

Jamming Method Based on Optimal Power Difference for LMS-GPS Receivers

Fulai Liu^{1, 2}, Yadong Wang^{1, 2, *}, Ling Yue^{1, 2}, Xiaodong Kan^{1, 2}, and Hui Song^{1, 2}

Abstract—Jamming and anti-jamming techniques for global position systems (GPS) play important roles in electronic countermeasure. Least mean square (LMS)-based anti-jamming algorithm is widely used in GPS receivers, since it can avoid matrix inversion and has low complexity. For convenience, we call them LMS-GPS receivers. To improve the anti-jamming performance of the LMS-GPS receivers, it is very meaningful to study the jamming technique. Considering that existing jamming signals are easily suppressed by LMS-GPS receivers, a new jamming method named as optimal power difference jamming is proposed in this paper to improve the jamming effect further. Specifically, the analytical relationship between jamming-to-signal ratio (JSR) and the power difference of two interference signals is firstly given. Then, the conclusion that there is always an optimal power difference where the JSR can take the extreme value is drawn. Finally, the optimal power difference is derived as about 22 dB for single-tone interference and 29 dB for band-limited Gaussian noise interference. Simulation results show that the proposed method with optimal power difference is able to improve the JSR remarkably.

1. INTRODUCTION

Recently, the global position system (GPS) has become an indispensable part of daily life, which provides the position, velocity and timing information to enable many applications used in our daily life. Since the satellites are over 20,000 km away and are powered by solar cells, GPS signals have very low power to reach earth, thus, it is particularly vulnerable to intentional or unintentional interference [1]. Least mean square (LMS) algorithm is one of the most widely used anti-jamming algorithm because of its low complexity and better convergence performance. For convenience, we refer to a GPS receiver that uses the LMS algorithm for anti-jamming as an LMS-GPS receiver. In order to improve the anti-jamming performance of the LMS-GPS receiver, it is necessary to study the impact of interference.

Several anti-jamming algorithms have been proposed for GPS [2–9]. The affine combination of two LMS filters has a better performance compared to a single LMS filter, however, its computation cost is not attractive [2]. The performance of LMS algorithm without jamming is analyzed for different SNR environment in [3]. Variable step method is used to reach the fast convergence and a low steady state error in [4], which does not consider the interference problem. An approximate expression is presented in [5], which shows that the smaller LMS step size can decrease the mis-adjustment. Unfortunately, it may also cause a longer convergence time. A numerical analysis of the effect of carrier frequency on the signal-tone interference performance is given in [6]. A new variable step-size LMS algorithm is introduced in [7], which can provide fast convergence by adjusting the step size, however, the performance of this algorithm is highly depend on the criterion used to adjust the step size. The proportionate normalized LMS (PNLMS) adaptation algorithm is proposed in [8], which can improve the initial convergence by

Received 13 November 2018, Accepted 22 December 2018, Scheduled 8 January 2019

* Corresponding author: Yadong Wang (wangyadong_neu@163.com).

¹ Engineer Optimization & Smart Antenna Institute, Northeastern University at Qinhuangdao, China. ² School of Computer Science and Engineering, Northeastern University, Shenyang, China.

adjusting the step size of each filter coefficient. The anti-jamming LMS algorithms mentioned above ignore the influences of the interference signal power on the performance of these algorithms.

Meanwhile, most previous interference methods do not attach enough importance to the LMS-GPS receivers. They concentrate on producing code-tracking error, influencing acquisition performance, etc. For example, the influence of continuous wave (CW) interference on the acquisition performance of GPS receivers is analysed in [9]. Wideband interference has the best jamming effect on signal acquisition and tracking of the GPS receiver, which can cover the received signals and influence the performance of the GPS receiver [10]. In [11], the influence of CW jamming for the GPS receiver is analyzed, and the influence of Doppler frequency and integral duration is considered. The effect of band-limited Gaussian noise interference with different bandwidths, the influence of integration time and early-late spacing is assessed in [12].

In this paper, we propose a novel jamming method for LMS-GPS receivers. Firstly, the characteristics of the LMS anti-jamming algorithm are analyzed. And then, in view of the characteristics that the LMS algorithm will suppress higher power signal preferentially, the power difference between the two suppressed interference signals is adjusted to maximize output jamming-to-signal ratio (JSR) of the LMS-GPS receivers, which can ensure that the two interference signals have higher power in the received data of the LMS-GPS receivers. The proposed jamming method can obtain larger output JSR than one interference signal or two interference signals with the same power.

The rest of the paper is organized as follows. In Section 2, the system model of an LMS-GPS receiver is presented. The proposed jamming method is developed in detail and followed by the problem formulation in Section 3. Simulations have been performed to analyze the impact of different parameters on the power difference between the two suppressed interference signals in Section 4. Section 5 provides a concluding remark to summarize the paper.

2. SYSTEM MODEL

Consider a GPS receiver using the LMS anti-jamming algorithm as shown in Fig. 1, in which $\mathbf{x} = [x_1(t), x_2(t), \dots, x_M(t)]^T$ denotes the received signal of array antenna; $\mathbf{w} = [w_1, w_2, \dots, w_M]^T$ represents the array weight vector; $y(t)$ is the output signal of the received signal processed by the LMS algorithm, which is multiplied by the local carrier and the local C/A code to demodulate and disperse, and the data code of the GPS signal can be recovered by the integrate and dump.

The LMS algorithm is an adaptive beam forming algorithm, which has been widely used in the GPS receivers [13]. The optimal weights are obtained by iterative operations according to the minimum mean square error (MMSE). In general, the progress of the adaptive algorithm can be summarized as

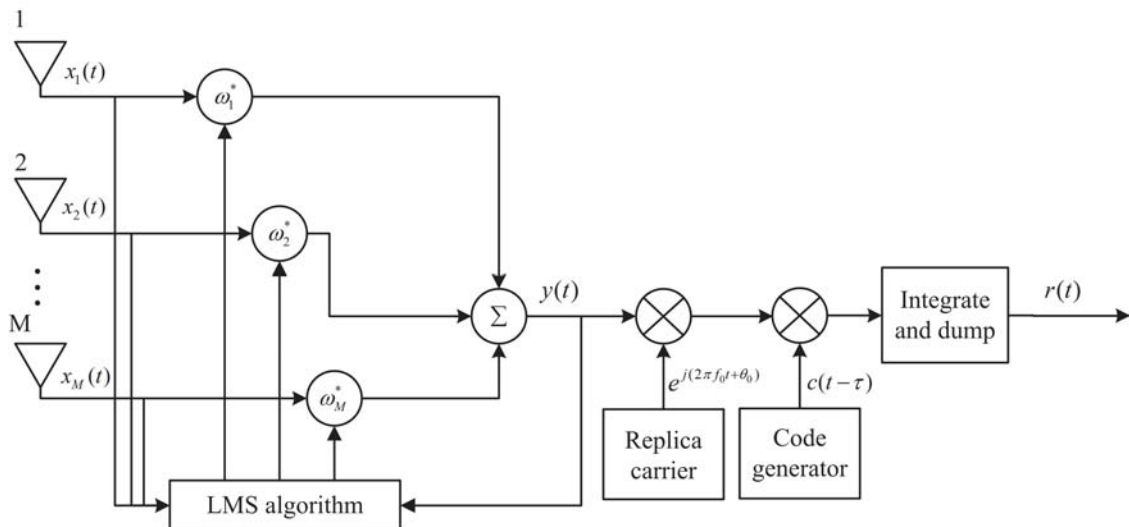


Figure 1. The model of an LMS-GPS receiver.

follows: 1) Calculating the output of the array based on the received signal and the current weight vector. 2) Obtain the error between the output signal and the desired signal. 3) Adjusting the weight vector based on a certain rule. The above process is a continuous iterative process until meeting the requirements and reaching the steady state. The LMS algorithm can be described by the following equations

$$\varepsilon(n) = d(n) - \mathbf{w}^H(n)\mathbf{x}(n) \quad (1)$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu\mathbf{x}(n)\varepsilon(n) \quad (2)$$

where $d(n)$ is the desired signal; $\mathbf{x}(n)$ denotes the input signal vector at sampling time n ; $\varepsilon(n)$ stands for the deviation error; $\mathbf{w}(n)$ represents the filter coefficients vector; and μ is equal to variable step [14]. μ plays an important role in the performance of the proposed algorithm, which can influence the steady state error and convergence rate of LMS.

3. ALGORITHM FORMULATION

The received signal vector of array $\mathbf{x}(t)$ consists of one GPS signal, two interference signals and one noise signal, and it can be represented as

$$\mathbf{x}(t) = \mathbf{a}_0(\theta_0)s_0(t) + \mathbf{a}_1(\theta_1)j_1(t) + \mathbf{a}_2(\theta_2)j_2(t) + \mathbf{v}(t) \quad (3)$$

where $\mathbf{a}_0(\theta_0)$, $\mathbf{a}_1(\theta_1)$ and $\mathbf{a}_2(\theta_2)$ denote the steering vectors of the GPS signal $s_0(t)$, two interference signals $j_1(t)$ and $j_2(t)$, respectively. $\mathbf{v}(t)$ stands for the array noise vector. $j_1(t)$ and $j_2(t)$ have the following forms

$$j_1(t) = \sqrt{P_1}s_1(t) \quad (4)$$

$$j_2(t) = \sqrt{P_2}s_2(t) \quad (5)$$

where P_1 and P_2 denote the power of $j_1(t)$ and $j_2(t)$, respectively; $s_1(t)$ and $s_2(t)$ are power normalized interference signal; $j_1(t)$ is uncorrelated with $j_2(t)$. Introducing the notation $\Delta P = P_1 - P_2$, $j_2(t)$ can be rewritten as $j_2(t) = \sqrt{P_1 - \Delta P}s_2(t)$.

Considering the relationship between the received signal and ΔP , $\mathbf{x}(t)$ can be given by

$$\mathbf{x}(t) = \mathbf{a}_2\sqrt{(P_1 - \Delta P)}s_2(t) + \mathbf{c} \quad (6)$$

where $\mathbf{c} = \mathbf{a}_0s_0(t) + \mathbf{a}_1\sqrt{P_1}s_1(t) + \mathbf{v}(t)$.

Above all, we give the following Theorem.

Theorem 1: For an LMS-GPS receiver, assume that the received signal includes a desired signal with a steering vector \mathbf{a}_0 and two interference signals with a power difference ΔP . If the receiver's weight vector \mathbf{w} satisfies $|\mathbf{w}^H\mathbf{a}_0| > 0$, then, there must be an interference signal power difference ΔP_{opt} such that the following equation holds

$$\left. \frac{\partial O_{JSR}}{\partial \Delta P} \right|_{\Delta P = \Delta P_{\text{opt}}} = 0 \quad (7)$$

where O_{JSR} stands for the JSR of the received signal.

Proof: The desired signal's output power P_{0out} can be expressed as

$$\begin{aligned} P_{0out} &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |s_{dout}(t)|^2 dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |\mathbf{w}^H(n)\mathbf{a}_0s_0(t)|^2 dt \\ &= |\mathbf{w}^H(n)\mathbf{a}_0|^2 \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |s_0(t)|^2 dt \\ &= |\mathbf{w}^H(n)\mathbf{a}_0|^2 P_0 \end{aligned} \quad (8)$$

where $s_{dout}(t) = \mathbf{w}^H(n)\mathbf{a}_0s_0(t)$, $\mathbf{w}(n)$ denotes the weight vector given by the n th iteration of the LMS algorithm; \mathbf{a}_0 represents the desired signal steering vector; P_0 stands for the input power of the GPS signal.

Similarly, the output powers of two interference signals are

$$\begin{aligned} P_{1out} &= |\mathbf{w}^H(n)\mathbf{a}_1|^2 P_1 \\ P_{2out} &= |\mathbf{w}^H(n)\mathbf{a}_2|^2 P_2. \end{aligned} \quad (9)$$

Combining Eqs. (8) and (9), the output JSR of signal is

$$\begin{aligned} O_{JSR} &= \frac{P_{1out} + P_{2out}}{P_{0out}} = \frac{|\mathbf{w}^H(n)\mathbf{a}_1|^2 P_1 + |\mathbf{w}^H(n)\mathbf{a}_2|^2 P_2}{|\mathbf{w}^H(n)\mathbf{a}_0|^2 P_0} \\ &= \frac{|\mathbf{w}^H(n)\mathbf{a}_1|^2 P_1 + |\mathbf{w}^H(n)\mathbf{a}_2|^2 (P_1 - \Delta P)}{|\mathbf{w}^H(n)\mathbf{a}_0|^2 P_0}. \end{aligned} \quad (10)$$

Assuming that $P_0, P_1, \mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$ are invariant, the relationship between signal output JSR and ΔP can be expressed as

$$O_{JSR} = \frac{-\mathbf{a}_2^H \mathbf{w}(n) \mathbf{w}^H(n) \mathbf{a}_2 \Delta P + [|\mathbf{w}^H(n)\mathbf{a}_1|^2 + |\mathbf{w}^H(n)\mathbf{a}_2|^2] P_1}{|\mathbf{w}^H(n)\mathbf{a}_0|^2 P_0}. \quad (11)$$

Considering the iterative solution process of the LMS algorithm, $\mathbf{w}(n)\mathbf{w}^H(n)$ can be written as

$$\begin{aligned} \mathbf{w}(n)\mathbf{w}^H(n) &= \mathbf{w}(n-1)\mathbf{w}^H(n-1) + 2\mu\varepsilon(n-1)\mathbf{x}(n-1)\mathbf{w}^H(n-1) \\ &\quad + 2\mu\varepsilon^*(n-1)\mathbf{w}(n-1)\mathbf{x}^H(n-1) \\ &\quad + 4\mu^2\varepsilon(n-1)\varepsilon^*(n-1)\mathbf{x}(n-1)\mathbf{x}^H(n-1) \\ &= \mathbf{w}(0)\mathbf{w}^H(0) + \sum_{j=1}^n \mathbf{f}(j) \end{aligned} \quad (12)$$

where

$$\begin{aligned} \mathbf{f}(j) &= \mathbf{A}_0(j)(P_1 - \Delta P) + \mathbf{B}_0(j)\sqrt{P_1 - \Delta P} + \mathbf{C}_0(j) \\ \mathbf{A}_0(j) &= \sum_{i=1}^j 4\mu^2\varepsilon(j-1)\varepsilon^*(j-i)\mathbf{a}_2\mathbf{a}_2^H s_2(j-1)s_2^*(j-i) \\ &\quad + \sum_{i=2}^j 4\mu^2\varepsilon^*(j-1)\varepsilon(j-i)\mathbf{a}_2\mathbf{a}_2^H s_2(j-i)s_2^*(j-1) \\ \mathbf{B}_0(j) &= 2\mu\varepsilon(j-1)\mathbf{a}_2s_2(j-1)\mathbf{w}^H(0) + 2\mu\varepsilon^*(j-1)\mathbf{w}(0)\mathbf{a}_2^H s_2^*(j-1) \\ &\quad + \sum_{i=1}^j 4\mu^2\varepsilon(j-1)\varepsilon^*(j-i)(\mathbf{a}_2\mathbf{c}^H s_2(j-1) + \mathbf{c}\mathbf{a}_2^H s_2^*(j-i)) \\ &\quad + \sum_{i=2}^j 4\mu^2\varepsilon^*(j-1)\varepsilon(j-i)(\mathbf{a}_2\mathbf{c}^H s_2(j-i) + \mathbf{c}\mathbf{a}_2^H s_2^*(j-1)) \\ \mathbf{C}_0(j) &= 2\mu\varepsilon(j-1)\mathbf{c}\mathbf{w}^H(0) + 2\mu\varepsilon^*(j-1)\mathbf{w}(0)\mathbf{c}^H \\ &\quad + \sum_{i=1}^j 4\mu^2\varepsilon(j-1)\varepsilon^*(j-i)\mathbf{c}\mathbf{c}^H + \sum_{i=2}^j 4\mu^2\varepsilon^*(j-1)\varepsilon(j-i)\mathbf{c}\mathbf{c}^H \end{aligned} \quad (13)$$

Substituting Eq. (12) into Eq. (11), we have

$$O_{JSR} = \frac{\mathbf{A}_1(n)(P_1 - \Delta P)^2 + \mathbf{B}_1(n)(P_1 - \Delta P)^{\frac{3}{2}} + \mathbf{C}_1(n)(P_1 - \Delta P) + \mathbf{D}_1(n)(P_1 - \Delta P)^{\frac{1}{2}} + \mathbf{E}_1(n)}{\mathbf{F}_1(n)(P_1 - \Delta P) + \mathbf{H}_1(n)(P_1 - \Delta P)^{\frac{1}{2}} + \mathbf{M}_1(n)} \quad (14)$$

where

$$\begin{aligned} \mathbf{A}_1(n) &= \mathbf{a}_2^H \sum_{m=1}^n A_0(m) \mathbf{a}_2 \\ \mathbf{B}_1(n) &= \mathbf{a}_2^H \sum_{m=1}^n \mathbf{B}_0(m) \mathbf{a}_2 \\ \mathbf{C}_1(n) &= \mathbf{a}_2^H \mathbf{w}(0) \mathbf{w}^H(0) \mathbf{a}_2 + \mathbf{a}_1^H \sum_{m=1}^n \mathbf{A}_0(m) \mathbf{a}_1 P_1 + \mathbf{a}_2^H \sum_{m=1}^n \mathbf{C}_0(m) \mathbf{a}_2 \\ \mathbf{D}_1(n) &= \mathbf{a}_1^H \sum_{m=1}^n \mathbf{B}_0(m) \mathbf{a}_1 P_1 \\ \mathbf{E}_1(n) &= \mathbf{a}_1^H \mathbf{w}(0) \mathbf{w}^H(0) \mathbf{a}_1 P_1 + \mathbf{a}_1^H \sum_{m=1}^n \mathbf{C}_0(m) \mathbf{a}_1 P_1 \\ \mathbf{F}_1(n) &= \mathbf{a}_0^H \sum_{m=1}^n \mathbf{A}_0(m) \mathbf{a}_0 P_0 \\ \mathbf{H}_1(n) &= \mathbf{a}_0^H \sum_{m=1}^n \mathbf{B}_0(m) \mathbf{a}_0 P_0 \\ \mathbf{M}_1(n) &= \mathbf{a}_0^H \mathbf{w}(0) \mathbf{w}^H(0) \mathbf{a}_0 P_0 + \mathbf{a}_0^H \sum_{m=1}^n \mathbf{C}_0(m) \mathbf{a}_0 P_0. \end{aligned} \quad (15)$$

For convenience, we define $P_1 - \Delta P = x^2$, and O_{JSR} can be rewritten as

$$O_{JSR} = \frac{\mathbf{A}_1(n)x^4 + \mathbf{B}_1(n)x^3 + \mathbf{C}_1(n)x^2 + \mathbf{D}_1(n)x + \mathbf{E}_1(n)}{\mathbf{F}_1(n)x^2 + \mathbf{H}_1(n)x + \mathbf{M}_1(n)}. \quad (16)$$

The derivative of O_{JSR} can be given by

$$\frac{\partial O_{JSR}}{\partial x} = \frac{ax^5 + bx^4 + cx^3 + dx^2 + ex + f}{(\mathbf{F}_1(n)x^2 + \mathbf{H}_1(n)x + \mathbf{M}_1(n))^2} \quad (17)$$

where

$$\begin{aligned} a &= 2\mathbf{A}_1(n)\mathbf{F}_1(n), b = \mathbf{B}_1(n)\mathbf{F}_1(n) + 3\mathbf{A}_1(n)\mathbf{H}_1(n), c = 2\mathbf{B}_1(n)\mathbf{H}_1(n), \\ d &= 2\mathbf{B}_1(n)\mathbf{H}_1(n) + 4\mathbf{A}_1(n)\mathbf{M}_1(n), e = 2\mathbf{C}_1(n)\mathbf{M}_1(n) - 2\mathbf{E}_1(n)\mathbf{F}_1(n), \\ f &= \mathbf{D}_1(n)\mathbf{M}_1(n) - \mathbf{E}_1(n)\mathbf{H}_1(n). \end{aligned} \quad (18)$$

Since the denominator in Eq. (17) is always greater than zero, and the highest power of the numerator polynomial is 5, i.e., it must have 5 roots, it is easy to know that the numerator polynomial has at least one real root because the complex roots appear in pairs. Therefore, there is at least one real number in Eq. (17) such that the first-order derivative of O_{JSR} is zero. Thus we have

$$\left. \frac{\partial O_{JSR}}{\partial x} \right|_{x=x_0} = 0 \quad (19)$$

Owing to $\Delta P = P_1 - x^2$, there must be a ΔP satisfying the following expression

$$\left. \frac{\partial O_{JSR}}{\partial \Delta P} \right|_{\Delta P = \Delta P_{\text{opt}}} = 0 \quad (20)$$

This concludes the proof.

Theorem 3.1 shows that in Eq. (16), there must be a ΔP_{opt} satisfying Eq. (20), and the ΔP_{opt} is called Stationary Point. Therefore, we can reasonably speculate that the stationary point may be a maximum or minimum point.

Next, we will use the simulation to further study the relationship between output JSR and ΔP . Assume that the uniform linear array (ULA) consists of five sensors ($M = 5$) spaced by half-wavelength, and the number of snapshots is $L = 400$. Assuming that there are one GPS signal and two single-tone interference signals, their center frequencies and directions are $f_0 = 1.575$ GHz, $\theta_0 = 20^\circ$, $f_1 = 1.574$ GHz, $\theta_1 = -40^\circ$, $f_2 = 1.5755$ GHz, $\theta_2 = 60^\circ$, respectively. In this simulation, the signal-to-noise-ratio (SNR) is set to -20 dB and 60 dB for the GPS signal and the first interference signal, respectively, and the SNR of the second interference signal changes from 0 dB to 60 dB.

Figure 2 plots the the output JSR versus ΔP . It can be seen that there is a maximum when ΔP changes from 0 dB to 60 dB, and this result matches Theorem 1. On the other hand, Fig. 2 shows that the optimal power difference of single-tone interference is about 22 dB.

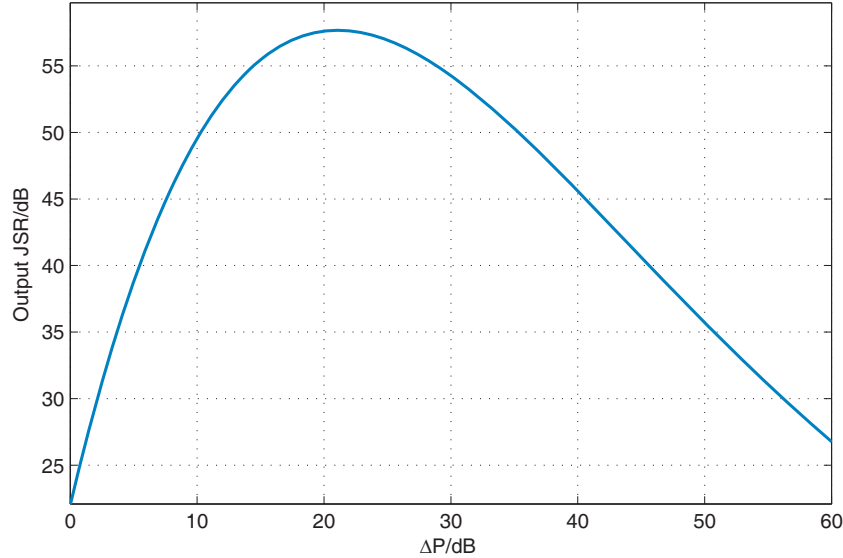


Figure 2. The output JSR versus ΔP .

Through the above theoretical analysis and simulation result of Fig. 3, the following conclusion can be drawn: we can maximize the signal output JSR by finding an optimal power difference ΔP_{opt} . That is, when two interference signals with different powers are used to interfere with the LMS-GPS receiver, there must be an optimal interference power difference so that the received signal has the largest output JSR after being processed by the LMS algorithm. From the conclusion, we give an interference method named as jamming method based on optimal power difference (OPD-LMS), in which two interference signals are utilized, and the power difference between the two interference signals is set to ΔP_{opt} .

4. SIMULATION RESULTS

In this section, we construct several simulations to demonstrate the effectiveness of the proposed algorithm. Consider a ULA with M antenna elements, in which $M = 5$, $d/\lambda = 1/2$, d is element spacing, and λ is wave length. Assuming that there are three signals impinging on the antenna array, the frequency of the desired signal is $f_1 = 1.575$ GHz, and the center frequencies of the two interferer signals are $f_2 = 1.574$ GHz and $f_3 = 1.5755$ GHz, respectively. Their DOAs are $\theta_1 = 20^\circ$, $\theta_2 = -40^\circ$, and $\theta_3 = 60^\circ$, respectively.

Figure 3 shows the output JSR versus single-tone interference's input power. Since GPS is a direct sequence spread spectrum system, it has an anti-interference ability of about 36 dB for single-tone

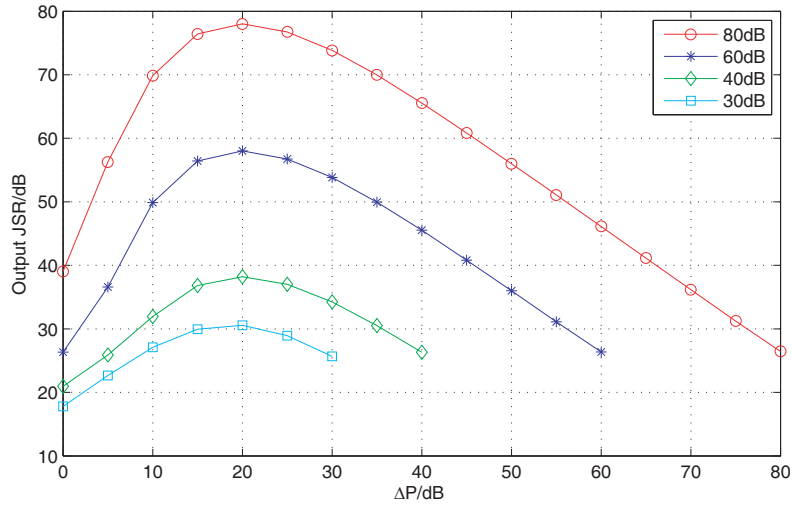


Figure 3. The output JSR versus single-tone interference’s input power.

interference. We set the first interference power as $P_1 = 30$ dB, 40 dB, 60 dB and 80 dB, respectively, and the ΔP changes from 0 dB to P_1 . It is easy to know that for different values of P_1 , there is always a maximum in output JSR. Therefore, we can conclude that though P_1 affects the output JSR of the signal processed by the LMS algorithm, the optimal interference power difference ΔP_{opt} is almost not changed, that is to say, P_1 has no effect on ΔP_{opt} .

Figure 4 plots the output JSR versus snapshots. In brief, the snapshots impact the output JSR of signal significantly. On the one hand, when the snapshots are 400, 600, 1000, respectively, as the ΔP increases, there is a maximum point in their output JSR, but their value is different. On the other hand, the maximum output JSR depends on the number of snapshots, that is, as the snapshots increase, the maximum JSR decreases. The reason may be that the anti-interference performance of the LMS algorithm significantly increases as the snapshots increase, which causes the output JSR to decrease. Therefore, we can conclude that, as the snapshots increase, the output JSR decreases, and ΔP_{opt} increases, but it can also get an ideal output JSR when $\Delta P = 22$ dB.

Figure 5 gives the output JSR versus band-limited Gaussian noise interference’s input power.

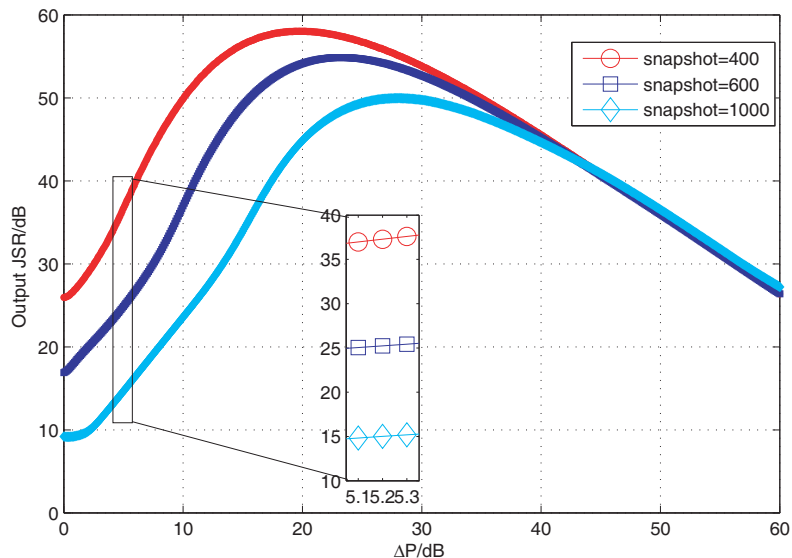


Figure 4. The output JSR versus snapshot.

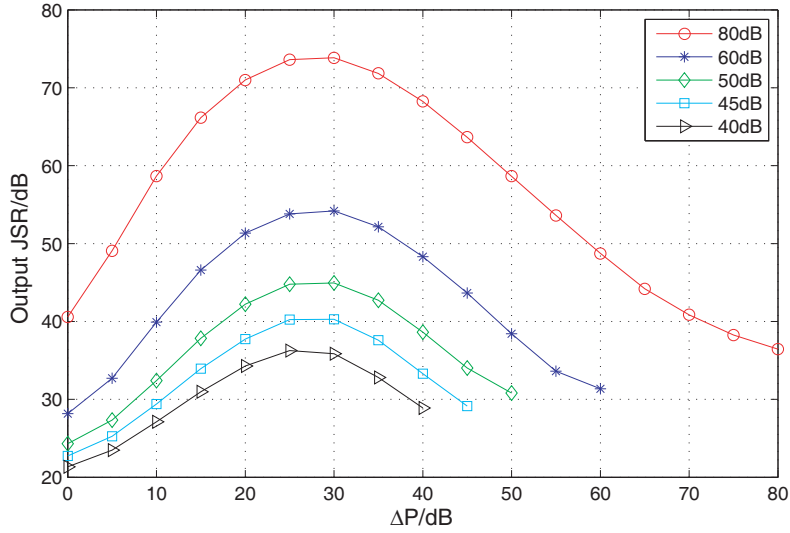


Figure 5. The output JSR versus band-limited Gaussian noise interference’s input power.

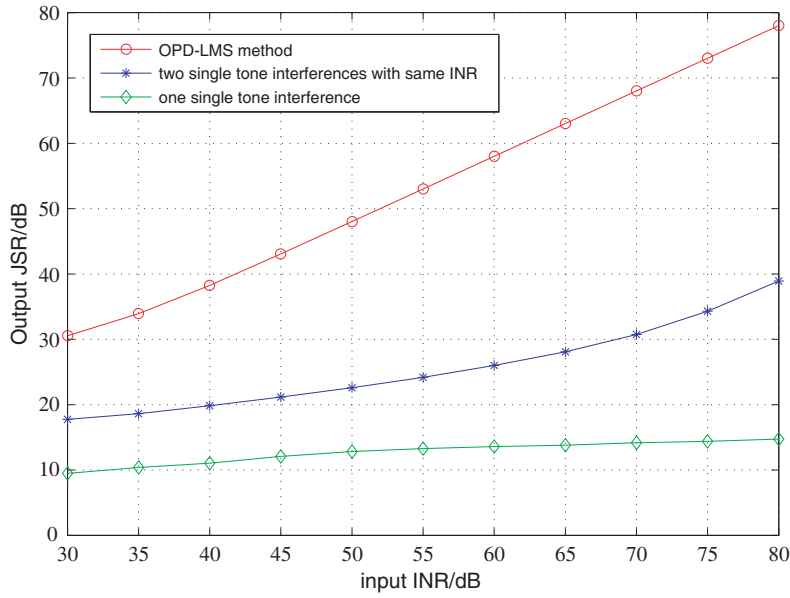


Figure 6. The output JSR versus the input INR in different interference methods.

Similar to Fig. 3, we also consider $P_1 = 40$ dB, 45 dB, 50 dB, 60 dB and 80 dB, respectively, and the ΔP changes from 0 dB to P_1 . When the interference signal is band-limited Gaussian noise interference, we can see that they all have the maximum, but unlike Fig. 3, the optimal power difference is about 29 dB. Therefore, we have the following conclusion: the optimal power difference of band-limited Gaussian noise interference is about 29 dB.

Figure 6 shows the output JSR versus the input interference-to-noise ratio (INR) in different interference methods. It is observed that the proposed method always outperforms the other methods and offers a continuously rising gain over the whole range of the input INR, while the two single-tone interference method experiences relatively smooth gain. Especially when $INR = 80$ dB, the proposed method can consistently offer about 40 dB and 65 dB gain compared with one single-tone interference method and two single-tone interferences method with the same INR, respectively. Based on the above discussion, we verify the superiority of the proposed scheme under various system settings.

5. CONCLUSION

In this paper, a new interference method for the LMS-GPS receiver is presented. The proposed jamming method includes two interference signals, and they have a optimal power difference ΔP_{opt} . Through theoretical and simulation analysis, the optimal power difference ΔP_{opt} is derived as about 22 dB for single-tone interference and 29 dB for band-limited Gaussian noise interference. Compared to one interference signal and two identical power interference signals, the presented method gets a higher JSR and has better jamming performance.

ACKNOWLEDGMENT

This work was supported by the Natural Science Foundation of Hebei Province (No. F2016501139), by the Fundamental Research Funds for the Central Universities under Grant (Grant No. N172302002, Grant No. N162304002), and by the National Natural Science Foundation of China (Grant No. 61501102).

REFERENCES

1. Pinker, A. and C. Smith, "Vulnerability of the GPS Signal to Jamming," *GPS Solutions*, Vol. 3, No. 2, 19–27, 1999.
2. Kamatham, Y., B. Kinnara, and M. K. Kartan, "Mitigation of GPS multipath using affine combination of two LMS adaptive filters," *IEEE International Conference on Signal Processing, Informatics, Communication and Energy Systems*, Vol. 35, 1–4, 2015.
3. Ahmad, Z., M. Tahir, and I. Ali, "Analysis of beamforming algorithms for antijams," *2013 XVIIIth International Seminar/Workshop on Direct and Inverse Problems of Electromagnetic and Acoustic Wave Theory (DIPED)*, 89–96, 2013.
4. Chan, S. C. and Y. Zhou, "Improved generalized-proportionate stepsize LMS algorithms and performance analysis," *IEEE International Symposium on Circuits and Systems*, 2325–2328, 2006.
5. Gardner, W., "Nonstationary learning characteristics of the LMS algorithm," *IEEE Transactions on Circuits and Systems*, Vol. 34, No. 10, 1199–1207, 2003.
6. Luo, H., "Accurate analysis of processing gain in direct sequence spread spectrum communication systems under single-tone and narrowband interference," *Telecommunication Engineering*, 2014.
7. Pazaitis, D. I. and A. G. Constantinides, "A novel kurtosis driven variable step-size adaptive algorithm," *IEEE Transactions on Signal Processing*, Vol. 47, No. 3, 864–872, 1999.
8. Duttweiler, D. L., "Proportionate normalized least-mean-squares adaptation in echo cancelers," *IEEE Transactions on Speech and Audio Processing*, Vol. 8, No. 5, 508–518, 2002.
9. Ye, F., H. Tian, and F. Che, "CW interference effects on the performance of GPS receivers," *2017 Progress In Electromagnetics Research Symposium — Fall (PIERS — FALL)*, 66–72, 2017.
10. Mao, Y. and C. Guo, "Analysis of interference effect on signal acquisition and tracking of GPS receiver," *IEEE International Conference on Communication Problem-Solving*, 592–595, 2014.
11. Balaei, A. T., A. G. Dempster, and L. L. Presti, "Characterization of the Effects of CW and Pulse CW Interference on the GPS Signal Quality," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 45, No. 4, 1418–1431, 2009.
12. Betz, J. W. and K. R. Kolodziejcki, "Generalized theory of code tracking with an early-late discriminator Part II: Noncoherent processing and numerical results," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 45, No. 4, 1557–1564, 2009.
13. Liu, F., R. Du, and X. Bai, "A virtual space-time adaptive beamforming method for space-time antijamming," *Progress In Electromagnetics Research M*, Vol. 58, 183–191, 2017.
14. Turan, C., M. S. Salman, and A. Eleyan, "A new variable step-size block LMS algorithm for a non-stationary sparse systems," *International Conference on Electronics Computer and Computation*, 1–4, 2016.