Fast Calculation of Monostatic Radar Cross Section of Conducting Targets Using Hierarchical Characteristic Basis Function Method and Singular Value Decomposition

Can Xia[1, 2], Wanqing You[2], and Yufa Sun[2, *]

Abstract—A novel hierarchical characteristic basis function method (HCBFM) is proposed to calculate monostatic radar cross section based on singular value decomposition characteristic basis function method. In order to reduce the number of incident plane waves and accelerate the generation of characteristic basis functions (CBFs), an improved CBFs construction method is studied in this paper. Firstly, the target is partitioned with hierarchical approach, and at each incident plane wave, the high-level CBFs defined in large blocks are expressed as a linear combination of the previously generated low-level CBFs defined in the corresponding small blocks. Finally, the high-level CBFs in large blocks are orthogonalized by using singular value decomposition at multiple excitations, and a set of linearly independent CBFs can be obtained. Numerical results are given to demonstrate the accuracy and high efficiency of the proposed method.

1. INTRODUCTION

The method of moments (MoM) [1] is preferred in the solution of electromagnetic scattering problems, which converts integral equations into a dense linear matrix system. However, it places a considerable burden on the computational complexity and memory requirement when the electrically large target is analyzed. Recently, some iterative solvers, such as the multilevel fast multipole method (MLFMM) [2] based on addition theorem for spherical harmonics, the adaptive integral method (AIM) [3]: a fast iterative integral-equation solver through splitting the impedance matrix into near-field and far-field components, and the adaptive cross approximation (ACA) algorithm [4], which is a low-rank decomposition method, are developed to improve the calculation of the MoM. Unfortunately, these iterative methods suffer from convergence problems of ill-conditional matrices for electrically large targets under analysis. Especially, when the monostatic scattering problems are considered, the iterative process need be repeated for each excitation.

The characteristic basis function method (CBFM) [5, 6] is an iteration-free method, which can speed up the direct solution of the MoM matrix equations by dividing the target into multiple adjacent blocks. For each block, the CBFM constructs macro basis functions called characteristic basis functions (CBFs) which indicate the current distribution of each block. And the number of CBFs is smaller than that of the RWG basis functions proposed by Rao et al. [7] in 1982. In [8], a singular value decomposition based CBFM (SVD-CBFM) is proposed to solve multiple excitation electromagnetic scattering problems. An improved SVD-CBFM [9] is presented to further reduce the number of incident plane waves and CBFs. However, as the electrical size of target increases, the size of each block will become large with retaining a proper number of blocks. This will lead to the increase of unknowns of each block, and the construction of CBFs becomes very time consuming.

* Corresponding author: Yufa Sun (yfsun_ahu@sina.com).
[1] Maanshan Teacher's College, Maanshan 243041, China. [2] Key Lab. of Intelligent Computing & Signal Processing, Ministry of Education, Anhui University, Hefei 230601, China.

In order to accelerate generation of CBFs and calculation of the reduced matrix equation, hierarchical CBFM (HCBFM) is proposed in this paper. The target is firstly divided with hierarchical partitioning approach. For each incident plane wave, the CBFs defined in large blocks are expressed as a linear combination of the previously generated low-level primary characteristic basis functions (PCBFs) and secondary characteristic basis functions (SCBFs) defined in the relatively small blocks, which can avoid directly solving the CBFs in the large blocks. Finally, the high-level CBFs in large blocks are orthogonalized by using singular value decomposition (SVD) at multiple excitations, and a set of linearly independent CBFs can be obtained. Besides, the ACA algorithm is also employed to fill the impedance matrix of far field, which can further accelerate the matrix-vector multiplication procedure of generating SCBFs and constructing the reduced matrix.

2. FORMULATION

2.1. Conventional SVD-CBFM

The surface integral equation is usually transformed into the following matrix equation using MoM.

$$\mathbf{Z} \cdot \mathbf{J} = \mathbf{E} \tag{1}$$

where $\mathbf{Z}$ is an $N \times N$ impedance matrix; $\mathbf{J}$ and $\mathbf{E}$ are $N \times 1$ current vector and excitation vector, respectively; and N is the number of unknowns. The CBFM firstly divides the target into M adjacent blocks then constructs characteristic basis function on each block. Eq. (1) can be expressed as

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \ldots & \mathbf{Z}_{1M} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \ldots & \mathbf{Z}_{2M} \\ \vdots & \vdots & \ldots & \vdots \\ \mathbf{Z}_{M1} & \mathbf{Z}_{M2} & \ldots & \mathbf{Z}_{MM} \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ \vdots \\ \mathbf{J}_M \end{bmatrix} = \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \\ \vdots \\ \mathbf{E}_M \end{bmatrix} \tag{2}$$

where $\mathbf{Z}_{ij}$ $(i = 1, 2 \ldots, M; j = 1, 2, \ldots M)$ is an $N_i \times N_j$ impedance sub-matrix, and $\mathbf{J}_i$ and E_i are the surface current vector and excitation vector, respectively.

In order to obtain a set of completely orthogonal CBFs, the SVD-CBFM needs to set large number of incident plane wave excitations in the direction of elevation and azimuth (θ and φ). Let N_θ and N_φ indicate the number of samples in θ and φ, respectively. Considering two kinds of polarization, the total number of incident excitations is $N_p = 2N_\theta N_\varphi$, which are arranged in matrix $\mathrm{E}_{ii}^{N_p}$. The PCBFs for each block under multiple plane wave excitations can be obtained by solving the following equation.

$$\mathbf{Z}_{ii}^e \mathbf{J}_{ii}^P = \mathbf{E}_{ii}^{N_p} \quad (i = 1, 2, 3 \ldots M) \tag{3}$$

where $\mathbf{Z}_{ii}^e$ is an $N_i^e \times N_i^e$ extended self-impedance matrix, and $\mathbf{J}_{ii}^P$ and $\mathbf{E}_{ii}^{N_p}$ are $N_i^e \times N_p$ current matrix and excitation matrix, respectively.

Usually, the number of plane wave excitations used to generate the CBFs will exceed the number of degrees of freedom associated with the block, and it is desirable to remove the redundancy in the basic functions by applying SVD. This is done by expressing the CBFs as

$$\mathbf{J}_{ii}^P = \mathbf{U}\mathbf{W}\mathbf{V}^T \tag{4}$$

where $\mathbf{U}$ is an $N_i^e \times r_i$ orthogonal matrix, a complete set of CBFs of block i. $\mathbf{W}$ is an $r_i \times r_i$ diagonal matrix, and $\mathbf{V}$ is an $r_i \times N_p$ orthogonal matrix. Supposing that $\mathbf{J}_i^k$ is the kth column vector of $\mathbf{U}$, the current $\mathbf{J}_i$ of block i can be expressed as a linear combination of $\mathbf{J}_i^k$ as follows:

$$\mathbf{J}_i = \sum_{k=1}^{r_i} \alpha_i^k \mathbf{J}_i^k \ (i = 1, 2, 3 \ldots M) \tag{5}$$

where $k = 1, 2, 3 \ldots r_i$, α_i^k is an unknown coefficient to be determined. Multiplying both sides of Eq. (2) by the transpose of $\mathbf{J}_i$, the reduced coefficient matrix can be expressed as

$$\mathbf{Z}^R \cdot \alpha = \mathbf{E}^R \tag{6}$$

where $\mathbf{Z}^R$ is a $\sum_{i=1}^{M} r_i \times \sum_{i=1}^{M} r_i$ reduced matrix, and α and $\mathbf{E}^R$ are $\sum_{i=1}^{M} r_i \times 1$ coefficient vector and excitation vector, respectively. The coefficient vector can be solved after the LU decomposition [10] to the reduced matrix, then the total current distribution of the target can be obtained.

2.2. The Improved SVD-CBFM

In order to reduce the number of incident plane waves, accelerate the generation of CBFs and the filling speed of the reduced matrix, the construction of CBFs is improved in [9]. In this method, the coupling effect among the different blocks is taken into account through calculating the SCBFs of each block, which greatly reduces the number of incident plane wave excitations. Let H_θ and H_φ indicate the numbers of samples in θ and φ, respectively. Considering two kinds of polarization, the total number of incident plane wave excitations is $H_p = 2H_\theta H_\varphi (H_p \ll N_p)$. The PCBFs for each block under each new plane wave excitation can be obtained by solving the following equation:

$$\mathbf{Z}_{ii}^e \mathbf{J}_i^P = \mathbf{E}_{ii} \, (i = 1, 2 \ldots M) \tag{7}$$

where $\mathbf{Z}_{ii}^e$ is an $N_i^e \times N_i^e$ extended self-impedance matrix, and $\mathbf{J}_i^P$ and $\mathbf{E}_{ii}$ are $N_i^e \times 1$ current vector and excitation vector, respectively. According to the Foldy-Lax multiple scattering equation, the first-order SCBFs of block i are calculated by scattered fields due to the PCBFs on all blocks except from itself. Similarly, we can calculate the additional second-order SCBFs. These SCBFs can be written as follows [11]:

$$\mathbf{Z}_{ii}^e \mathbf{J}_i^{S1} = -\sum_{j=1(j \neq i)}^{M} \mathbf{Z}_{ij} \mathbf{J}_j^P \, (i = 1, 2 \ldots M) \tag{8}$$

$$\mathbf{Z}_{ii}^e \mathbf{J}_i^{S2} = -\sum_{j=1(j \neq i)}^{M} \mathbf{Z}_{ij} \mathbf{J}_j^{S1} \, (i = 1, 2 \ldots M) \tag{9}$$

Therefore, the real CBFs of block i can be obtained by solving Eqs. (7), (8), (9) and removing the extended part. Then a set of perfectly orthogonal CBFs for each block can be obtained by using the SVD procedure under H_p incident wave excitations.

2.3. The Hierarchical CBFM

It is well known that the target needs to be first divided into multiple blocks when the CBFM is applied, but the size of the blocks has a great impact on computational time and storage consumption. In order to achieve the minimum calculation time, the numbers of blocks and unknowns N should be satisfied with $M \approx 0.9N^{1/3}$ according to [12]. However, when the electrically large target is analyzed, it is usually divided into large blocks for improving the efficiency of calculation. The larger the size of blocks is, the smaller the reduction matrix which can be easily solved [13] is. Inevitably large subdomain contains more unknowns, and the procedure of generating CBFs will become very time consuming. In order to mitigate this problem, a novel HCBFM is proposed in this paper. In HCBFM, the electrically large target is firstly divided with hierarchical partitioning approach, and at each incident plane wave, the high-level CBFs defined in large block are expressed as a linear combination of the previously generated low-level CBFs defined in the corresponding small block, which can avoid directly solving the CBFs in the large block. Suppose that the target is firstly divided into M blocks, then each block is subdivided into N subdomains, and these subdomains can be further divided until the lowest level subdomains contain a few of RWG basis functions. For instance, as shown in Fig. 1, the target is firstly divided into four ($M = 4$) blocks called the second-level. Each block is subdivided into nine ($N = 9$) subdomains, called the first-level. The solid dot denoted as $1\{9\}$ in Fig. 1 stands for the ninth subdomain of the first block.

It is assumed that the target is also illuminated by H_p incident wave excitations. Supposing that the second-level block i contains N subdomains, for each specific plane wave excitation, the PCBFs of

Figure 1. Hierarchical analysis of a target.

first-level extended small subdomain $i\{k\}$ ($i = 1, 2 \ldots M$; $k = 1, 2 \ldots N$) are expressed as:

$$\mathbf{Z}_{i\{k\}i\{k\}}\mathbf{J}_{i\{k\}}^{P} = \mathbf{E}_{i\{k\}}^{r} \tag{10}$$

where $\mathbf{Z}_{i\{k\}i\{k\}}$ is the self-impedance of the extended subdomain k in the block i, and $\mathbf{E}_{i\{k\}}^{r}$ ($r = 1, 2 \ldots 2H_\theta H_\varphi$) is the rth incident plane wave excitation vector. Considering the mutual effects with other first-level small subdomains, the first-order SCBFs and the second-order SCBFs of first-level extended small subdomain k in the block i are generated as

$$\mathbf{Z}_{i\{k\}i\{k\}}\mathbf{J}_{i\{k\}}^{S1} = -\sum_{h=1(h\neq k)}^{N}\mathbf{Z}_{i\{k\}i\{h\}}\mathbf{J}_{i\{h\}}^{P} \tag{11}$$

$$\mathbf{Z}_{i\{k\}i\{k\}}\mathbf{J}_{i\{k\}}^{S2} = -\sum_{h=1(h\neq k)}^{N}\mathbf{Z}_{i\{k\}i\{h\}}\mathbf{J}_{i\{h\}}^{S1} \tag{12}$$

where $\mathbf{Z}_{i\{k\}i\{h\}}$ is a mutual impedance matrix between subdomain k and h ($h = 1, 2, 3 \ldots N$) in the block i. The linear combination of the PCBFs and SCBFs after removing the extended portion can indicate the characteristic basis function of subdomain k in the block i under the rth incident wave excitation

$$\mathbf{J}_{i\{k\}}^{r} = a_{i\{k\}}\mathbf{J}_{i\{k\}}^{P} + b_{i\{k\}}\mathbf{J}_{i\{k\}}^{S1} + c_{i\{k\}}\mathbf{J}_{i\{k\}}^{S2} \tag{13}$$

Then constructing a reduced matrix equation of N small subdomains according to Eq. (6), the current coefficients $a_{i\{k\}}$, $b_{i\{k\}}$ and $c_{i\{k\}}$ can be obtained by directly solving the reduced matrix equation. $a_{i\{k\}}$, $b_{i\{k\}}$, and $c_{i\{k\}}$ are substituted into Eq. (13), and the real current of the subdomain $i\{k\}$ can be obtained under the rth incident wave excitation. Therefore, superimposing the currents of N small subdomains, the current response of the second-level block i under the rth incident wave excitation can be achieved as:

$$\mathbf{J}_{i}^{r} = \sum_{k=1}^{N}\mathbf{J}_{i\{k\}}^{r} \tag{14}$$

Then $\mathbf{J}_i$ of the second-level block i under multiple incident excitations are compressed by using SVD, and a set of linearly independent CBFs can be obtained. Finally, constructing a reduced matrix equation of M blocks on the second level in the same way, we can get the surface current of the target by direct solver.

In the HCBFM, the matrix-vector multiplication procedures of generating SCBFs and constructing the reduced matrix are also time consuming. In order to further improve the efficiency of calculation, the ACA algorithm [14] is also used to speed up the calculation of each level impedance matrix of far field.

3. NUMERICAL RESULTS AND DISCUSSIONS

In this section, several numerical examples are given to demonstrate the accuracy and efficiency of the proposed method. All the results are computed on the Intel®Core®CPU E5-2630 V4@2.2 GHz,

256 GB RAM PC. The compiler used is Code::Blocks. The ACA and SVD threshold are set to 10^{-3}, and the second-order SCBFs are chosen in the paper. We use the relative error of the RCS to estimate the accuracy of the proposed method, and the relative error is defined as follows:

$$\text{Err}(\%) = 100 \times \sqrt{\sum_n \left| \text{RCS}^{pm} - \text{RCS}^{ref} \right|^2} \Big/ \sqrt{\sum_n \left| \text{RCS}^{ref} \right|^2} \tag{15}$$

where RCS^{pm} is the RCS provided by the proposed method, and RCS^{ref} is the RCS provided by the FEKO.

Firstly, monostatic RCS for a 252.3744-mm PEC almond is calculated at the incident frequency of 7 GHz, and the incident angle is set to $\theta_i = 0° - 180°$, $\varphi_i = 90°$. The almond is divided into 6236 triangular patches, and the number of unknowns is 15950, which is divided into 48 first-level blocks and 12 second-level blocks, and each block is extended 0.05λ on the first-level and 0.15λ on the second-level in all directions. The monostatic RCSs in VV polarization calculated by the improved SVD-CBFM and HCBFM are shown in Fig. 2. In the two methods, the number of incident plane waves is set to $N_\theta = N_\varphi = 8$. When the improved CBFM is applied, the number of generated CBFs is 384 on each block, which can obtain about 82 CBFs after SVD, and the dimension of the reduced matrix is 988×988. But when the HCBFM is applied, the number of generated CBFs is 128 on each second-level block, which can obtain about 75 CBFs after SVD, and the dimension of the reduced matrix is 900×900, which is decreased about 9% compared to the previous method. It can be seen from Fig. 2 that the RCSs calculated by the two methods agree well with calculation results of the FEKO.
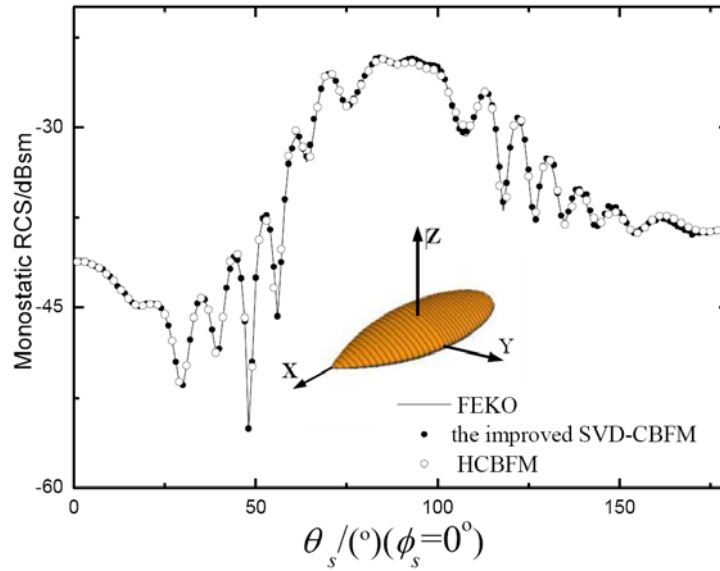
**Figure 2.** Monostatic RCS of the PEC almond in VV polarization.

Table 1 shows the computational time and relative error of the two methods. It can be found that the HCBFM outperforms the improved SVD-CBFM in computation time under the equivalent precision.

Finally, monostatic RCS for 256 discrete conducting cubs with 0.25 m side length at the incident frequency of 300 MHz is calculated. The incident angle is set to $\theta_i = 0° - 180°$, $\varphi_i = 90°$. The distance between two adjacent targets is 0.25 m. The geometry is divided into 22778 triangular patches, and the number of unknowns is 34176, which is divided into 64 first-level blocks and 16 second-level blocks, and each block is extended 0.05λ on the first-level and 0.15λ on the second-level in all directions. The monostatic RCSs in HH polarization calculated by the improved SVD-CBFM and HCBFM are shown in Fig. 3. In the two methods, the number of incident plane waves is set to $N_\theta = 10$, $N_\varphi = 8$. When the improved CBFM is applied, the number of generated CBFs is 480 on each block, which can obtain

Table 1. CPU time and relative error of the PEC almond.

Methods	CBFs generation	reduced matrix filling	total time	relative error (%)
Improved SVD-CBFM	1157s	2739s	3903s	0.18
HCBFM	557s	2023s	2585s	1.09

about 114 CBFs after SVD, and the dimension of the reduced matrix is 1824×1824. But when the HCBFM is applied, the number of generated CBFs is 160 on each second-level block, which can obtain about 106 CBFs after SVD, and the dimension of the reduced matrix is 1692×1692, which is decreased efficiently compared to the previous method. It can be seen from Fig. 3 that the RCSs calculated by the two methods agree well with calculation results of the FEKO. The computational time and relative error of the two methods are shown in Table 2.
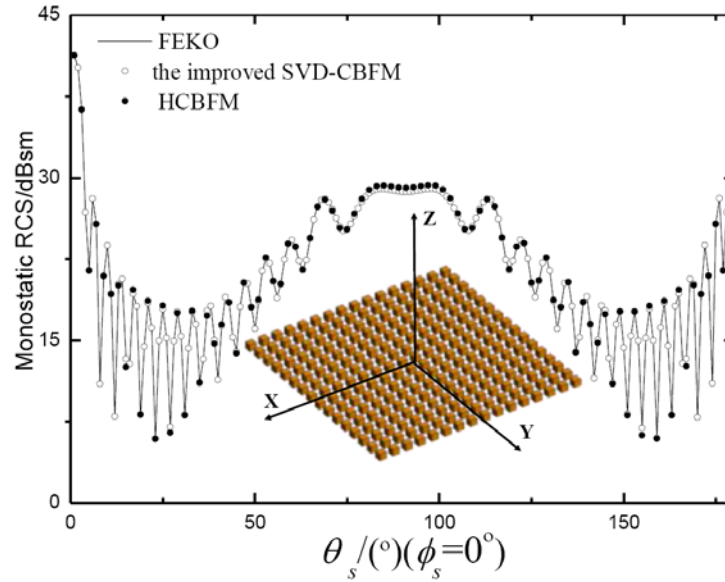
Figure 3. Monostatic RCS of 256 discrete conducting cubs in HH polarization.

Table 2. CPU time and relative error of 256 discrete conducting cubs.

Methods	CBFs generation	reduced matrix filling	total time	relative error (%)
Improved CBFM	4504s	36809s	41345s	0.55
HCBFM	1933s	13811s	15776s	1.06

4. CONCLUSION

In this paper, the HCBFM is proposed to solve electromagnetic scattering problem efficiently under multiple incident excitations. This proposed method can reduce the time of CBFs generation and reduced matrix filling significantly with a small number of incident plane wave excitations compared to the SVD-CBFM. Besides, the ACA algorithm is also used to accelerate the calculation of impedance matrix of far field efficiently. Numerical results demonstrate that the proposed method is accurate and efficient.

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