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Impedance Synthesis of 2D Antenna Arrays of Slotted Spherical Radiators

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Abstract—An impedance synthesis problem of 2D antenna arrays consisting of slotted spherical radiators, whose geometric centers are located at nodes of a flat rectangular grid with double periodicity, has been solved. The problem is formulated as follows: to determine complex impedances distributed over surfaces of the spherical radiators which allows us to steer the radiation pattern (RP) of the antenna array to given directions. Analytical solution of the impedance synthesis problem (as an alternative to numerical solution) was obtained under the assumption that spherical radiators are excited by axially symmetric magnetic currents with equal amplitudes. The approach was verified by simulation of the five-element linear antenna array. The possibility of RP scanning in a wide range was confirmed by using the synthesized distributions of complex impedances.

1. INTRODUCTION

A distinctive functional characteristic of antenna arrays is electric spatial scanning of their RPs (see, e.g., [1]). The scanning is usually controlled by varying amplitude-phase distributions of currents in array elements. For impedance vibrator arrays, there exists a possibility to control the phase distribution by varying surface impedances of vibrator elements [2, 3]. In this case, effective electric lengths of individual vibrators and, hence, the amplitude-phase distribution of currents over the entire array are varied by adjusting vibrator impedances.

Using this approach, we have earlier investigated one-dimensional and two-dimensional vibrator arrays and justified this approach [4, 5]. For symmetric excitation of equidistant vibrator arrays by voltage generators, the problem was solved in an analytical form. This article is aimed at the analytic solution of the impedance synthesis problem for the 2D plane antenna array consisting of slotted spherical radiators with small diffraction radii. We could not find the problem solution in the literature.

It should be noted that the electrodynamic characteristics of a slotted spherical antenna consisting of spherical scatterer with a narrow equatorial slot can be analyzed using the theory of coplanar antennas [6]. However, such an analysis by direct numerical methods is difficult to accomplish even for an isolated ideally conducting sphere and especially for an array of spherical radiators with impedance coatings.

Arrays of small impedance spherical resonators can also be considered as elements of tunable metamaterials for monitoring terahertz (THz) radiation [7]. For example, such materials based on dielectric resonators which do not use conductive materials and, hence, eliminate some loss); they can be important for THz applications [8, 9]. Such structures with innovative impedance coatings can be used for the detection of THz waves, and they can be used in integrated elements of photodetectors [10]. In any case, such applications confirm the practical significance of the results presented in this paper.

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2. FORMULATION OF THE IMPEDANCE SYNTHESIS PROBLEM

Let a circular slot be cut in a sphere parallel to its equatorial section as shown in Fig. 1. The sphere radius is R, and the slot width measured along the slot ark is d. The coordinate of the slot center is θ_0. In the spherical coordinate system (r', θ', φ'), the slot occupies the area defined by coordinates $\varphi' \in [0; 2\pi]$ and $\theta' \in [\theta_0 - d/(2R); \theta_0 + d/(2R)]$. The sphere surface is characterized by distributed isotropic impedance $\bar{Z}_S$ normalized to free space resistance $Z_0 = 120\pi$ [Ohm].

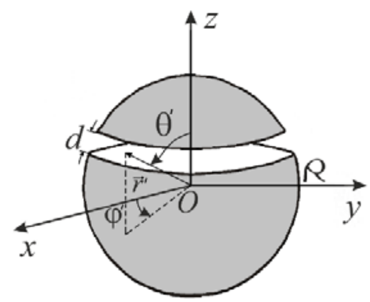

Figure 1. The slotted spherical antenna.

Let us consider the most interesting for practical application axially symmetric excitation of narrow $(d \ll R)$ circular slot. We may assume that the slot electric field has only a meridional component with a constant distribution along the slot axis. Then, within the framework of the above approximations, the density of equivalent magnetic $\vec{J}^m$ and electric $\vec{J}^e$ currents on the slot aperture can be represented as [11]

$$\vec{J}^m = -\frac{V_0}{d}\delta\left(r' - R\right)\vec{\varphi}^0, \quad \vec{J}^e = \frac{V_0}{Z_S d}\delta\left(r' - R\right)\vec{\theta}^0, \tag{1}$$

where $\vec{\varphi}^0$ and $\vec{\theta}^0$ are the unit vectors of the spherical coordinate system; $\delta(r' - R)$ is the Dirac delta function; V_0 is the complex voltage amplitude, and $Z_S = \bar{Z}_S Z_0$.

The electromagnetic fields excited by impedance spherical antennas in free space can be obtained based on components of the Green's functions by using pair representations for currents in Eq. (1) on the spherical scatterer surface [12]. In the case of monochromatic excitation, when time dependence is $e^{i\omega t}$ (ω is the circular frequency), electric field components in the spherical coordinate system (r, θ, φ) can be written as

$$\begin{aligned}
E_r(r,\theta) &= \frac{V_0 R \sin\theta_0}{2rd}\sum_{j=1}^{\infty} A_j \left[\mathrm{ctg}\theta P_j^1(\cos\theta) + \frac{dP_j^1(\cos\theta)}{d\theta}\right] \\
&\quad \times \left\{h_j^{(2)}(kr) + \frac{\bar{Z}_S}{ikr}\left[(j-1)\,h_j^{(2)}(kr) - kr\,h_{j+1}^{(2)}(kr)\right]\right\}, \\
E_\theta(r,\theta) &= \frac{V_0 k R \sin\theta_0}{2d}\sum_{j=1}^{\infty} A_j \left\{P_j^1(\cos\theta)\left[h_{j+1}^{(2)}(kr) - h_j^{(2)}(kr)\left(i\bar{Z}_S + \frac{j+1}{kr}\right)\right]\right. \\
&\quad \left. - \frac{\bar{Z}_S}{i(kr)^2} h_j^{(2)}(kr)\left[j(j+1)\left(P_j^1(\cos\theta) + \frac{dP_j^1(\cos\theta)}{d\theta}\right) - \frac{1}{\sin^2\theta}\left(\frac{dP_j^1(\cos\theta)}{d\theta} - 2\mathrm{ctg}\theta P_j^1(\cos\theta)\right)\right]\right\}, \\
E_\varphi(r,\theta) &= 0,
\end{aligned} \tag{2}$$

where $k = 2\pi/\lambda$ is the wave number; λ is the free space wavelength; $h_j^{(2)}(kr) = \sqrt{\frac{\pi}{2kr}} H_{j+1/2}^{(2)}(kr)$ are the Hankel's spherical functions of the second kind; $P_j^l(\cos\theta)\Big|_{l=0,1}$ are the Legendre associated functions of the first kind; and coefficients A_j are defined as follows

$$A_j = \frac{(2j+1)}{j\ (j+1)} \times \frac{P_j\left(\cos\left(\theta_0 + d/(2R)\right)\right) - P_j\left(\cos\left(\theta_0 - d/(2R)\right)\right)}{Q_j\left(h_j^{(2)}(kR)\right)}, \tag{3}$$

where $Q_j(h_j^{(2)}(kR)) = \frac{(j+1)\left[j\bar{Z}_S - ikR\left(1+\bar{Z}_S^2\right)\right] - 2\bar{Z}_S k^2 R^2}{ikR\left(1+\bar{Z}_S^2\right)} h_j^{(2)}(kR) + kRh_{j+1}^{(2)}(kR)$. The formulas (2) will be used for further analysis.

Consider the 2D antenna array consisting of the slot spherical antennas (Fig. 1) located in free space at the plane $(x0y)$ of the Cartesian coordinate system (x, y, z) as shown in Fig. 2. We will also define a spherical coordinate system, which polar axis coincides with the axis $\{0z\}$, and the angle φ is measured from the axis $\{0x\}$. Let d_y be the distance between adjacent rows of the array, d_x be the distance between the centers of spherical elements in a row, N_y be the number of rows, and N_x be the number of elements in a row. Without loss of generality, we will assume that the parameters of the spherical radiators: sphere radii R, d, and θ_0 are equal for all $N = N_z \times N_x$. The sphere impedances, $\bar{Z}_{S,nm}$ ($n \in [1, N_z]$ and $m \in [1, N_x]$), are different.

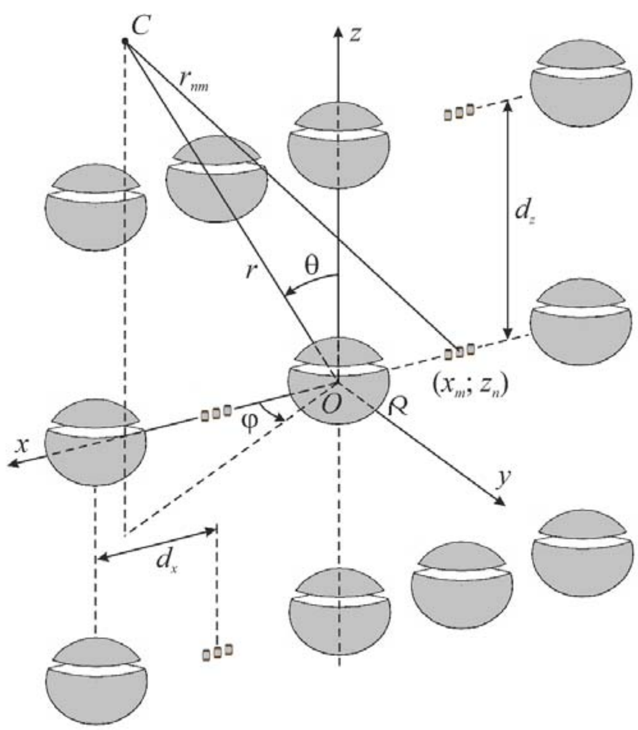

Figure 2. Geometry of the 2D antenna array.

Taking into account the formulas (2) and the asymptotics of the spherical functions $h_j^{(2)}(kr) \approx \frac{1}{kr} i^{j+1} e^{-ikr}$ under conditions $|kr| \gg j$, $(kr) \to \infty$, we can write the main component of the radiated electric field in the wave zone by the spherical antenna with indices (n, m) in the following form

$$E_{nm}^{\theta} = -\frac{V_{nm} kR \sin\theta_0}{2d} \cdot \frac{e^{-ikr_{nm}}}{kr_{nm}} \sum_{j=1}^{\infty} A_{j,nm} P_j^1(\cos\theta)\, i^j \left[1 - \bar{Z}_{S,nm}\right], \tag{4}$$

where $A_{j,nm} = A_j|_{\bar{Z}_s \to \bar{Z}_{s,nm}}$.

If terms proportional to $(1/r)^2$ and$(1/r)^3$ are eliminated from formulas (2), the components of the electric fields E_φ and E_r of a single spherical radiator in the wave zone are equal to zero. Let all spherical radiators in the array be resonantly tuned by selecting their internal resistances and parameters d_x/λ, R/d_x, d_y/λ, R/d_y. In this case, the mutual influence of elements can be compensated, and the total radiation field of the array $E_\theta(r,\theta,\varphi)$ can be presented as the sum of the radiation fields of all radiators. If the phases of these fields arriving at the observation point $C(r,\theta,\varphi)$ are taken into account, the resulting field can be written as

$$E_\theta\left(r,\theta,\varphi\right) = -\frac{kR\sin\theta_0}{2d}\sum_{n=1}^{N_z}\sum_{m=1}^{N_x}\frac{e^{-ikr_{nm}}}{r_{nm}}V_{nm}\sum_{j=1}^{\infty}A_{j,nm}P_j^1\left(\cos\theta\right)i^j\left[1-\bar{Z}_{S,nm}\right]. \quad (5)$$

Thus, the problem of impedance synthesis is reduced to defining the impedances of each radiator, $\bar{Z}_{s,nm}$, which, for a given direction of the maximum radiation in the wave zone $(\theta_{\max},\varphi_{\max})$, is determined based on formula (5).

3. SOLUTION OF THE PROBLEM OF IMPEDANCE SYNTHESIS

First of all, let us convert expression (4) for the radiation field of the single spherical radiator in another form. For perfectly conducting spherical radiators ($\bar{Z}_S = 0$), the coefficients in Eq. (3) can be presented as

$$A_j|_{\bar{Z}_S=0} = A_j^0 = \frac{(2j+1)}{j\ (j+1)}\times\frac{P_j\left(\cos\left(\theta_0+d/(2R)\right)\right)-P_j\left(\cos\left(\theta_0-d/(2R)\right)\right)}{-(j+1)h_j^{(2)}\left(kR\right)+kRh_{j+1}^{(2)}\left(kR\right)}. \quad (6)$$

Since according to the impedance concept $|\bar{Z}_{S,nm}|\ll 1$, the expression for the functional $Q_j(h_j^{(2)}(kR))$ up to terms proportional $\bar{Z}_{S,nm}^2$ can be rewritten as

$$Q_j(h_j^{(2)}\left(kR\right)) \approx -(j+1)h_j^{(2)}\left(kR\right)+kRh_{j+1}^{(2)}\left(kR\right)-i\frac{\bar{Z}_{S,nm}}{kR}\left(j(j+1)-2k^2R^2\right)h_j^{(2)}\left(kR\right).$$

Multiplying the numerator and denominator of expression (3) by $[-(j+1)h_j^{(2)}(kR)+kRh_{j+1}^{(2)}(kR)$ $+i\frac{\bar{Z}_{S,nm}}{kR}(j(j+1)-2k^2R^2)h_j^{(2)}(kR)]$ we obtain with accuracy up to $\bar{Z}_{S,nm}^2$

$$A_{j,nm}\approx A_j^0\left[1+i\frac{\bar{Z}_{S,nm}}{kR}\cdot\frac{j(j+1)-2k^2R^2}{kRh_{j+1}^{(2)}\left(kR\right)-(j+1)h_j^{(2)}\left(kR\right)}h_j^{(2)}\left(kR\right)\right]. \quad (7)$$

Then, formula (4) takes the following form

$$\begin{aligned}E_{nm}^\theta \approx\ & -\frac{V_{nm}kR\sin\theta_0}{2d}\cdot\frac{e^{-ikr_{nm}}}{kr_{nm}}\sum_{j=1}^{\infty}A_j^0P_j^1\left(\cos\theta\right)i^j\\ &\times\left[1-\bar{Z}_{S,nm}\left(1-\frac{ih_j^{(2)}\left(kR\right)}{kR}\frac{j(j+1)-2k^2R^2}{kRh_{j+1}^{(2)}\left(kR\right)-(j+1)h_j^{(2)}\left(kR\right)}\right)\right].\end{aligned} \quad (8)$$

Formula (8) can be presented as the sum of two terms. The first term defines the radiation pattern (RP) of the annular slot cut in a perfectly conducting sphere [13],

$$Fs(\theta) = -\frac{kR\sin\theta_0}{2kd}\sum_{j=1}^{\infty}A_j^0P_j^1\left(\cos\theta\right)i^j, \quad (9)$$

while the second defines variation of the radiation field of the spherical antenna, due to influence of the sphere impedance coating. Thus, we obtain

$$E_{nm}^\theta = V_{nm}\frac{e^{-ikr_{nm}}}{r_{nm}}\left[Fs(\theta)-\bar{Z}_{S,nm}\left(Fs(\theta)+\Delta Fs(\theta)\right)\right], \quad (10)$$

where the term $\Delta Fs(\theta) = \frac{\sin\theta_0}{2kd}\sum\limits_{j=1}^{\infty} A_j^0 \frac{j(j+1)-2k^2R^2}{kRh_{j+1}^{(2)}(kR)-(j+1)h_j^{(2)}(kR)} P_j^1(\cos\theta)\, h_j^{(2)}(kR)\, i^{j+1}$. Finally, we can write

$$E_\theta(r,\theta,\varphi) = \sum_{n=1}^{N_z}\sum_{m=1}^{N_x} \frac{e^{-ikr_{nm}}}{r_{nm}} V_{nm}\left[Fs(\theta) - \bar{Z}_{S,nm}\left(Fs(\theta) + \Delta Fs(\theta)\right)\right]. \tag{11}$$

Taking into account the geometric path difference for the wave propagating from neighboring array elements to the observation point $C(r,\theta,\varphi)$, we can rewrite the expression (11) for the field $E_\theta(r,\theta,\varphi)$ in wave zone as

$$E_\theta(\theta,\varphi) = e^{-i(N_z+1)u/2} e^{-i(N_x+1)v/2} \sum_{n=1}^{N_z}\sum_{m=1}^{N_x} V_{nm}\left[Fs(\theta) - \bar{Z}_{S,nm}\left(Fs(\theta) + \Delta Fs(\theta)\right)\right] e^{i(nu+mv)}. \tag{12}$$

where $u = kd_z\cos\theta$ and $\nu = kd_x\sin\theta\cos\varphi$. If the spherical radiators are perfectly conducting, $\bar{Z}_{S,nm} = 0$, and amplitudes of excitation voltages are equal, $V_{nm} = V_0$, expression (12) can be reduced to

$$E_\theta(\theta,\varphi) = V_0 e^{-i(N_z+1)u/2} e^{-i(N_x+1)v/2} Fs(\theta) \sum_{n=1}^{N_z} e^{inu} \sum_{m=1}^{N_x} e^{imv}. \tag{13}$$

In this case, the maximum of the RP of the array is reached under conditions $u = v = 0$, and it is directed along the axis $\{0y\}$ ($\theta = \pi/2$, $\varphi = \pi/2$). As can be seen, expression (13) is the product of two multipliers consisting of independent series. These multipliers present the RP of the linear array of spherical antennas which axes are directed along the axes $\{0z\}$ and $\{0x\}$.

As known from the antenna array theory (see, e.g., [14]), scanning of the array RP in space can be achieved by linear phase shifts between currents of the adjacent array radiators. If the phase shifts between adjacent rows and adjacent radiators in a row are equal to Δv and Δu, the direction of the maximum array radiation can be determined by the angles $(\theta_{\max};\varphi_{\max})$ defined by the formulas

$$\cos\theta_{\max} = \Delta u/(kd_z) \quad \text{and} \quad \sin\theta_{\max}\cos\varphi_{\max} = \Delta v/(kd_x). \tag{14}$$

Then expression (13) can be represented as

$$E_\theta(\theta,\varphi) = V_0 e^{-i(N_z+1)u/2} e^{-i(N_x+1)v/2} Fs(\theta) \sum_{n=1}^{N_z}\sum_{m=1}^{N_x} e^{i(nu+mv)} e^{-i[(n-1)\Delta u + (m-1)\Delta v]}. \tag{15}$$

As can be seen, formulas (12) and (15) become identical if the relations

$$1 - e^{-ik[(n-1)d_z\cos\theta + (m-1)d_x\sin\theta\cos\varphi]} = \bar{Z}_{S,nm}\left(1 + \frac{\Delta Fs(\theta)}{Fs(\theta)}\right) \tag{16}$$

are valid for all indices n and m.

If the impedances of the array radiators are complex quantities $\bar{Z}_{S,nm} = \bar{R}_{nm} + i\bar{X}_{nm}$, Equation (16) can be represented as follows

$$1 - \cos(k\gamma_{nm}) + i\sin(k\gamma_{nm}) = \left(\bar{R}_{nm} + i\bar{X}_{nm}\right)\left(1 + \frac{\Delta Fs(\theta_{\max})}{Fs(\theta_{\max})}\right). \tag{17}$$

Relations (16) are equations for array beam scanning, whose solution uniquely defines the matrix of unknown impedances $\{\bar{Z}_{S,nm}\}$ for the predefined angles $(\theta_{\max};\varphi_{\max})$. It is easy to verify that if $d_z = 0$ or $d_x = 0$, Equation (16) can be reduced to equations for one-dimensional arrays.

Taking into account Eqs. (9) and (11), we obtain, based on Eq. (17), the final formulas for the real and imaginary parts of the surface impedances

$$\begin{aligned} \bar{R}_{nm} &= \frac{C_2\sin(k\gamma_{nm}) + C_1\left[1-\cos(k\gamma_{nm})\right]}{C_1^2 + C_2^2}, \quad n = 1,2,\ldots,N_z; \quad m = 1,2,\ldots,N_x; \\ \bar{X}_{nm} &= \frac{C_1\sin(k\gamma_{nm}) - C_2\left[1-\cos(k\gamma_{nm})\right]}{C_1^2 + C_2^2}, \end{aligned} \tag{18}$$

where $C_1 = \mathrm{Re}\{1 + \frac{\Delta Fs(\theta_{\max})}{Fs(\theta_{\max})}\}$ and $C_2 = \mathrm{Im}\{1 + \frac{\Delta Fs(\theta_{\max})}{Fs(\theta_{\max})}\}$.

It should be noted that formulas (18) are valid for any number of radiators and for arbitrary distance between them. However, in the general case, the values of the impedances $\bar{Z}_{S,nm} = \bar{R}_{nm} + i\bar{X}_{nm}$ thus obtained as effective physical quantities cannot guarantee that the conditions $\bar{R}_{nm} \geq 0$ hold true. These conditions, put forward by energy considerations, determine the possibility of practical realization in the form of the internal impedance $\bar{Z}_{S,nm} = \bar{R}_{nm} + i\bar{X}_{nm}$ of the spherical radiator. In such cases, the conditions $\bar{R}_{nm} \geq 0$ can be fulfilled after substitutions $V_{nm} \to -V_{nm}$ and $\bar{Z}_{S,nm} \to -\bar{Z}_{S,nm}$.

4. NUMERICAL RESULTS

In order to verify the approach, we present simulation results for a one-dimensional equidistant array consisting of five identical spherical radiators located along the axis $\{0z\}$ at the distance $d_z = \lambda/2$ from each other (Fig. 2). The array parameters are as follows: $N_z = 5$, $d_x = 0$, $d_z = \lambda/2$, $\theta_0 = \pi/2$, $R = \lambda/20$, and $d = \pi\lambda/500$. Such an array geometry simplifies the analysis and demonstration of beam scanning, since, in this case, the RP does not depend on the angular coordinate φ. In other words, the array radiation fields should be calculated only in the E-plane, considering the geometric difference of the wave paths from the adjacent radiators to the observation point, $d_z \cos\theta$, in the wave zone.

Simulation results, the value of required complex impedances $\bar{Z}_{S,n} = \bar{R}_n + i\bar{X}_n$, are listed in Table 1 for the angles $\theta_{\max} = 90°; 70°; 50°; 30°$. The results are obtained based on Eq. (18), taking into account only ten terms in series of the formulas (9) and (10), which is sufficient for the spherical antennas of small radii.

As expected, all spherical radiators should be perfectly conducting, i.e., $\bar{Z}_{S,n} = 0$ when $\theta_{\max} = 90°$. The normalized RP for this case is shown in Fig. 3 as solid curve 1. As can be seen, the basis array

Table 1. Estimated values of the complex impedances $\bar{Z}_{S,n} = \bar{R}_n + i\bar{X}_n$.

$n\backslash\theta_{\max}$	$\theta_{\max} = 30°$	$\theta_{\max} = 50°$	$\theta_{\max} = 70°$
1	$0 + i0$	$0 + i0$	$0 + i0$
2	$0.117 - i0.28$	$0.177 - i0.192$	$0.148 - i0.054$
3	$-0.104 - i0.072$	$-0.072 - i0.268$	$0.171 - i0.211$
4	$0.183 - i0.171$	$-0.033 - i0.01$	$0.044 - i0.305$
5	$-0.119 - i0.198$	$0.183 - i0.157$	$-0.099 - i0.239$

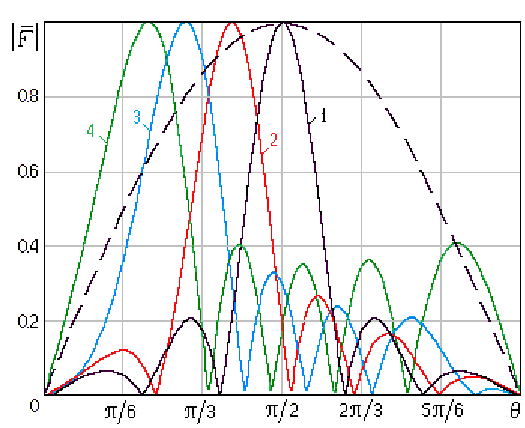

Figure 3. The E-plane RP of linear antenna array: 1 — $\theta_{\max} = 90°$; 2 — $\theta_{\max} = 70°$; 3 — $\theta_{\max} = 50°$; 4 — $\theta_{\max} = 30°$; dotted line presents the RP of the single radiator.

element, the radiator with index $n = 1$, should be perfectly conducting for any other $\theta_{\max}$. A similar effect was also observed for vibrator arrays [4].

As can be seen from Fig. 3, the shape of the angular dependence of the electric field intensity $|\bar{F}|$ varies slightly when the RP maximum is varied: the greater is the deviation of the $\theta_{\max}$ angle from the vertical, the more noticeable are these variations. The simulation results have shown that impedances of the array elements calculated for the sector $\theta_{\max} \in [90° \pm 45°]$ allow to accurately steer the array RP in any predefined direction in this sector. In other angular sectors, the direction defined by the angle $\theta_{\max}$ and the direction of the RP maximum may differ because the radiation of a single spherical slot antenna is ineffective in these angular sectors (dashed curve 1 in Fig. 3).

5. CONCLUSION

The new method of impedance synthesis of the RP for vibrator structures [4, 5] is generalized for the case of 2D flat antenna arrays of slotted spherical radiators. The problem of impedance synthesis for antenna array with double periodicity is solved analytically. Formulas for direct calculation of the impedances of each array radiator which allow to steer the maximum of the antenna RP in a predefine direction are presented. The formulas can be used as basis for development of algorithms intended to control the array radiation, for example, for spatial scanning of the RP. In addition, we may state that this problem cannot be solved by using commercial program packages. The approach was verified by simulation of the five-element linear array. The possibility of RP scanning in a wide range was shown by using the synthesized distributions of complex impedances. The results obtained by using this approach can also be applied to the simulation of adaptive antenna arrays, in which the dynamic parameters and characteristics are adjusted depending on influence of various external factors.

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