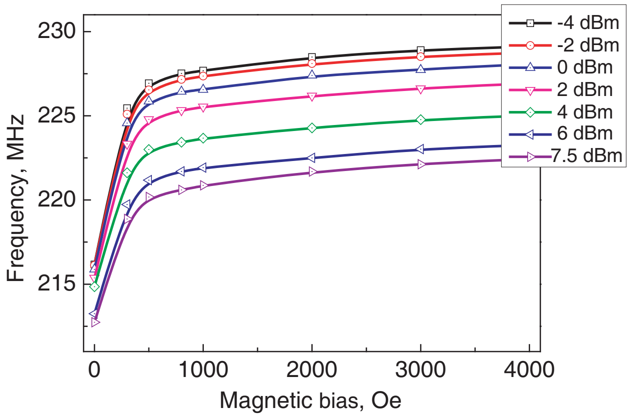


**Figure 7.** (Color online) Shift of the SRR resonance with regards to the magnetic bias at various values of the input power (experiment).

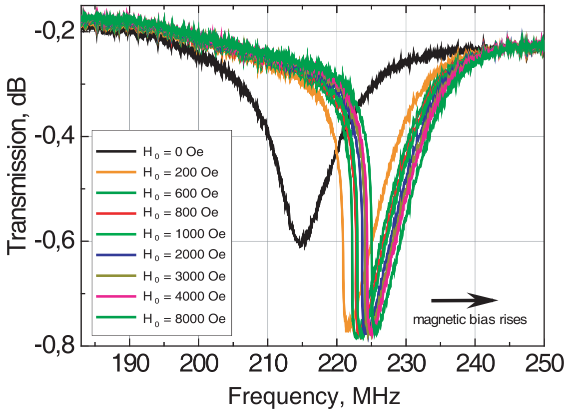


**Figure 8.** (Color online) The dependence of the resonance curve position on the magnetic bias at the input power level 4 dBm (experiment).

Fig. 8 shows the effect of the magnetic bias on the position and shape of the resonance curve with the input power 4 dBm. It can be seen that the influence of the magnetic bias is stronger in this case. This occurs most likely due to the increase of the volume of the magnetic element. Namely, when the resonance frequency of the SRR shifts by 2 MHz (from 215 MHz to 217 MHz) in the case of a rectangular parallelepiped (Fig. 4), and the magnetic bias varies from 0 to 200 Oe, for the case of a disc (Fig. 8), the resonance frequency of the SRR shifts by 3 times larger factor — 6 MHz (from 215 MHz to 221 MHz) with the same variations of the magnetic bias.

The bistability loop for the transmission through the meta-atom with a ferrite element in the form of a disc is shown in Fig. 9 ($f_0 = 221\,\mathrm{MHz}$, $H_0 = 800\,\mathrm{Oe}$). Here $f_0$ is the SRR resonance frequency at the input power 5 dBm.

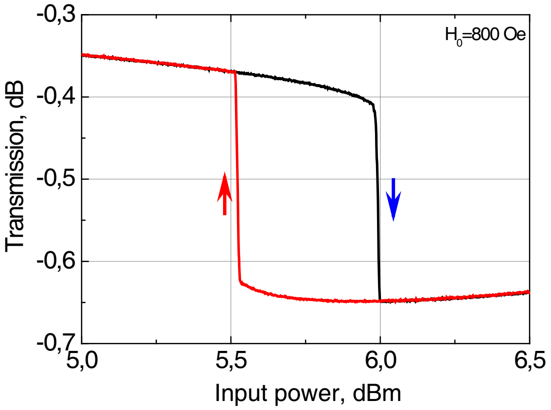


**Figure 9.** (Color online) Hysteresis loop for the ferrite disc $H_0 = 800\,\mathrm{Oe}$ at $f_0 = 221\,\mathrm{MHz}$ (experiment).

Figure 9 demonstrates a similar behavior of the bistability loop to the one shown in Fig. 6. The difference is that with the increase of input power, the jump-down (jump-up) of the output power is observed at lower values of the input power (5.5–6.0 dBm). This most likely indicates a greater contribution to the nonlinearity by the ferrite element. Since the frequency has not practically been changed, it can be seen that the nonlinearity in the second case is clearly greater, which is apparently caused by the greater volume of the magnet taking part in the resonance oscillations of this structure. Thus, it is evident that one can significantly vary the nonlinear features of a meta-atom by changing the magnetic properties of the magnetic element made from the strong ferromagnet.

## 3. CONCLUSIONS

In this work,

- we have experimentally demonstrated the possibility of the resonance frequency adjustment for the SRR loaded with the ferrite elements of different shapes by means of not only the input power but also the magnetic bias;
- we have shown that as the magnetic bias increases to a certain value, the resonance curve of nonlinear meta-atom demonstrates bistability.
- it has been demonstrated that the volume of the ferrite element affects the resonance frequency of the meta-atom in a predictable way;
- we have achieved a significant enhancement of the nonlinearity of the whole structure by means of the meta-atom nonlinear properties enhancement using both the ferrite element and varactor diode.

Thus, a device designed on the basis of the given meta-atom can be used more efficiently as a nonlinear element under the conditions of very small values of microwave power and magnetic biases. However, it is necessary to choose those ranges of frequencies and magnetic biases for which both sources of nonlinearity are the most effective and can produce the maximum increase of the nonlinear coefficients in the motion equation of the meta-atom.

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