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On Localized Antenna Energy in Electromagnetic Radiation

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Abstract—We provide a general and rigorous formulation of antenna localized electromagnetic radiation energy in generic antenna systems based on Poynting flow instead of the spectral approach proposed earlier. The main theory is first developed using the principles of energy-momentum conservation and the center-of-energy theorem, culminating in the derivation of a direct localized energy expression. It is rigorously established that this expression satisfies the main features expected of physical energy, mainly positive definiteness and regularity. The obtained formula involves only the radiated fields (no current or charge) source and is easier to compute using specialized direct time-domain EM solvers. The proposed approach is expected to play a role in understanding energy localization in coupled antennas and shed light on gain enhancement methods.

1. LOCALIZED ENERGY: MOTIVATIONS AND GENERAL CONSIDERATIONS

The subject of localized energy has emerged in recent years as one of the potential critical domains projected to play increasingly important roles in various applications. For example, we mention 5G and mmWave technologies, which are accompanied by interactions and energy exchange at multiple physical scales, often in dense and complex electromagnetic environments; indeed, systems like Ultra-Dense Networks (UDN) and massive MIMO involve interactions in the near-zone and hence increased localization [1, 2]. Also, it is now established that near-field nano-optics, subwavelength imaging, plasmonics, and optical antennas all involve highly nontrivial processes of local energy enhancement [3, 4]. Electromagnetic energy localization has also been studied in depth within the context of condensed matter physics [5] and is a familiar subject in periodic structures like photonic crystals. Energy localization is now seen as a fundamental factor in electromagnetic therapy in general [6] and microwave hyperthermia in particular [7]. However, while looking into the electromagnetic energy in the near-field of antennas, the applied electromagnetics community mostly focuses the attention towards the "reactive energy" for antennas [8–15]. It should be noted here that near-field focusing is another classical antenna application where energy localization is essential [16–18], and the same is true for the closely related subject of wireless power transfer [19]. Also in traditional RF and microwave antenna engineering, localized energy has been singled out as one of the main principles behind ultra-wide band (UWB) performance [20, 21]. Therefore, it has become essential to look into the near-fields of the antennas more comprehensively and focus on aspects like localized energy, moving beyond the standard notions of reactive-energy and antenna-Q factors [22–27].

The subject of localized energy was recently treated from a more general perspective, that of the Weyl (plane-wave) expansion [28, 29], which was used to propose a spacetime spectral theory of localized energy in [30, 31]. Localized energy in the near-field interaction zone was also analyzed and estimated in [32] for strongly-coupled wire antennas. In contrast to the previous spectral approach to localized energy proposed in [30], this paper provides a justification of localized electromagnetic energy

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in radiation problems based on the Poynting flow, especially in comparison with its sister more familiar concept of reactive energy. Our main goal is to attain some conceptual clarity relating to how localized energy can be separated from the total field by relying on relative motion of energy (propagating field) with respect to non-propagating fields (fields at rest with respect to the antenna frame of reference.) In order to effect this separation in a rigorous manner, we use an argument based on the energy-momentum conservation and center of energy theorem. The derived expression is then demonstrated to be positive definite and regular.

The paper is organized as follows. The motivation of this research is given in Sec. 2, where comparison with some previous attempts in the spectral domain are highlighted. The main conceptual and theoretical aspects of Poynting localized energy are discussed in Sec. 3, where the major result of this paper is given by expression (20). The obtained localized energy is then briefly analyzed in Sec. 4 and shown to obey most of the important physical characteristics of energy, including positive definiteness and regularity. Also, comparison is made with reactive energy and the relativistic approach to further distinguish the two concepts. Finally, conclusions are given.

2. MOTIVATION AND BACKGROUND

It has been suggested before (see [22, 30, 31]) that there are three, fundamentally distinct types of electromagnetic energy pertinent to antennas, i.e., radiation, problems; these are reactive Energy, localized Energy, and stored Energy. *Reactive* energy, the energy type most commonly addressed in applied electromagnetics, relates to that energy stored in the *circuit representation* of the input port of the antenna system. Indeed, it is well known that the frequency-domain input impedance of any antenna has the structure of [33]

$$R + i(X_L - X_C),$$

where $R, X_L, X_C \geq 0$. Physically, R is interpreted as the radiation resistance, while X_L and $-X_C$ are taken as the *inductive* and *capacitive* reactances, respectively. As noted in [31], since this RLC circuit is merely a *model* of the physically measurable input impedance, it cannot be considered an actually existing physical medium inside which electromagnetic energy is stored. On the other hand, *electromagnetic* stored energy is best treated as a real physical property of the *fields*, i.e., this energy does not reside in the circuit model of the input port but is somehow distributed within the global structure of the *field* configuration of the entire system, including most prominently the *near-field zone.* Consequently, it was suggested in [30, 31] that stored energy and reactive energy should be distinguished from each other. A more useful definition of stored energy briefly proposed in [30] and further developed in [25–27] was to define stored energy as *recoverable energy*, i.e., the energy that can be retrieved from the antenna system after switching off its steady-state source. In this paper, we focus on the *second* energy type listed above, *localized* energy, which we consider an independent energy form distinct from both reactive and stored energy. Intuitively, we define localized energy density, hereafter w_{loc}, as *the spatial density of electromagnetic energy confined or concentrated in space around the source, i.e., the energy that potion of energy that tends not to flow or propagate away from the source, but instead is concatenated within a bounded spatial domain enclosing the radiating source.*

There are two possible senses for the term 'localized energy', depending on how energy movement is defined. If we focus on energy *propagation*, the natural definition of w_{loc} will be that effected in terms of nonpropagating modes. Since in vacuum nonpropagating modes can be shown to be strictly a superposition of evanescent modes, the original definition and derivation of w_{loc}, originally given in [30], involved spectral integrals including all possible evanescent modes added together in order to estimate the total localized energy W_{loc} given by

$$W_{\mathrm{loc}}(t) = \int_V d^3r \, w_{\mathrm{loc}}(\mathbf{r}, t), \tag{1}$$

where the integration region V is the field support. A different approach to localized energy is to use the idea of *power flow* instead of energy propagation. In this case, instead of performing a generalized Fourier analysis through the Weyl expansion as done in [30] in order to decompose the total field into propagating and evanescent parts, one use the Poynting theorem to describe how electromagnetic energy move around in vacuum. Unfortunately, there is some evidence that the two pictures are not isomorphic.

Indeed, it was found in [31] that differential equations describing energy propagation according to the Weyl expansion and the Poynting theorem, respectively, do not lead to the same physical description.

In this paper, we adopt the Poynting picture of localized energy, in contrast to the Weyl picture advocated in [31]. There are several motivations for doing this, which we summarize as follows. First, although exact and rigorous expressions for localized energy where derived in [30], these involve quite complicated spectral integrals with potential pole and branch point singularities. To our best knowledge, no attempt has been made so to numerical compute localized energy based on the expressions derived in [30], though some examples for the scalar case were presented in [31]. Second, although the Weyl picture in terms of propagating and nonpropagating (evanescent) modes is rigorous and more physically appealing, most *current* engineering applications tend to formulate energy flow in terms of power, relying mainly on the Poynting theorem to establish connection with electromagnetic fields. Third, even though the Weyl picture adopted in [31] is an exact spacetime approach to energy flow, it remains important for completeness to develop the second theory (the Poynting picture) in order to gather enough material to compare the two energy localization concepts together.

To recap, there are two possible localized energy types:

(i) Weyl Localized Energy (Localization = No spacetime).

(ii) Poynting Localized Energy (Localization = No power flow).

It is important to emphasize that we do not claim that the two localized energy types are the same; in fact, we anticipate they are different. However, very little is known about both and we hope that by focusing in the present paper on the Poynting type, we can provide some contribution to the topic.

3. POYNTING LOCALIZED ENERGY: THE FUNDAMENTAL THEORY

We assume an arbitrarily-shaped antenna supporting a continuous electric current density $\mathbf{J}(\mathbf{r},t)$ defined on region V_s. The current radiates into an infinite isotropic and homogeneous medium with permittivity ε and permeability μ. For simplicity, in this paper we focus on non-dispersive media, i.e., we include neither temporal nor spatial dispersion. In fact, we consider the medium of the radiation field to be essentially like the vacuum. In this case, the speed of electromagnetic wave propagation c and group velocity are the same, and both are equal to

$$|\mathbf{v}_g| = c = 1/\sqrt{\varepsilon\mu}. \tag{2}$$

Our goal here is to find a formula suitable for calculating time-dependent localized energy by utilizing the *Poynting flow* to characterize the energy localization process. In order to achieve this, a compromise between the Weyl and Poynting pictures is attempted based on the use of the *center-of-energy theorem*, which also utilizes the Poynting theorem. The idea of the center of energy is to bestow upon Poynting flow a spacetime flavour allowing it to come closer to the pure spacetime approach of the Weyl localized energy. Indeed, since the latter depends on treating moving energy as essentially the energy of the pure propagating modes, while localized energy is that of the evanescent nonpropgating modes, it follows that a treatment of power flow through a concept like "moving center of energy" or "electromagnetic mass" can provide an intuitive bridge between the two energy types.

To derive the Poynting localized energy, we first consider the traditional total electromagnetic energy density given by [34–38],

$$w_{\mathrm{em}}(\mathbf{r},t) = \frac{1}{2}\varepsilon\left|\mathbf{E}(\mathbf{r},t)\right|^2 + \frac{1}{2}\mu\left|\mathbf{H}(\mathbf{r},t)\right|^2. \tag{3}$$

It is well known that the total radiation energy in an unbounded exterior region is infinite. That is, if the exterior domain is denoted by V_R, where R is the maximum dimension of V_R, then the following divergence holds for any radiating antenna [11, 31, 25, 26]

$$\lim_{R\to\infty}\int_{V_R} d^3r\, w_{\mathrm{em}}(\mathbf{r},t) = \infty. \tag{4}$$

Reactive, localized, and stored energies all have to deal with this problem, which is the reason why a careful derivation of a "subtraction term" required to regularize (4) is needed, e.g., see various different approaches in [8–15]. In order to find the regularization term suitable for subtraction from (3), we turn toward the center-of-energy theorem for help.

We start by noting that in general, the fields radiated by any source can be decomposed into propagating and nonpropagating modes using the Weyl expansion (also known as the *plane-wave spectrum* method, see [28]). That is, we have

$$\mathbf{E}\left(\mathbf{r},t\right)=\mathbf{E}_{\text{propag.}}\left(\mathbf{r},t\right)+\mathbf{E}_{\text{nonpropag.}}\left(\mathbf{r},t\right),\quad \mathbf{H}\left(\mathbf{r},t\right)=\mathbf{H}_{\text{propag.}}\left(\mathbf{r},t\right)+\mathbf{H}_{\text{nonpropag.}}\left(\mathbf{r},t\right). \tag{5}$$

From this it follows that the total electromagnetic energy can be decomposed as

$$w_{\text{em}}\left(\mathbf{r},t\right)=w_{\text{loc}}\left(\mathbf{r},t\right)+w_{\text{propag}}\left(\mathbf{r},t\right), \tag{6}$$

where $w_{\text{loc}}(\mathbf{r},t)$ and $w_{\text{propag}}(\mathbf{r},t)$ are the localized and propagating energy densities, respectively. Note that there is also a *third* energy type, that of due to *interaction* between propagating and nonpropagating energy. However, the analysis in [30] has suggested that this interaction energy is essentially localized, and hence we include it with the nonpropagating energy density to give the total localized energy according to the prescription

$$w_{\text{loc}}\left(\mathbf{r},t\right)=w_{\text{nonpropag}}\left(\mathbf{r},t\right)+w_{\text{interaction}}\left(\mathbf{r},t\right). \tag{7}$$

In this manner, the determination of localized energy will be completed by finding the propagating energy density $w_{\text{propag}}(\mathbf{r},t)$ since from (6) we have

$$w_{\text{loc}}\left(\mathbf{r},t\right)=w_{\text{em}}\left(\mathbf{r},t\right)-w_{\text{propag}}\left(\mathbf{r},t\right). \tag{8}$$

Next, consider a *source-free region*, i.e., a vacuum configuration where only pure propagating waves exist; in other words, we focus now on the case

$$\mathbf{E}\left(\mathbf{r},t\right)=\mathbf{E}_{\text{propag.}}\left(\mathbf{r},t\right),\quad \mathbf{H}\left(\mathbf{r},t\right)=\mathbf{H}_{\text{propag.}}\left(\mathbf{r},t\right), \tag{9}$$

which corresponds to the energy configuration

$$w_{\text{em}}\left(\mathbf{r},t\right)=w_{\text{propag}}\left(\mathbf{r},t\right),\ w_{\text{loc}}\left(\mathbf{r},t\right)=0. \tag{10}$$

This will serve as the extreme limit scenario in which energy localization is *non*existent. In this case, electromagnetic energy is understood to be set by a source "at infinity" into a perpetual motion in which local accumulation of energy is prohibited. We would like now to describe how such purely propagating energy "moves in spacetime" via the idea of Poynting power flow.

Let us focus on an electromagnetic pulse belonging to the pure propagating wave configuration of Eqs. (9) and (10). Let the electromagnetic energy density corresponding to this pulse be denoted by $u_{em}(\mathbf{r},t)$. Schwinger defined the *center-of-energy* of this pulse as [34]

$$\left\langle \mathbf{r}\left(t\right)\right\rangle :=\int_{V_{\text{pulse}}} d^3r\,\mathbf{r}\,u_{\text{em}}\left(\mathbf{r},t\right), \tag{11}$$

where V_{pulse} is the pulse support. Therefore, the center-of-energy in electromagnetics plays a role analogous to the one played by *center of mass* in rigid-body mechanics. It intuitively gives an idea about where the focal point of the moving electromagnetic energy is located within this energy distribution itself. The usefulness of the center of energy concept is that it allows us to visualize (and quantify) how energy is moving in space by tracing how the point $\langle\mathbf{r}(t)\rangle$ changes location with time.

Based on the Poynting theorem, the following differential equation, often referred to as the center-of-energy theorem, can be solved to obtain the trajectory of energy centriod [34, 37]

$$\mathbf{v}_E W_{\text{em}}(t)=c^2\mathbf{P}(t). \tag{12}$$

Here, $\mathbf{v}_E$ is the speed of the center of energy defined as

$$\mathbf{v}_E:=\frac{d}{dt}\left\langle \mathbf{r}\left(t\right)\right\rangle. \tag{13}$$

The total *electromagnetic momentum* $\mathbf{P}$ in Eq. (12) can be computed by

$$c^2\mathbf{P}(t)=\int_{V_{\text{pulse}}} d^3r\,\mathbf{S}\left(\mathbf{r},t\right), \tag{14}$$

where $\mathbf{S}$ is the time-dependent Poynting vector $\mathbf{S}(\mathbf{r},t) = \mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)$. The total propagating electromagnetic energy itself is found by

$$W_{\rm em}(t) := \int_{V_{\rm pulse}} d^3r\, u_{\rm em}\left(\mathbf{r},t\right). \tag{15}$$

However, it is not possible to directly use the center-of-energy theorem (12) in order to estimate the propagating energy density since the latter is a *local* quantity. The reason is that Eq. (12) is not a local (point) function relation in space, but an integrated (or spatial average) of such relation. In order to obtain the local version, we utilize the fact that in its original derivation, the center of energy theorem does not impose any restriction on the field region $V_{\rm pulse}$ appearing in integrals like Eqs. (14) and (15). We then choose an infinitesimal volume $V \rightarrow dv$ and approximate these integrals as

$$W_{\rm propag.} \simeq w_{\rm em}\left(\mathbf{r},t\right) dv, \quad c^2\mathbf{P} \simeq \mathbf{S}\left(\mathbf{r},t\right) dv. \tag{16}$$

Consequently, the center-of-energy theorem (12) becomes

$$\mathbf{v}_E w_{\rm em}\left(\mathbf{r},t\right) = \mathbf{S}\left(\mathbf{r},t\right), \tag{17}$$

which is the local relation we need. It can be interpreted as the center-of-energy theorem applied to a highly-concentrated spatial pulse centered at the spacetime point $(\mathbf{r},t)$. Conversely, by using partition of unity techniques in differential topology, one can also build any large region $V_{\rm pulse}$ out of a finite number of overlapping pulses each with very small support such that Eq. (17) applies to every one of these infinitesimal pulses, and then recover Eq. (12) from Eq. (17) using proper spatial averaging process.

The final step is to estimate the center-of-energy velocity. For that matter, we make use of the fact that it is the pure propagating field configuration (9) what is under consideration. In this case, Fourier analysis of the field (wave packet analysis) reveals that the speed of energy propagation is equal to group velocity, that is, we have the classic result [38]

$$|\mathbf{v}_E| = |\mathbf{v}_g| = c. \tag{18}$$

Substituting Eq. (18) into Eq. (17), we arrive at

$$w_{\rm propag}\left(\mathbf{r},t\right) = \frac{1}{c}\left|\mathbf{S}\left(\mathbf{r},t\right)\right|, \tag{19}$$

where the first relation in Eq. (10) was used. Deploying Eq. (19) in Eq. (8) results in

$$w_{\rm loc}\left(\mathbf{r},t\right) = \frac{1}{2}\left(\varepsilon\mathbf{E}\cdot\mathbf{E} + \mu\mathbf{H}\cdot\mathbf{H} - 2\sqrt{\mu\varepsilon}|\mathbf{E}\times\mathbf{H}|\right). \tag{20}$$

This is the main result of the present work. It expresses the *spacetime* localized energy density fully in terms of spatiotemporal electromagnetic fields by utilizing the Poynting vector. The key motivating idea behind the proposed definition of localized energy using Eq. (20) is the use of the Poynting vector $\mathbf{S}$ in Eq. (19) to estimate the propagating energy density $w_{\rm propag}$ and *only* this density type. In other words, the signature of the Ponying picture of electromagnetic energy localization is that $\mathbf{S}$ contributes nothing to nonpropagating energy. This is intuitively clear within the framework of the usual interpretation of the Poynting theorem since one expects that nonpropgating energy (stationary energy) has $\mathbf{S} = 0$.

4. GENERAL CHARACTERISTICS OF LOCALIZED ENERGY

A comprehensive analysis and investigation of the Poynting localized energy given by Eq. (20) is beyond the scope of the present paper but will be tackled by the authors in other places. For the purpose of the present work, we provide a quick overview of what we consider the most salient features of the energy type addressed by Eq. (20).

4.1. Positive Definiteness

We rigorously prove that the term $w_{\mathrm{loc}}(\mathbf{r},t)$ is *strictly positive semi-definite*, i.e., the relation

$$w_{\mathrm{loc}}(\mathbf{r},t) \geqslant 0, \quad W_{\mathrm{loc}}(t) \geqslant 0, \tag{21}$$

which is the minimum necessary condition for any energy form to correctly models energy phenomena. The proof runs as follows.

We start with the vector identity

$$|\mathbf{A}\times\mathbf{B}|^2 = |\mathbf{A}|^2|\mathbf{B}|^2 - (\mathbf{A}\cdot\mathbf{B})^2. \tag{22}$$

By direct computations, this can be expanded in the form

$$|\mathbf{A}\times\mathbf{B}|^2 = \left(\frac{|\mathbf{A}|^2+|\mathbf{B}|^2}{2}\right)^2 - \left[\left(\frac{|\mathbf{A}|^2-|\mathbf{B}|^2}{2}\right)^2 + (\mathbf{A}\cdot\mathbf{B})^2\right]. \tag{23}$$

We immediately infer from Eq. (23) that

$$|\mathbf{A}\times\mathbf{B}|^2 \leqslant \left(\frac{|\mathbf{A}|^2+|\mathbf{B}|^2}{2}\right)^2, \tag{24}$$

where equality holds if and only if the following two conditions are satisfied

$$\mathbf{A}\cdot\mathbf{B}=0, \quad |\mathbf{A}|^2=|\mathbf{B}|^2. \tag{25}$$

Next, we insert the following substitutions $\mathbf{A}=\sqrt{\varepsilon}\mathbf{E}$, $\mathbf{B}=\sqrt{\mu}\mathbf{H}$, into Eq. (24), resulting in

$$\sqrt{\varepsilon\mu}\,|\mathbf{E}\times\mathbf{H}| \leqslant \left(\frac{1}{2}\varepsilon|\mathbf{E}|^2+\frac{1}{2}\mu|\mathbf{H}|^2\right). \tag{26}$$

In other words,

$$\frac{1}{2}\varepsilon|\mathbf{E}|^2+\frac{1}{2}\mu|\mathbf{H}|^2-\sqrt{\varepsilon\mu}\,|\mathbf{E}\times\mathbf{H}| \geqslant 0,$$

and from Eq. (20) the relation in Eq. (21) immediately follows, which is the required statement of localized energy positive definiteness.

Note that according to Eq. (25), the equality in Eq. (21) holds if and only if the following condition is satisfied:

$$\mathbf{E}\cdot\mathbf{H}=0, \quad \frac{1}{2}\varepsilon|\mathbf{E}|^2=\frac{1}{2}\mu|\mathbf{H}|^2. \tag{27}$$

Clearly, these conditions hold in the case of pure propagating electromagnetic waves in vacuum, which confirms the intuitive foundations of localized energy. Indeed, only in the case of pure propagating modes does the localized energy identically vanishes.

4.2. Energy Regularity

The next important general characteristic is that localized energy must satisfy is *regularization*, which in our case can be expressed by the condition

$$\lim_{R\to\infty}\int_{V_R} d^3r\, w_{\mathrm{loc}}(\mathbf{r},t) < \infty. \tag{28}$$

This says that the total localized energy obtained by means of Eq. (1) must be finite when computed within an unbounded radiation domain contained within the antenna exterior region. A rigorous general proof of Eq. (28) is lengthy and will be reported elsewhere.

The proof requires developing a time-dependent power series expansion for w_{loc} in terms of $1/r$, using a time-dependent version of the frequency-domain Wilcox expansion [39]. By expanding both electric/magnetic energy and the Ponyting vector in powers of $1/r$, one can estimate the asymptotic behaviour of localized energy when $r\to\infty$ using techniques similar to those developed in [22]. The main difficulty in completing the proof resides in moving from the frequency domain, as was originally done in [31], to the time domain to show that all radial terms $1/r^n$ with power $n < 3$ are exactly zero, i.e., they do not contribute to total energy. That will ensure that the total energy infinite radial integral in Eq. (1) is actually finite. However, the presence in Eq. (20) of the absolute value of $\mathbf{S}$ instead of direct quadratic quantities as in electric and magnetic energy density considerably complicates the calculations but the main idea above remains the same. The authors will report these detailed and lengthy details elsewhere.

4.3. Comparison with Reactive Energy

Next, we briefly compare localized energy with the more popular reactive energy. For the determination of $W_{\mathrm{loc}}(t)$ using Eq. (20), one subtracts all the radial energy-flow terms having spatial variation function $1/r^n$ ($n = 2m$, $m \neq 1$) [22]. The presence of such terms can be predicted using the Wilcox expansion principle [39]. Thus, this localized energy $W_{\mathrm{loc}}(t)$ is *not* same as the reactive antenna energy $W_{\mathrm{react}}(t)$. Moreover, the following condition holds for any arbitrary antenna system:

$$W_{\mathrm{loc}}(t) < W_{\mathrm{react}}(t). \tag{29}$$

A convenient FDTD model developed in [40, 41] was applied to the problem of calculating reactive energy in [42]. Using this FDTD simulation paradigm, some initial result relating to this relation between localized energy and reactive energy is reported here in Figure 1, which is in perfect agreement with Eq. (29). Additional numerical results will be reported for array systems in strong and weak mutual coupling scenarios.

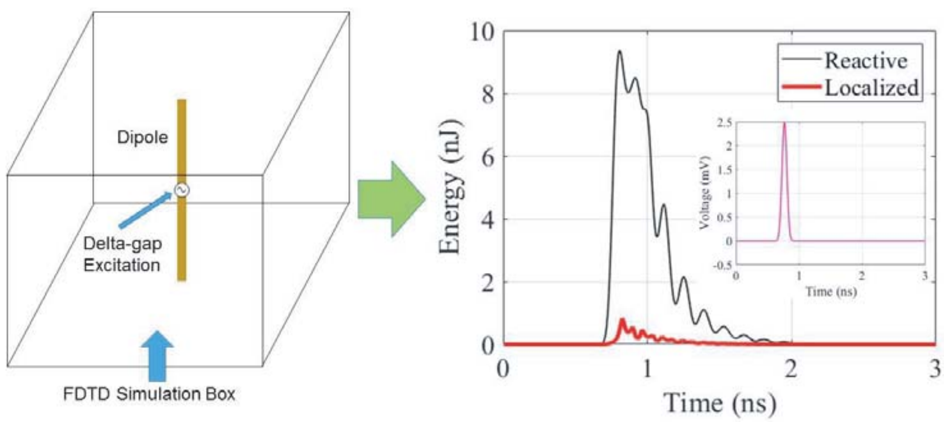

Figure 1. Comparison between localized and reactive energy for a half-wavelength thin-wire dipole having impedance matching point at 3.73 GHz with impedance bandwidth 11.06%. The input excitation impulse is shown in the inset.

4.3.1. Comparison with the Relativistic Approach

Finally, we briefly comment on some relativistic considerations in electrodynamics [34–38] relating to localized energy as defined here. Like total electromagnetic energy and reactive energy, the localized energy (20) is not relativistic, which is the typical case in antennas since the presence of a classic source term destroys Lorentz invariance (the frame at rest with the classic current becomes the preferred inertial coordinate system.) However, it is instructive to compare our formulation with some manifestly relativistic energy formulation proposed earlier. In particular the approach [43, 44], motivated by relativistic considerations, defines an equivalent "electromagnetic mass" of propagating waves in terms of the fields. From this mass, the "energy at rest" of the field is defined by means of the familiar relativistic formal $E = mc^2$, is taken as an estimation of "localization energy," given by

$$w_{\mathrm{inertial}}(\mathbf{r}, t) = \sqrt{|w_{\mathrm{em}}(\mathbf{r}, t)|^2 - |\mathbf{S}(\mathbf{r}, t)|^2}.$$

This expression, however, is essentially local (like Eq. (20)). It was not investigated in connection with global energy (total energy in the entire space or in large spatial regions). Moreover, the author in [43] defines reactive energy as essentially the inertia energy above, and hence is not in agreement with our approach, which treats reactive and localized energies as distinct. In fact, the inertia energy and ours do not agree numerically (see Figure 2), which is not surprising given that they address different physical phenomena. For example, the Bateman theory is expected to apply to relativistic particle dynamics scenarios, while ours is connected with classical fields in antennas and does not cover such situations. On the other hand, our localized energy approach goes beyond traditional reactive energy in conventional electromagnetics and explore new phenomenon like spatial energy localization around radiators in global domains, which is currently being actively investigated for applications like high gain antennas and energy harvesting.

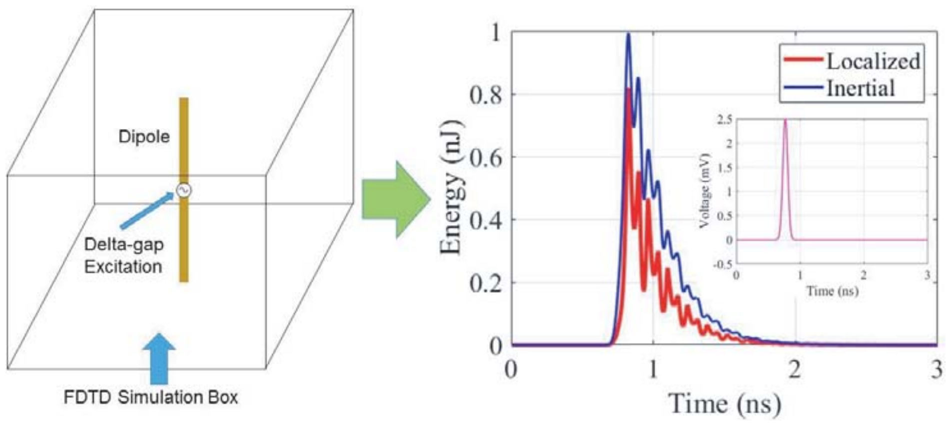

Figure 2. Comparison between localized and inertial energy for a half-wavelength thin-wire dipole having impedance matching point at 3.73 GHz with impedance bandwidth 11.06%. The input excitation impulse is shown in the inset.

5. CONCLUSION

Antenna Localized energy, distinct from both reactive and stored energy, was investigated in this paper from a very general perspective and without restriction to specific antenna type by developing a concept of Poynting localized energy in contrast to the previously proposed spectral approach. A simple derivation of a direct expression for time-dependent localized energy density in terms of total electromagnetic fields was given based on energy-momentum conservation and the center-of-energy theorem. The obtained localized energy was shown to enjoy many of the important characteristics of physical energy, including positive definiteness and regularity. The authors believe that suitable time-domain full-wave solvers could be developed to compute reactive and localized energy based on this will be reported by the authors in future work for several antenna types.

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