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Wideband Transmitting Adaptive Digital Beamforming Based on Sub-Band Multiple Linear Constrained Minimum Variance Method

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Abstract—This paper describes wideband transmitting adaptive digital beamforming (ADBF) scheme with nulls in the direction of interference. The scheme partitions the wideband transmit signal into independent sub-bands using an analysis filter bank behind each array element. In each channel, sub-band ADBF weight vector is computed based on the minimum variance criterion with multiple linear constraints to form the sub-band transmit beam. Finally, a wideband transmit adaptive beam is reconstructed through the synthesis filters. Theoretical analysis and simulation experiments show that this algorithm can form a wideband transmit beam with deep nulls, and the pointing direction of the null keeps unchanged regardless of frequency. The algorithm proposed in this paper has little computation load and is efficient to implement in engineering applications.

1. INTRODUCTION

The adaptive arrays deployed in modern radar systems tend to attenuate external interfering signals in the antenna receive pattern, such as hostile jammers, unintentional electromagnetic interference, or ambient clutter. The technology of signal processing at the radar receive pattern is becoming more and more efficient, and it is becoming more and more difficult to improve the detection performance by optimizing the radar receiving signal processing algorithm. Therefore, recent research institutions have developed the technology for adaptive digital beamforming (ADBF) in the transmit pattern of the antenna as well. The advantage is that the antenna can impose a significant two-way loss on interfering signal. However, most transmit ADBF algorithms developed are only suitable for narrowband applications [1–3], and the weights applied at each array element are only computed for the center frequency of the array. Therefore, the phase shifter behind each element is only calibrated for the array center frequency. When a wideband signal is transmitted, the actual phase shift of the signals transmitted by each array element will deviate with the actual frequency, which leads to the change of transmit null pointing direction over the entire signal bandwidth.

The early wideband beam forming method divides the wideband signal into several frequency bands and separately transmits the signals of each frequency band through the subarray antenna of different apertures to compensate the aperture crossing of the wideband array signal [4, 5]. If we want to improve the accuracy, we need to divide a large number of bands, and the implementation is more complex.

In order to solve above problem, Vouras and Tran proposed a robust transmit nulling (RTN) in wideband arrays beamforming algorithm [6], which derives the signal-to-interference-plus-noise ratio (SINR) function for frequency integration in order to maximize the SINR of the transmitted signal. The SINR function is solved by conjugate gradient algorithm to get the optimal weight vector. Because the conjugate iterative step size of this algorithm is difficult to compute, in order to get the optimal

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solution, many conjugate iterations are often needed. So the amount of computation is increased greatly, which is not conducive to the implementation of the project.

The linearly constrained minimum variance (LCMV) method, which allows multiple linear constraints, is an extension of the classical minimum variance distortionless response filter. The main innovation introduced in this paper is a wideband transmitting adaptive digital beamforming (ADBF) scheme that the optimal ADBF weight vector is computed according to the multiple linear constrained minimum variance (MLCMV) criterion on the basis of a wideband array architecture, so that the nulls direction of the wideband transmitting adaptive beam does not deviate from the actual frequency. In order to avoid temporal aliasing, the sampling frequency of the tapped delay line (TDL) is often set to two times the maximum frequency of the wideband signal, and the high sampling frequency is not conducive to the realization of the radar system. While the band partition technique partitions the wideband signal into several sub-bands, it can reduce the sampling frequency of the TDL, which is more conducive to the implementation of the project. At the same time, the sub-band partition technology has local optimum characteristics, so that the depth of the nulls is deeper, and the interference suppression performance is superior.

2. WIDEBAND TRANSMISSION ARRAY ARCHITECTURE

[7–10] present some methods that use optimization algorithms to solve electromagnetic and antenna problems. The architecture of a wideband transmission array is shown in Fig. 1. The antenna is a uniform linear array with N elements. The distance between array elements is d, and behind each array element is a TDL with J real taps spaced T_s seconds apart. Supposing that the input signal of the wideband array is $x[t]$, the lowest frequency is f_L, and the maximum frequency is f_H. In order to avoid spatial aliasing, $d = c/(2f_H)$ should be set, and c is the speed of light in free space, then the transmit signal of the output of the nth element is

$$x_n(t) = \sum_{k=0}^{J-1} x(t - kT_s) w_n[k] \tag{1}$$

where $w_n[k]$ is the kth tap coefficient of the nth array element, and $k = 0, 1, \ldots, J-1$. In order to avoid temporal aliasing, T_s is chosen to be $1/(2f_H)$. The tap delay chain vector of each array element can be expressed as

$$\mathbf{e}(f) = \left[1, e^{-j2\pi f T_s}, \ldots, e^{-j2\pi f T_s (J-1)}\right]^T \tag{2}$$

where $[\bullet]^T$ is a transposed operator.

If the wideband signal transmission direction is θ_0, the delay of the adjacent array elements transmitting signal to the target is $\tau = d \sin\theta_0/c$, and the phase difference is $\phi = 2\pi df \sin\theta_0/c$. The frequency dependent array steering vector of the wideband transmitting signal is

$$\mathbf{v}(\theta_0, f) = [1, \exp(j2\pi f d \sin\theta_0/c), \ldots, \exp(j2\pi f d(N-1)\sin\theta_0/c)]^T \tag{3}$$

Therefore, the frequency dependent space-time steering vector of the wideband transmitting signal can be written as

$$\mathbf{V}_{st}(\theta_0, f) = \mathbf{v}(\theta_0, f) \otimes \mathbf{e}(f) \tag{4}$$

where $\otimes$ represents the Kronecker product of the vector.

3. SUB-BAND MLCMV METHOD

3.1. Band Partitioning

The band partition filter bank usually involve two sets of filter bank, one of which is the analysis filter bank, mainly used for the partition of wideband signals. The sub-bands after partition can be processed individually, such as beamforming. The other set is the synthesis filter bank, which is mainly used for the reconstruction of wideband signal [11, 12].

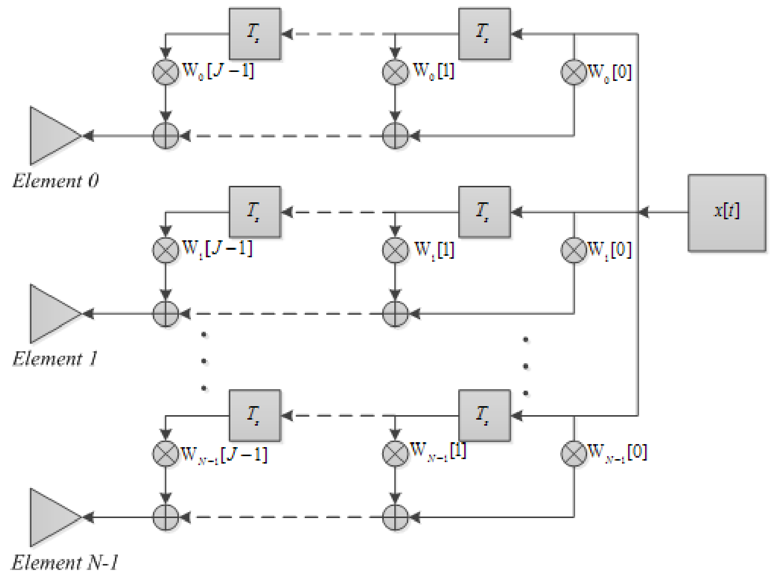

Figure 1. Wideband array structure of TDL.

Assuming that each element has an M sub-band channel, the ideal analysis filter bank is composed of several bandpass filters, which can partition the wideband signal with a bandwidth of B into a sub-band signal of M with a bandwidth of B/M, and the TDL tap sampling frequency of each sub-band can be decimated to the original $1/M$ at most. The Discrete Fourier Transform Filter Bank (DFTFB) can be used for band partition and reconstruction of wideband signals [13]. The analysis filter for each sub-band channel can be seen as a translation of a low pass prototype filter $H_0(z)$ with a length of P and a bandwidth of B/M. In other words,

$$H_m(z) = H_0(zW^{m+i}), \quad m = 1, 2, \ldots, M \tag{5}$$

$$H_0(z) = 1 + z^{-1} + \ldots + z^{-(P-1)} \tag{6}$$

where $W = e^{-j2\pi/P}$, $H_m(z)$ is z-transform of the mth sub-band channel analysis filter and the complex variable $z = e^{j\omega}$. The length of the filter is $P = f_s/(B/M)$, while $f_s = 2f_H$. Frequency offset $i = (f_L - B/2M)/(B/M)$. Fig. 2 is a diagram of the amplitude frequency response of DFTBF.

The synthesis filters are related by

$$F_m(z) = W^{-(m+i)} F_0(zW^{m+i}) \tag{7}$$

where $F_0(z) = H_0(z)$, $F_m(z)$ is the z-transform of the mth sub-band channel synthesis filter. Thus each synthesis filter has the same magnitude response as the corresponding analysis filter. The reconstructed signal at the output of the filter bank is

$$y(t) = Mx(t - MT_s + 1) \tag{8}$$

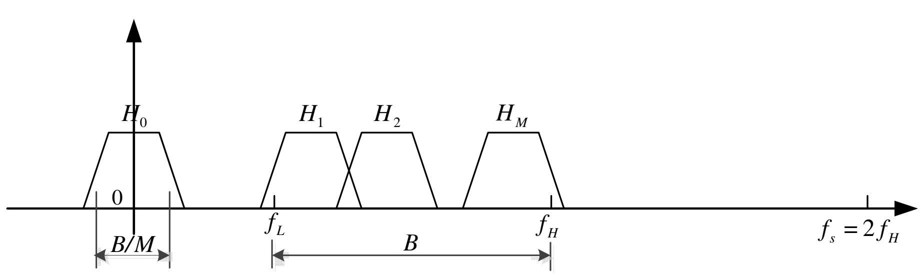

Figure 2. Diagram of the amplitude frequency response of DFTBF.

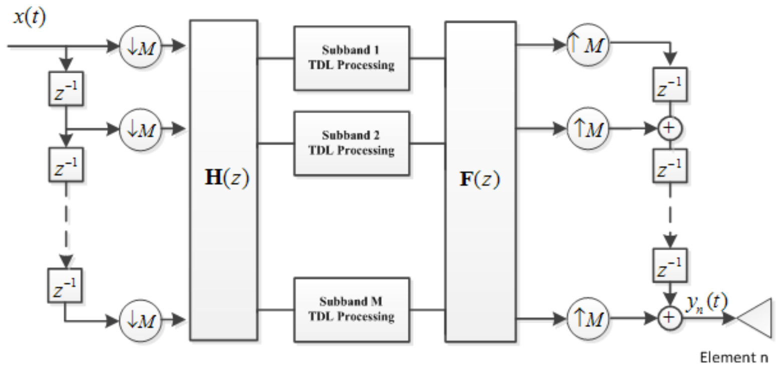

Figure 3. Band partition processing structure of array element.

The structure of the analysis filter and the structure of the synthesis filter can be equivalently transformed into a polyphase structure [11]. The polyphase structure reduces the amount of computation to the original $1/M$, so it is called high efficient structure. Thus, the band partition processing structure can be simplified as shown in Fig. 3.

3.2. Sub-Band ADBF via MLCMV

There are M sub-band processing channels behind each of the N array elements, and the TDL in each frequency sub-band has J taps. The bandwidth of the input wideband signal $x[t]$ is B; the minimum frequency is f_L; the maximum frequency is f_H. The bandwidth of the wideband signal is divided into K frequency points, namely $\{f_1, f_2, \ldots, f_K\} \in [f_L, f_H]$, and the set of the space-time steering vectors of the wideband signals at each frequency point can be expressed as

$$\mathbf{C} = [\mathbf{V}_{st}(\theta_0, f_1), \mathbf{V}_{st}(\theta_0, f_2), \ldots, \mathbf{V}_{st}(\theta_0, f_K)] \tag{9}$$

If there is interference in θ_1 direction of the radar, the interference noise covariance of the mth sub-band signal can be estimated from the maximum likelihood estimation (MLE)

$$\mathbf{N}_{st-m} = \frac{\beta}{K} \sum_{l=1}^{K} \mathbf{V}_{st-m}(\theta_1, f_l) \mathbf{V}_{st-m}^H(\theta_1, f_l) \left| H_m(f_l) F_m(f_l) \right| + \sigma^2 \mathbf{I} \tag{10}$$

where $[\bullet]^H$ is a conjugate transpose operator, σ^2 the power of a zero mean additive white noise Gaussian process, β a real positive scalar, $\mathbf{I}$ the identity matrix, f_l the lth frequency point, and $l = 1, 2, \ldots, K$. For the case where Q nulls in the array pattern are desired, the noise covariance matrix is written as

$$\mathbf{N}_{st-m} = \frac{\beta}{K} \sum_{l=1}^{K} \left(\sum_{j=1}^{Q} \mathbf{V}_{st-m}(\theta_j, f_l) \mathbf{V}_{st-m}^H(\theta_j, f_l) \right) \left| H_m(f_l) F_m(f_l) \right| + \sigma^2 \mathbf{I} \tag{11}$$

According to the LCMV criterion, $\mathbf{C}$ can be used as a multi-constraint matrix. The weight of the MLCMV filter of the mth sub-band should be satisfied

$$\begin{cases} s.t. \quad \mathbf{W}_m^H \mathbf{C} = \mathbf{F}^H \\ \min\limits_{\mathbf{W}_q} \quad \mathbf{W}_m^H \mathbf{N}_{st-m} \mathbf{W}_m, \end{cases} \tag{12}$$

The signal gain of the target direction at each frequency point can be limited to 1, so the response vector $\mathbf{F}$ is $K \times 1$ dimension all 1 vector. $\mathbf{W}_m$ is the $MJ \times 1$ dimension ADBF weight vector of the mth sub-band signal and

$$\mathbf{W}_m = \left[\mathbf{w}_{m0}, \mathbf{w}_{m1}, \ldots, \mathbf{w}_{mN-1} \right]^T, \tag{13}$$

where $\mathbf{w}_{mn}$ is the mth sub-band tap coefficient of the nth array element, and

$$\mathbf{w}_{mn} = [w_{mn}[0], w_{mn}[1], \ldots, w_{mn}[J-1]]^T. \tag{14}$$

We can use the Lagrange constant method to find the extreme value under the constraint condition of Eq. (12). The objective function is constructed as

$$L(\mathbf{W}_m) = \frac{1}{2}\mathbf{W}_m^H \mathbf{N}_{st-m}\mathbf{W}_m - \left[\mathbf{W}_m^H \mathbf{C} - \mathbf{F}^H\right]\lambda_m, \tag{15}$$

Find the derivative of $L(\mathbf{W}_m)$ about $\mathbf{W}_m$ and make it equal to 0. Useful relations for a matrix $\mathbf{A}$ and vectors x and y are

$$\frac{d(x^T\mathbf{A}x)}{dx} = 2Ax, \quad \frac{d(x^T\mathbf{A}y)}{dx} = Ay, \quad \frac{d(x^T\mathbf{A}y)}{dy} = A^T x. \tag{16}$$

We can get $\mathbf{N}_{st-m}\mathbf{W}_m - \mathbf{C}\lambda_m = 0$, and the optimal ADBF weight vector is $\mathbf{W}_m = \mathbf{N}_{st-m}^{-1}\mathbf{C}\lambda_m$. From the constraint condition $\mathbf{W}_m^H\mathbf{C} = \mathbf{F}^H$, we can get

$$\lambda_m = (\mathbf{C}^H\mathbf{N}_{st-m}^{-1}\mathbf{C})^{-1}\mathbf{F}. \tag{17}$$

Therefore, the optimal ADBF weight vector of the mth sub-band MLCMV filter is

$$\mathbf{W}_{\text{opt-m}} = \mathbf{N}_{st-m}^{-1}\mathbf{C}(\mathbf{C}^{\mathrm{H}}\mathbf{N}_{st-m}^{-1}\mathbf{C})^{-1}\mathbf{F}. \tag{18}$$

3.3. Sub-Band Synthesis

We have obtained the optimal ADBF weight vector of each sub-band. After the sub-band transmit adaptive beamforming and the reconstruction of the synthesis filter bank, the frequency domain expression of the signal output at the mth element can be expressed as

$$Y_n(e^{j\omega}) = \sum_{m=1}^{M}\sum_{k=0}^{J-1} w_{mn}[k]X(e^{j\omega})H_m(e^{j\omega})F_m(e^{j\omega}), \tag{19}$$

where $X(e^{j\omega})$ represents the frequency domain form of the input signal.

The final output of the wideband signal transmitting beam antenna pattern with a main beam direction of θ_0 is

$$P(\theta, f) = \sum_{m=0}^{M} v_{st}^H(\theta, f)\mathbf{W}_m H_m(f)F_m(f). \tag{20}$$

4. SIMULATION RESULTS

We will verify the performance of the wideband transmitting ADBF based on sub-band MLCMV algorithm with band partition by computer simulation. In the experiment, the array is a uniform linear array of elements $N = 32$. Each array element has $M = 5$ sub-band processing channels, and the TDL in each frequency sub-band has $J = 3$ taps. The maximum frequency of the input wideband signal is 1500 MHz; the minimum frequency is 1000 MHz; the element spacing is $d = 0.1$ m. So the tap sampling frequency of TDL before the band partition is 3000 MHz, and the TDL tap sampling frequency of each sub-band is 600 MHz after band partitioning.

The bandwidth of the input signal is 500 MHz, which is divided into $M = 5$ sub-bands, and the bandwidth of each sub-band is 100 MHz. Therefore, the length of the filter should be $P = f_s/(B/M) = 30$, and the frequency offset should be $i = (f_L - B/2M)/(B/M) = 9.5$. Then the amplitude frequency response of the analysis filter of the mth sub-band is

$$H_m(z) = H_0(zW^{m+9.5}), \tag{21}$$

$$H_0(z) = 1 + z^{-1} + \ldots + z^{-29}, \tag{22}$$

where $W = e^{-j2\pi/30}$.

Figure 4 is the transmit antenna pattern of wideband RTN algorithm. The main beam direction is 0°, and the null direction is 20°. According to the calculation, the average null depth over the entire signal bandwidth is −43.04 dB; the deepest is −47.03 dB; the shallowest is −36.46 dB.

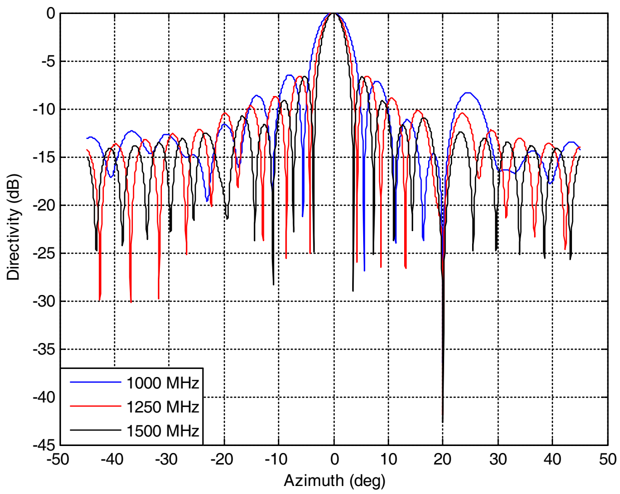

Figure 4. Transmit antenna pattern of wideband RTN algorithm.

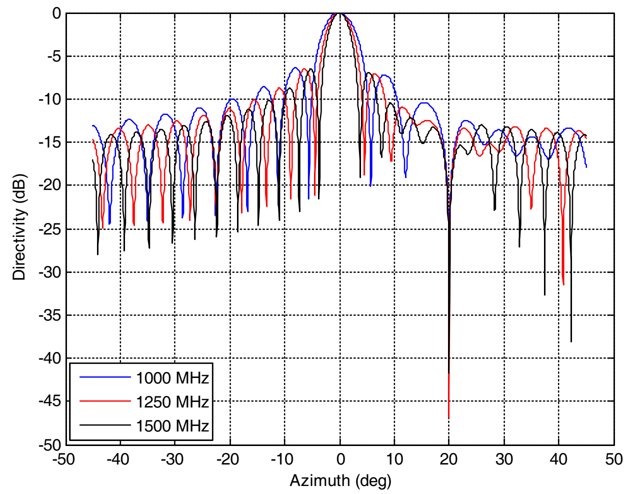

Figure 5. Transmit antenna pattern of wideband MLCMV algorithm.

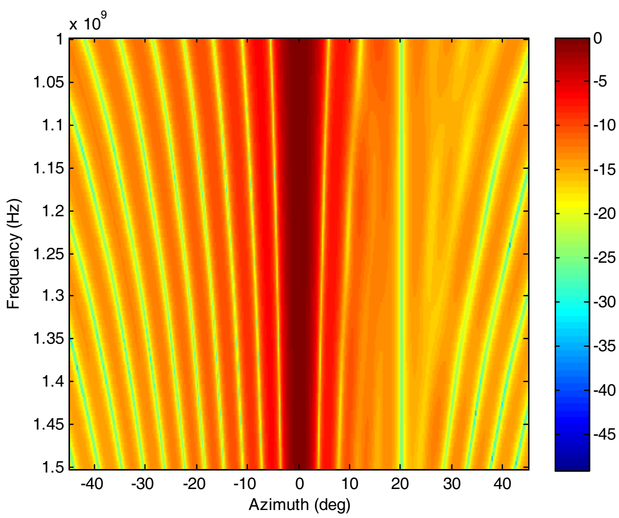

Figure 6. Antenna pattern as a function of frequency-wideband MLCMV algorithm.

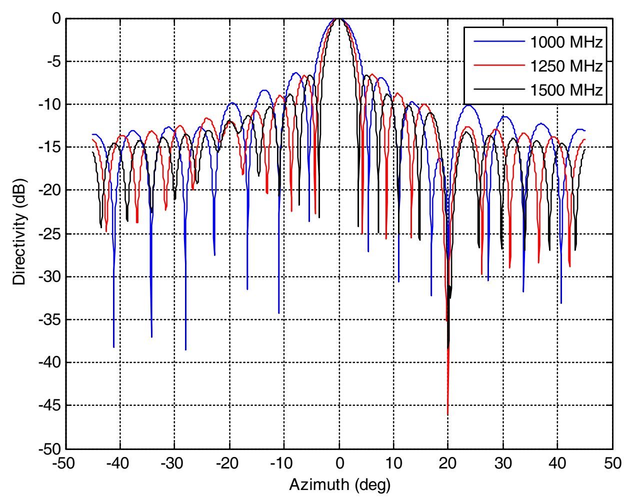

Figure 7. Wideband transmit antenna pattern of sub-band RTN algorithm.

Figure 5 is the transmit antenna pattern of wideband MLCMV algorithm proposed in this paper. The main beam direction is 0°, and the null direction is 20°. The pointing direction of the null does not change with frequency, as also seen in the frequency dependent array pattern of Fig. 6 where the color bar corresponds to directivity in dBi. The yellow line at 20° corresponds to the transmit null, and the frequency dependent slope of the line indicates that the direction of the null is a function of frequency. The average null depth over the entire signal bandwidth is −45.49 dB.

Figure 7 is the wideband transmit antenna pattern of sub-band RTN algorithm. The main beam direction is 0°, and the null direction is 20°. The average null depth over the entire signal bandwidth is −45.93 dB; the deepest is −49.89 dB; the shallowest is −33.27 dB.

Under the same conditions, the wideband transmit antenna pattern of sub-band MLCMV algorithm is shown in Fig. 8 and the antenna pattern as a function of frequency shown in Fig. 9. The pointing direction of the null clearly does not change with frequency. The average null depth over the entire signal bandwidth is −74.76 dB, and the average null depth is increased by 28.83 dB compared with sub-band RTN algorithm and increased by 29.27 dB compared with wideband MLCMV algorithm. Table 1 is the contrast of the null depths formed by different algorithms.

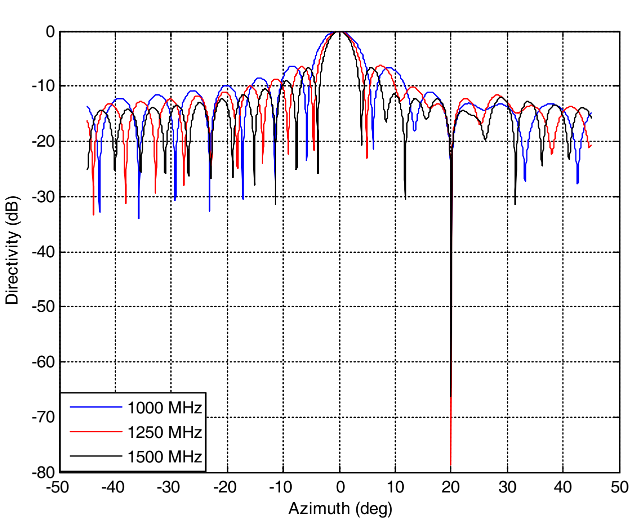

Figure 8. Wideband transmit antenna pattern of sub-band MLCMV algorithm.

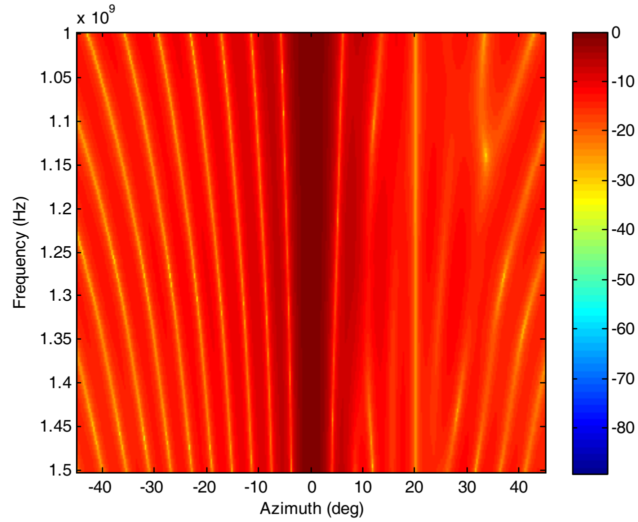

Figure 9. Antenna pattern as a function of frequency-sub-band MLCMV algorithm.

Table 1. The contrast of the null depths formed by different algorithms.

Null depth (dB)	Average	Deepest	Shallowest
Wideband RTN Algorithm	−43.04	−47.03	−36.46
Sub-band RTN Algorithm	−45.93	−49.89	−33.27
Wideband MLCMV Algorithm	−45.49	−49.07	−40.60
Sub-band MLCMV Algorithm	−74.76	−89.22	−64.64

In the simulation, the iterative step of the sub-band RTN algorithm to compute the ADBF weight vector is set to 10^{-8} that ensures convergence and the results obtained through multiple iterations. If the number of iterations is lower, the aperture fill effect cannot be effectively suppressed. The final simulation computation time by MATLAB platform is 18.73 s. Under the same computer simulation condition, the sub-band MLCMV algorithm to compute the ADBF weight vector by MATLAB platform takes 1.47 s. Its computation time is obviously shorter than the sub-band RTN algorithm.

5. CONCLUSION

This paper proposes a wideband transmit ADBF algorithm based on sub-band MLCMV criterion, which can effectively eliminate the aperture fill effect of wideband array and create deeper spatial nulls in the direction of interference. The simulation results verify the effectiveness of the proposed algorithm. Compared with RTN algorithm, the algorithm in this paper solves the optimal weight vector in each sub-band which requires less computation and is efficient to implement in engineering applications.

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