Progress In Electromagnetics Research Letters, Vol. 81, 21–28, 2019

Novel Directional Adaptive Relaxation Parameters for MUSIC-Like Algorithm

Narong Borijindargoon* and Boon Poh Ng

Abstract—An algorithm called MUSIC-like algorithm was originally proposed as an alternative method to the MUltiple SIgnal Classification (MUSIC) algorithm in order to circumvent requirement on subspace segregation. The relaxation parameter β, which was introduced into the formulation of the MUSIC-like algorithm, has enabled the algorithm to achieve high resolution performance comparable to the MUSIC algorithm without requiring explicit estimation of the signal and noise subspaces. An adaptive framework for the MUSIC-like algorithm was later developed under the α-stable distributed noise environment. In spite of great improvement on target's resolvability performance, a trade-off between such improvement and the estimation bias is inherent. In this letter, two novel directional adaptive β-selection methods for MUSIC-like algorithm under α-stable distributed noise are proposed. The proposed methods aim at reducing estimation bias and noise sensitivity which are inherent in prior adaptive β framework. Simulation results highlight noticeable reduction in the estimation bias as well as the noise sensitivity of the proposed methods without excessive compromise on target's resolvability performance compared with the original adaptive β framework.

1. INTRODUCTION

MUSIC algorithm [1] is a well-known eigenstructure-based method for spectral and direction-of-arrival (DOA) estimation with super resolution performance, which has been utilized in various applications from targets localization under different media [2–6] to sparse signal recovery under the multiple measurement vector (MMV) framework [7–9]. Despite its simplicity in terms of implementation, this subspace-based method requires either a priori knowledge on the number of targets or a model order estimated from the second-order moment of the obtained data in order to construct two orthogonal subspaces namely signal and noise subspaces. A pseudospectrum of MUSIC algorithm can be obtained as a function of a distance between each steering vector and the estimated noise subspace.

Signal model in array and statistical signal processing are often based on Gaussian assumption. In many practical scenarios, the signal and particularly the noise model may not admit Gaussian distribution. In this letter, we focus on impulse noise which can be modelled under α-stable distribution, a sub-class of stable distributions [10]. When an impulse noise is present, the model order estimators based on second order moment information such as the Akaike Information Criterion (AIC) and Minimum Description Length (MDL) [11, 12] are likely to produce erroneous estimations, and hence performance degradation of the MUSIC algorithm is expected [13].

Several techniques have been proposed in the literature to mitigate the above problem, all of which aim at extracting valuable information of the contaminated data matrix prior to the subspace decomposition process [14–16]. Despite the analysis based on fractional lower order moments framework, they can be regarded as a 1-step M-estimator which is a broader class of weighted covariance matrix [17]. Among various methods proposed for the impulse noise scenarios, the fractional lower order

Received 20 November 2018, Accepted 22 December 2018, Scheduled 7 January 2019
* Corresponding author: Narong Borijindargoon (narong001@e.ntu.edu.sg).
The authors are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore.

moment (FLOM) technique was reported to have comparable performance with the robust covariation technique [15]. To obtain the FLOM matrix, each element of an $M \times M$ FLOM matrix can be acquired by computing $C_{ik} = E[x_i(t)|x_k(t)|^{p-2}x_k^*(t)]$, where $1 < p < \alpha \leq 2$, $x_i(t)$ and $x_k(t)$ are the data samples obtained from the i^{th} and k^{th} sensors, and $E[\cdot]$ denotes the expectation operation. Note that the term $|x_k(t)|^{p-2}$ plays a key role in reducing the effect of the outlier produced by an impulse noise by mean of re-scaling each data point in $x_k^*(t)$. After obtaining the FLOM matrix, the MUSIC algorithm can be utilized where the signal and noise subspaces are estimated through the FLOM matrix. With less effect of the outlier within the FLOM matrix, an improved model order estimation can be achieved.

To avoid subspace estimation altogether, the MUSIC-like algorithm can be used as an alternative method. The algorithm was originally proposed as a means to circumvent model order estimation of the MUSIC algorithm, a step which is required when a priori knowledge on the number of targets is unavailable. A high resolution performance comparable to the MUSIC algorithm was shown to be attainable under a beamforming framework formulated in [18]. Detailed analysis of the algorithm was further carried out in [19] where the authors highlighted the role of a relaxation parameter β in regulating the weight vector solution. Aside from the theoretical investigation, additional experiments based on real data under controlled environment were also explored [4, 20]. Recently, an adaptive framework for β-selection method was proposed which provided additional improvement on target's resolvability performance over the fixed β [21]. However, it was noted that a trade-off between such improvement and estimation bias was inherent. It was suggested that the proposed adaptive framework could be beneficial in applications where target's resolvability is of the highest priority, and certain estimation bias is tolerable.

To further improve the performance of adaptive β, two novel adaptive β-selection methods are proposed in this letter. The rest of this letter is organized as follows. In Section 2, the signal model and beamforming formulation of the MUSIC-like algorithm are provided where the working principle of both fixed and adaptive β are briefly reviewed. Two novel adaptive β-selection methods are presented in Section 3, which include the formulation of both adaptive β and the principle idea behind them. In Section 4, simulation results comparing the performance of related algorithms are provided, and lastly, concluding remarks are drawn in Section 5.

2. SIGNAL MODEL AND LINKS BETWEEN RELATED ALGORITHMS

Consider an M-sensor linear array with half-wavelength spacing situated in the far field with K narrowband signal sources $\mathbf{s}(n) \in C^{K\times 1}$ which impinge along the direction $\Theta = [\theta_1, \ldots, \theta_K]^T$. The sensor's snapshot $\mathbf{x}(n)$ can be modelled as

$$\mathbf{x}(n) = \mathbf{A}(\Theta)\mathbf{s}(n) + \mathbf{v}(n), \tag{1}$$

where $\mathbf{A}(\Theta) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ is an array manifold matrix which comprises K steering vectors corresponding to each source direction; $\mathbf{a}(\theta)$ is a steering vector which is a function of direction θ; and $\mathbf{v}(n) \in \mathbb{C}^{M\times 1}$ denotes additive uncorrelated noise vector with zero mean. Under the assumption that the signal sources are uncorrelated, and infinite number of snapshots can be obtained, the covariance matrix $\mathbf{R} = E[\mathbf{X}\mathbf{X}^H]$ can be decomposed into two orthogonal subspaces, namely signal and noise subspaces by eigendecomposition. Given a matrix $\mathbf{U}_n$ which comprises eigenvectors that span the noise subspace, the spatial spectrum of MUSIC algorithm can be obtained by exploiting the orthogonality property between the signal and noise subspaces defined as $P_M(\theta) = 10\log_{10}\frac{1}{|\mathbf{a}(\theta)^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta)|}$.

It was shown in [22] that the MUSIC algorithm can be formulated under the beamforming framework. With this notion, the MUSIC-like algorithm was later proposed in [18] as an optimization problem for each look direction defined as

$$\begin{array}{ll} \min_{\mathbf{w}} & \mathbf{w}^H\mathbf{R}\mathbf{w} \\ s.t. & \mathbf{w}^H\mathbf{a}(\theta)\mathbf{a}(\theta)^H\mathbf{w} + \beta||\mathbf{w}||_2^2 = c, \end{array} \tag{2}$$

where $\mathbf{w}$ is the weight vector solution of the optimization problem in Eq. (2); a scalar value β is a control parameter which introduces certain relaxation into the constraint; and c is any constant value. It is shown that the weight vector solution $\mathbf{w}$ is an eigenvector corresponding to the minimum eigenvalue $\lambda_{\min}$ of the generalized eigenvalue problem $\mathbf{R}\mathbf{w} = \lambda(\mathbf{a}(\theta)\mathbf{a}(\theta)^H + \beta\mathbf{I})\mathbf{w}$, and the spatial spectrum of the

MUSIC-like algorithm can be obtained by $P_{Mlike}(\theta) = 10\log_{10}\frac{1}{|\mathbf{w}^H\mathbf{a}(\theta)|^2}$. The bound for β was proposed in [19] as

$$\underbrace{\max_{\theta\in\Theta}\frac{\lambda_{\mathbf{R},min}}{(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1}}}_{\beta_{\min}} < \beta < \underbrace{\min_{\theta\notin\Theta}\frac{\lambda_{\mathbf{R},min}}{(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1}}}_{\beta_{\max}}, \tag{3}$$

with the choice of β to be a value between $\beta_{\min}$ and $\beta_{\max}$ given as

$$\beta = (1-\xi)\beta_{\min} + \xi\beta_{\max}, \tag{4}$$

where $\xi \in (0,1)$ can be chosen by $\xi = \beta_{\min}/\beta_{\max}$. Detailed analysis regarding the upper bound $\beta_{\max}$ and lower bound $\beta_{\min}$ can be found in [19] Section 4 which can be summarized as follows. According to Eq. (26) in [19] Section IV, β value should be much larger than $\beta_{\min}$ along the source directions, and according to Eq. (29) in [19] Section IV, β value should be much smaller than $\beta_{\max}$ along the non-source directions. Note that the β value proposed in [19] is a fixed value throughout all look directions. The idea of β-selection method was further advanced in [21] under an adaptive framework where the value of β can be automatically readjusted corresponding to look directions (either source or non-source directions). The underlying notion was based on a distance parameter ξ_θ which approximates the distance between each steering vector and the anchor point $\mathbf{u}_M$, which is an eigenvector corresponding to the minimum eigenvalue $\lambda_{\mathbf{R},min}$ of the data covariance matrix $\mathbf{R}$. Each ξ_θ can be obtained by $\xi_\theta = 1 - |\mathbf{u}_M^H\mathbf{a}(\theta)|$, where $||\mathbf{a}(\theta)||_2^2 = 1$, and the value of ξ_θ is varied within the range of $\xi_\theta \in [0,1]$. The adaptive $\beta_P(\theta)$ can be obtained by substituting ξ_θ into Eq. (4), and the adaptive $\beta_P(\theta)$ can now be reexpressed for each look direction as

$$\beta_P(\theta) = \beta_{\max} - \delta\beta|\mathbf{u}_M^H\mathbf{a}(\theta)|, \tag{5}$$

where $\delta\beta = \beta_{\max} - \beta_{\min}$. The working principle of $\beta_P(\theta)$ can be briefly summarized as follows. The values of $\beta_P(\theta)$ along the source directions ($\theta \in \Theta$) are allowed to take up a relatively large values without exceeding the value of $\beta_{\max}$. On the other hand, the value of $\beta_P(\theta)$ along non-source directions ($\theta \notin \Theta$) should be kept low without extending lower than the value of $\beta_{\min}$. Detailed illustration on the performance improvement of $\beta_P(\theta)$ can be found in [21]. It was noted that in spite of great improvement on resolvability performance, the estimated direction of $\beta_P(\theta)$ is inherently biased. In the next section, two novel adaptive β-selection methods are proposed in order to reduce the estimation bias and noise sensitivity of $\beta_P(\theta)$.

3. NOVEL ADAPTIVE β SELECTION METHODS

3.1. The First Method: Adaptive $\beta_{\mathbf{C}}(\theta)$

With the result obtained from Eq. (5), it can be observed that the choice of $\beta_P(\theta)$ is associated with an inverse of Pisarenko's output [23], which is an inner product between an eigenvector $\mathbf{u}_M$ and the steering vector corresponding to each look direction $\mathbf{a}(\theta)$. Due to noise sensitivity characteristic of Pisarenko's method, we propose an alternative method with an objective to reduce the estimation bias of $\beta_P(\theta)$ while maintaining a satisfactory targets resolvability performance. A new adaptive $\beta_C(\theta)$ parameter based on a notion of re-scaled Capon's output is proposed, where $\beta_C(\theta)$ can be varied within the range of $[\beta_{\min}, \beta_{\max}]$. The new adaptive $\beta_C(\theta)$ is defined as

$$\beta_C(\theta) = \frac{(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1} - \min_{\theta\notin\Theta}(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1}}{\max_{\theta\in\Theta}(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1} - \min_{\theta\notin\Theta}(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1}}\delta\beta + \beta_{\min}, \tag{6}$$

The notion behind Eq. (6) can be summarized as follows. To obtain a re-scaled Capon's output within the range of $[\beta_{\min}, \beta_{\max}]$, the Capon's output is first obtained by computing $(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1}$ for each look direction [24]. Next, the minimum noise power $(\min_{\theta\notin\Theta}(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1})$ and maximum targets power $(\max_{\theta\in\Theta}(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1})$ are determined to obtain the range of spatial spectrum. $\beta_C(\theta)$ can now be obtained by the procedure denoted in Eq. (6) which comprises 4 steps as follows. Firstly, the Capon's output for each look direction is shifted down by the amount reflected in the minimum power of the noise floor. Secondly, it is normalized by the range of the spatial spectrum. Thirdly, it is re-scaled

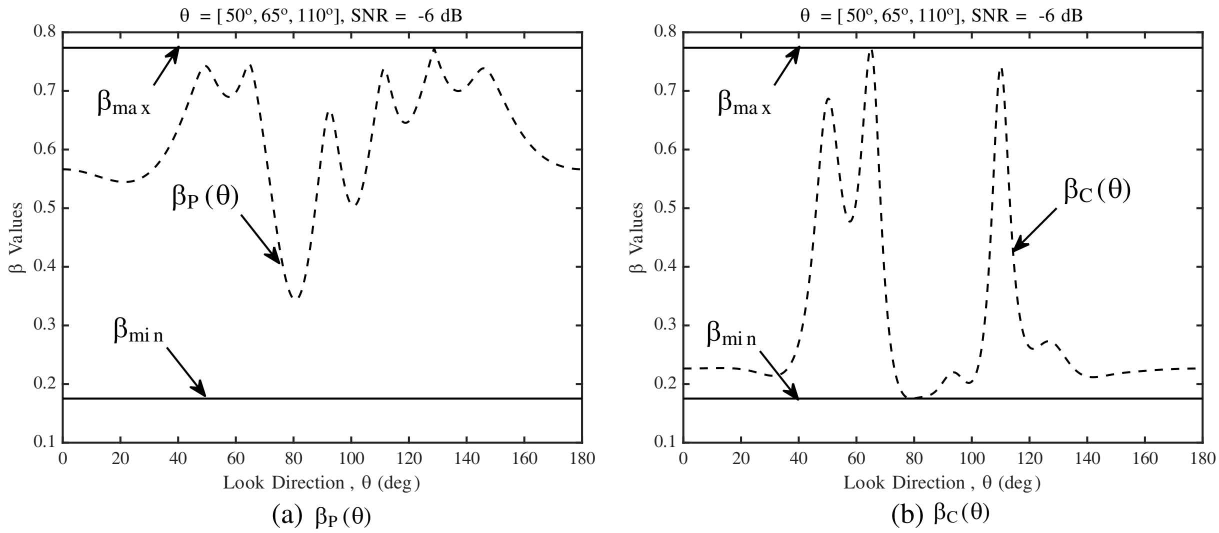

Figure 1. Variation of (a) $\beta_P(\theta)$ and (b) $\beta_C(\theta)$ correspond to each look direction.

up to the range of $[\beta_{\min}, \beta_{\max}]$ by $\delta\beta$. Lastly, it is shifted up so that the $\beta_C(\theta) \geq \beta_{\min}$. Consider a uniform linear array (ULA) of $M = 10$ sensors with 3 targets situated at $\Theta = [50°, 65°, 110°]$ where 100 data snapshots are obtained and SNR $= -6$ dB. The variation of $\beta_P(\theta)$ and $\beta_C(\theta)$ can be seen in Figs. 1(a) and (b), respectively. In Fig. 1(a), the variation of $\beta_P(\theta)$ along each look direction reflects noise sensitivity characteristic of this method. On the other hand, as can be seen in Fig. 1(b), the larger values of $\beta_C(\theta)$ are allowed along source directions while they are kept low along the non-source directions. As will be shown in the next section, this enhancement of $\beta_C(\theta)$ along the source directions together with a suppression along the non-source directions is able to lower the estimation bias without compromising the resolvability performance of the algorithm.

3.2. The Second Method: Adaptive $\boldsymbol{\beta_{\mathrm{LC}}(\theta)}$

With an objective to further optimize the values of $\beta_C(\theta)$ along source and non-source directions, it is observed that the values of $\beta_C(\theta)$ along non-source directions as shown in Fig. 1(b) can be further reduced. We now propose a new adaptive $\beta_{LC}(\theta)$ where the working principle is as follows. Along source directions, $\beta_{LC}(\theta)$ is allowed to take up maximum value ($\beta_{\max}$) while it should be kept minimum ($\beta_{\min}$) along non-source directions. This procedure can be accomplished through a generalized logistic function (GLF) which is defined as $Y(x) = A + (K - A)/(1 + \exp(-B(x - M)))$, where $Y(x)$ is the GLF output; A and K are the lower and upper asymptotes; B is the growth rate; and M is the location with maximum growth rate. To illustrate the GLF output, a plot of an arbitrary GLF is shown in Fig. 2(a) where A and K are set as 1 and 3, respectively; B is set to 2; and M is set to 4. The new $\beta_{LC}(\theta)$ can be obtained by passing Capon's output to the GLF where lower and upper asymptotes to be set as $\beta_{\min}$ and $\beta_{\max}$, respectively. By doing so, the value of $\beta_{LC}(\theta)$ will be limited within the range of $[\beta_{\min}, \beta_{\max}]$. The growth rate is now defined by $\beta_{\max}/\beta_{\min}$ which corresponds to the signal-to-noise ratio (SNR) of the obtained data. When SNR is high, the obtained data are with high fidelity, and hence the growth rate ($\beta_{\max}/\beta_{\min} = 1/\xi$) is automatically set to a high value, which is equivalent to a sharp slope in Fig. 2(a). This can be regarded as a sharp cutoff when considering if a particular direction has a high probability of being source direction. In contrast, when SNR is low, the obtained data are with low fidelity, and hence the growth rate will be automatically set to a lower value. The location for maximum growth rate works in a similar fashion as a location of the cutoff point, where any value that is greater than the cutoff point will be pushed toward the upper asymptote, and any value that is less than the cutoff point will be pushed toward the lower asymptote. The cutoff point is specified by $\chi \max_{\theta \in \Theta} (\mathbf{a}(\theta)^H \mathbf{R}^{-1} \mathbf{a}(\theta))^{-1}$, where $\chi = 0.8$ is used. Although it is not identical, this can be regarded in a similar fashion as a way to specify the angles of half-power-beamwidth in a spectral plot. The new adaptive $\beta_{LC}(\theta)$ can now be defined as

$$\beta_{LC}(\theta) = \beta_{\min} + \delta\beta/(1+\exp(-\frac{(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1} - \chi\max_{\theta\in\Theta}(\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta))^{-1}}{\xi})). \tag{7}$$

As can be seen in Fig. 2(b), $\beta_{LC}(\theta)$ along source directions were assigned to a relatively large value while it is kept to the minimum value along non-source directions. The discriminating power of $\beta_{LC}(\theta)$ can be observed by comparing Fig. 2(b) with Fig. 1(b).

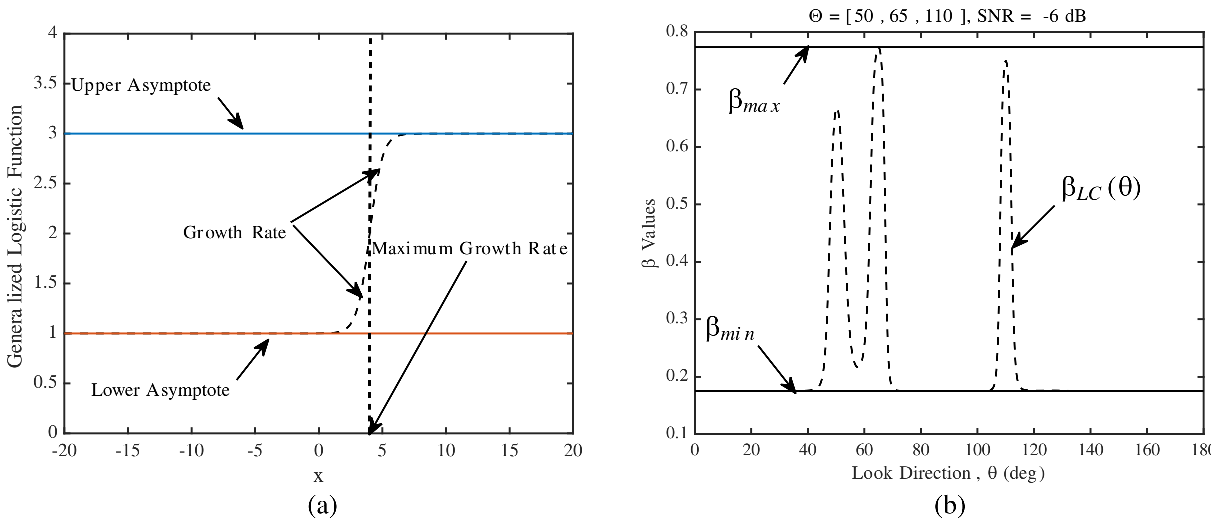

Figure 2. (a) Generalized logistic function and (b) Variation of $\beta_{LC}(\theta)$ corresponds to each look direction.

4. SIMULATION RESULTS

In this section, performances of the MUSIC-like algorithm based on the proposed $\beta_C(\theta)$ and $\beta_{LC}(\theta)$ are compared with the MUSIC, FLOM-MUSIC, and the MUSIC-like algorithm based on fixed β and $\beta_P(\theta)$. For signal source simulation, we follow the same configuration as presented in the previous section (ULA of $M = 10$ with 100 snapshots of data). The generalized signal-to-noise ratio (GSNR) will be used when $\alpha < 2$, which is defined as $\mathrm{GSNR(dB)} = 10\log_{10}(E[|s(t)|^2]/\gamma^\alpha)$. The impulse noise under the consideration is complex isotropic symmetric α-stable ($S\alpha S$) distributed. The α parameter specifies the likelihood of outlier occurrence as $\alpha < 2$ gets smaller. To obtain different GSNR levels, the γ parameter, which can be associated with the standard deviation under Gaussian distribution, is varied. To obtain the spatial spectrum of FLOM-MUSIC algorithm (when $\alpha < 2$), each element of an $M \times M$ FLOM matrix is first estimated prior to the subspace decomposition. This can be accomplished through the weighted covariance matrix $C_{ik} = E[x_i(t)|x_k(t)|^{p-2}x_k^*(t)]$, where $p = 1.1$ is set.

The spatial spectra of related algorithms are shown in Fig. 3 where two targets are situated at $\Theta = [70^\circ, 85^\circ]$ and $\mathrm{GSNR} = 0\,\mathrm{dB}$. As can be seen, the MUSIC algorithm fails under an impulse noise condition ($\alpha = 1.7$). However, the FLOM-MUSIC and the MUSIC-like with either fixed or adaptive β are able to identify all targets in non-Gaussian scenarios. Among MUSIC-like algorithms, note that $\beta_{LC}(\theta)$ has the lowest noise floor followed by $\beta_C(\theta)$, $\beta_P(\theta)$, and β.

A comparison between spatial spectra produced by the MUSIC-like algorithm with $\beta_C(\theta)$ and $\beta_{LC}(\theta)$ is shown in Fig. 4. As can be seen in Figs. 4(a) and 4(b), the spectra plots obtained from $\beta_{LC}(\theta)$ are less sensitive along non-source directions compared with the spectra plots obtained from $\beta_C(\theta)$. Additional arrows are provided in both figures to highlight the sharp response characteristic of $\beta_{LC}(\theta)$ over $\beta_C(\theta)$. Despite sharp response characteristic of $\beta_{LC}(\theta)$, it should be noted that this is accomplished through an extra step (GLF) taken while computing $\beta_{LC}(\theta)$.

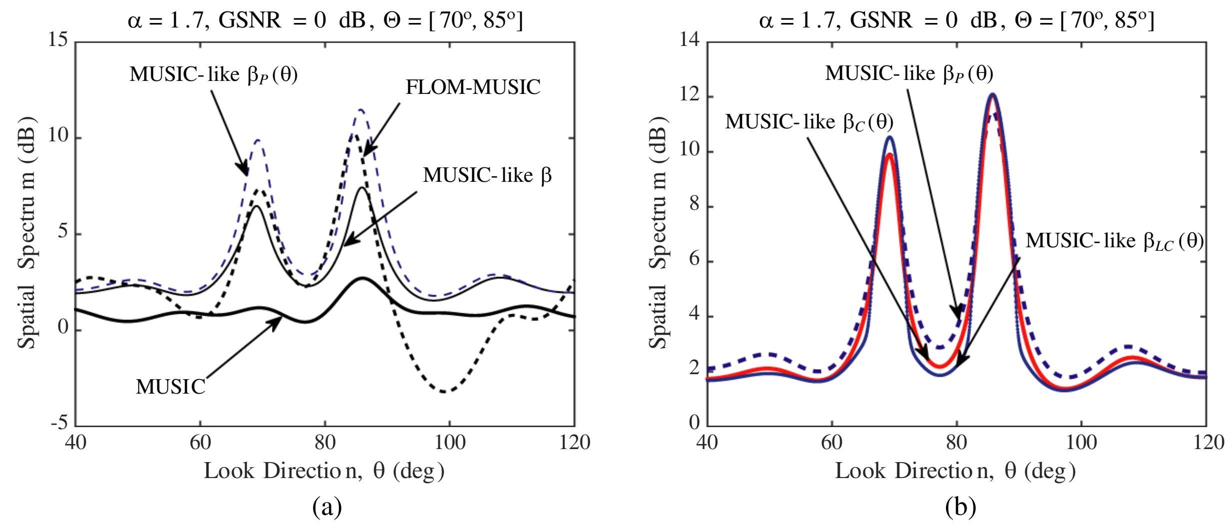

Figure 3. The spatial spectra of the related algorithms.

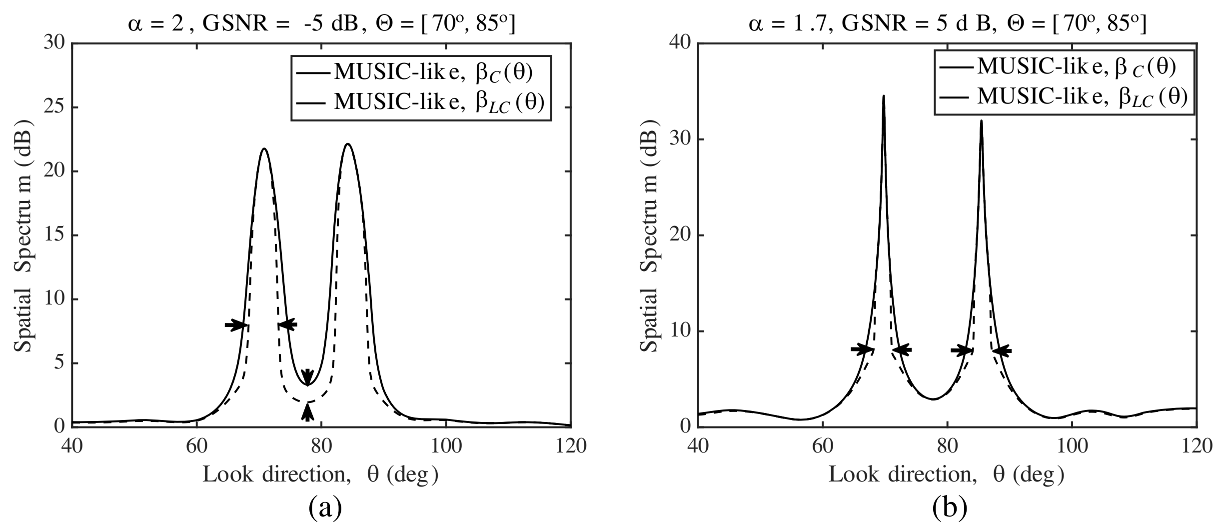

Figure 4. The spatial spectra of the MUSIC-like algorithm with $\beta_C(\theta)$ and $\beta_{LC}(\theta)$ where (a) $\alpha = 2$, GSNR $= -5$ dB and (b) $\alpha = 1.7$, GSNR $= 5$ dB.

Next, the performance parameters, probability of resolution and average root-mean-square error (RMSE) of related algorithms under Gaussian and non-Gaussian noise scenarios ($\alpha = 2$ and 1.7) at different levels of GSNR based on Monte Carlo simulation of 1000 trials with three targets situated at $\Theta = [50°, 65°, 110°]$ are shown in Figs. 5 and 6. Note that in the case of $\alpha = 2$, the complex isotropic $S\alpha S$ is reduced to Gaussian distribution, and hence FLOM-MUSIC will not be used for comparison. It can be seen in Figs. 5 and 6 that the proposed $\beta_C(\theta)$ is able to reduce estimation bias of $\beta_P(\theta)$ without compromising the targets' resolvability performance. Additional reduction on estimation bias can be obtained by $\beta_{LC}(\theta)$.

It can be seen in Fig. 6 that the estimation bias of $\beta_{LC}(\theta)$ is approaching the fixed β. However, the rate of targets resolvability of $\beta_{LC}(\theta)$ is slightly slower than the other methods in a certain range of GSNR levels as can be seen in Fig. 5. Note that both fixed and adaptive β-selection methods exhibit a robust characteristic which work in both Gaussian and non-Gaussian noises without requiring a preconditioned covariance matrix as the FLOM-MUSIC algorithm does.

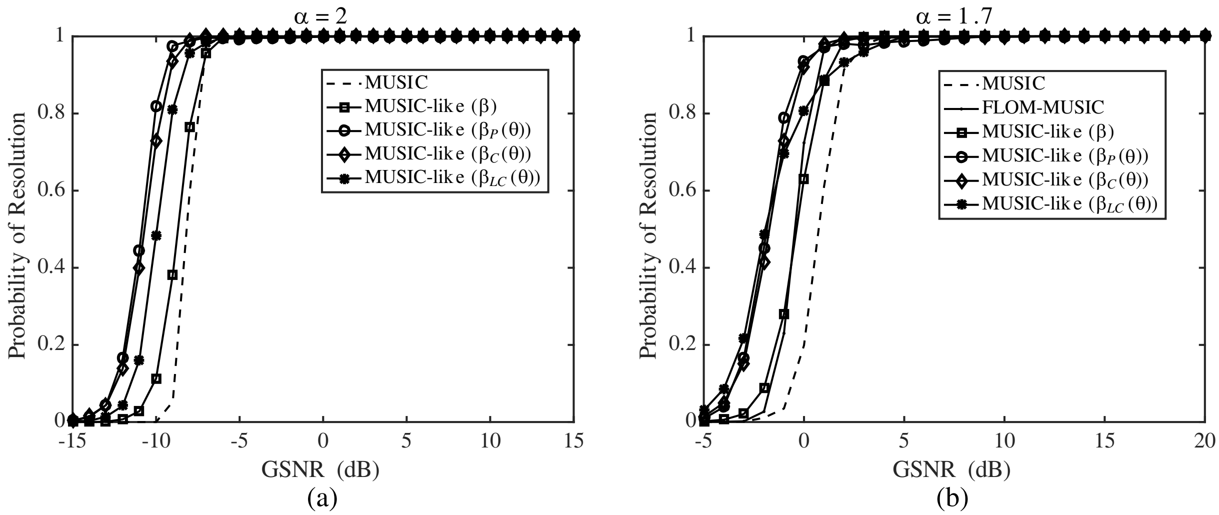

Figure 5. The probability of resolution of the related algorihtms where (a) $\alpha = 2$ and (b) $\alpha = 1.7$ were set.

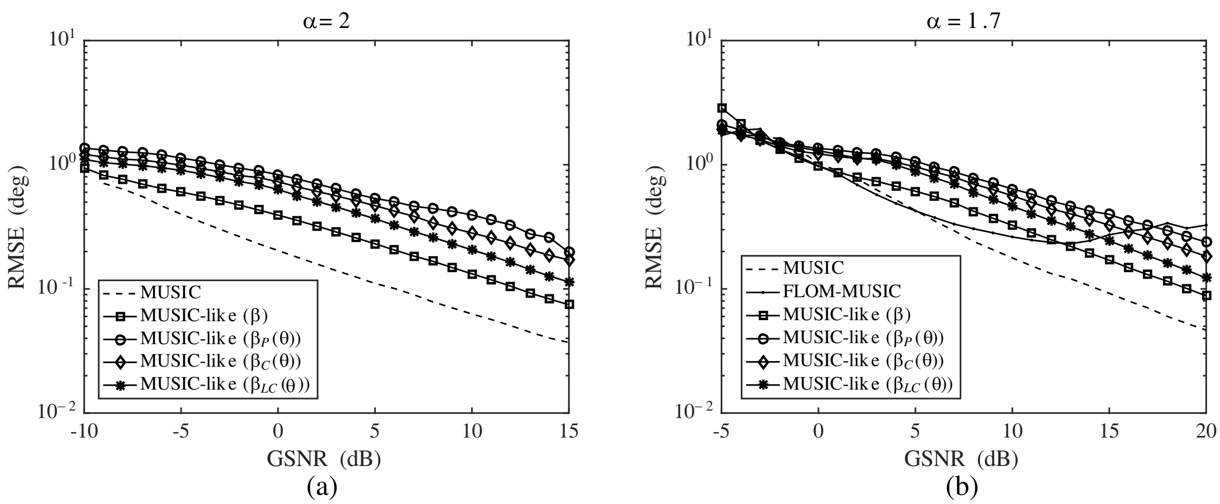

Figure 6. The RMSE of the related algorihtms where (a) $\alpha = 2$ and (b) $\alpha = 1.7$ were set.

5. CONCLUSION

In this letter, two novel adaptive β-selection methods ($\beta_C(\theta)$ and $\beta_{LC}(\theta)$) were proposed. Simulation results highlight a reduction in estimation bias and noise sensitivity characteristic, which is inherent in the original adaptive framework obtained form $\beta_P(\theta)$. In short, both $\beta_C(\theta)$ and $\beta_{LC}(\theta)$ are able to obtain lower estimation bias than $\beta_P(\theta)$. $\beta_C(\theta)$ is the best at balancing both performance parameters. Within the adaptive framework, $\beta_{LC}(\theta)$ has the lowest estimation bias with slight trade-off on the rate of targets resolvability performance.

REFERENCES

1. Schmidt, R. O., "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, Vol. 34, No. 3, 276–280, Mar. 1986.
2. Patole, S. M., M. Torlak, D. Wang, and M. Ali, "Automotive radars: A review of signal processing techniques," *IEEE Signal Process. Mag.*, Vol. 34, No. 2, 22–35, Mar. 2017.

3. Wan, L., X. Kong, and F. Xia, "Joint range-doppler-angle estimation for intelligent tracking of moving aerial targets," *IEEE Internet Things J.*, Vol. 5, No. 3, 1625–1636, Jun. 2018.
4. Borijindargoon, N., B. P. Ng, and S. Rahardja, "MUSIC-like algorithm for source localization in electrical impedance tomography," *IEEE Trans. Ind. Electron.*, 2018 (In Press).
5. Lee, O., J. Kim, Y. Bresler, and J. C. Ye, "Diffuse optical tomography using generalized MUSIC algorithm," *IEEE Int. Symp. Biomed. Imag.*, 1142–1145, Jun. 2011.
6. Scholz, B., "Towards virtual electrical breast biopsy: space-frequency MUSIC for trans-admittance data," *IEEE Trans. Med. Imag.*, Vol. 21, No. 6, 588–595, Jun. 2002.
7. Kim, J. M., O. K. Lee, and J. C. Ye, "Compressive MUSIC: Revisiting the link between compressive sensing and array signal processing," *IEEE Trans. Inf. Theory*, Vol. 58, No. 1, 278–301, Jan. 2012.
8. Davies, M. E. and Y. C. Eldar, "Rank awareness in joint sparse recovery," *IEEE Trans. Inf. Theory*, Vol. 58, No. 2, 1135–1146, Feb. 2012.
9. Lee, K., Y. Bresler, and M. Junge, "Subspace methods for joint sparse recovery," *IEEE Trans. Inf. Theory*, Vol. 58, No. 6, 3613–3641, Jun. 2012.
10. Shao, M. and C. L. Nikias, *Signal Processing With Alpha-Stable Distributions and Applications*, Wiley-Interscience, New York, 1995.
11. Akaike, H., "A new look at the statistical model identification," *IEEE Trans. Autom. Control*, Vol. 19, No. 6, 716–723, Dec. 1974.
12. Rissanen, J., "Modeling by shortest data description," *Automatica*, Vol. 14, No. 5, 465–471, Sept. 1978.
13. DjuriC, P. M., "A model selection rule for sinusoids in white Gaussian noise," *IEEE Trans. Signal Process.*, Vol. 44, No. 7, 1744–1751, Jul. 1996.
14. Liu, T. H. and J. M. Mendel, "A subspace-based direction finding algorithm using fractional lower order statistics," *IEEE Trans. Signal Process.*, Vol. 49, No. 8, 1605–1613, Aug. 2001.
15. Tsakalides, P. and C. L. Nikias, "The robust covariation-based MUSIC (ROC-MUSIC) algorithm for bearing estimation in impulsive noise environments," *IEEE Trans. Signal Process.*, Vol. 44, No. 7, 1623–1633, Jul. 1996.
16. Visuri, S., H. Oja, and V. Koivunen, "Subspace-based direction-of-arrival estimation using nonparametric statistics," *IEEE Trans. Signal Process.*, Vol. 49, No. 9, 2060–2073, Sept. 2001.
17. Adali, T. and S. Haykin, *Adaptive Signal Processing: Next Generation Solutions*, Vol. 55, John Wiley & Sons, 2010.
18. Zhang, Y. and B. P. Ng, "MUSIC-like DOA estimation without estimating the number of sources," *IEEE Trans. Signal Process.*, Vol. 58, No. 3, 1668–1676, Mar. 2010.
19. Reddy, V. V., B. P. Ng, and A. W. Khong, "Insights into MUSIC-like algorithm," *IEEE Trans. Signal Process.*, Vol. 61, No. 10, 2551–2556, May 2013.
20. Lim, H. S., B. P. Ng, and V. V. Reddy, "Generalized MUSIC-like array processing for underwater environments," *IEEE J. Ocean. Eng.*, Vol. 42, No. 1, 124–134, Jan. 2017.
21. Borijindargoon, N. and B. P. Ng, "Directional adaptive MUSIC-like algorithm under α-stable distributed noise," arXiv:1811.07110v1 [eess.SP], Nov. 2018.
22. Ng, B. P., M. H. Er, and C. Kot, "A MUSIC approach for estimation of directions of arrival of multiple narrowband and broadband sources," *Signal Process.*, Vol. 40, No. 2–3, 319–323, Nov. 1994.
23. Pisarenko, V. F., "The retrieval of harmonics from a covariance function," *Geophysical Journal International*, Vol. 33, No. 3, 347–366, 1973.
24. Capon, J., "High-resolution frequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, Vol. 57, No. 8, 1408–1418, Aug. 1969.