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Research on Analysis of High-Order Fractal Characteristics of Aircraft Echoes and Classification of Targets in Low-Resolution Radars

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Abstract—High-order fractal characteristics of low-resolution radar echoes provide a supplementary description of the dynamic characteristics of the echo structure of a target, which provides a new way for the classification and recognition of targets with low-resolution radars. On basis of introducing the definition of high-order fractal statistic — lacunarity as well as its calculation method and the lacunarity characteristics of a target echo under additive fractal clutter background, this paper analyzes the characteristics of the lancunarity parameter variation of target echoes from a surveillance radar at a VHF band, and puts forward a classification method for aircraft based on the feature of the echo lacunarity scale change rate from the viewpoint of pattern recognition. The target classification experiments using real recorded echo data show that, as a high-order fractal characteristic parameter, the lacunarity scale change rate can be used as an effective feature for aircraft target classification and recognition, and the proposed method has good classification performance.

1. INTRODUCTION

Most of active surveillance radars adopt the conventional low-resolution radar system, and their range resolution is generally much larger than the target size. Therefore, the target is often regarded as a point target, and the target information contained in the radar echo is limited, thus making it a key and difficult point in the field of radar target recognition to classify and identify all kinds of important military targets under the conventional radar system [1]. Aircraft are an important kind of target for the surveillance of air-defense surveillance radars. For the classification and identification of aircraft targets, it is generally based on the echo nonlinear modulations induced by the nonrigid body vibration and rotation of the aircraft rotating parts. To some extent, these kinds of nonlinear modulations reflect the complicated micro-motion modulation effects of various parts of aircraft and contain target attribute information, such as the geometric structure and material composition [2].

So far, many scholars have studied the theoretical models for aircraft echoes from low-resolution radars and proposed many feature extraction methods for echo modulation characteristics, such as complex cepstrum, SVD eigenvalue decomposition, empirical mode decomposition, and fractional Fourier transform [3–7]. Most of these existing features are time domain or Doppler domain features. Although they can reflect the modulation characteristics of target echo to a certain extent, when the beam irradiation time is short, the echo contains less information, and the resolution of Doppler spectrum will decrease. While the pulse repetition frequency is low, the sampling rate of the time domain signal is low, and the echo Doppler spectrum aliasing may be caused, which will lead to the degradation of the classification performance of these features. For this reason, in recent years, many articles [8–11] proposed to introduce nonlinear analysis methods such as fractal, multifractal and multifractal correlation into the characteristic analysis of aircraft echoes from the conventional low-resolution radars

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in time or frequency domain, apply these fractal features to the target classification and identification, and achieve better classification performance under specific experimental conditions. However, the fractal dimension only explains the complexity of a fractal set from a macro perspective. When people study various fractal sets, it is found that several significantly different fractal sets may have the same or similar fractal dimensions. At this time, it is difficult to accurately describe them with the fractal dimension and other low-order fractal parameters, and new fractal parameters must be introduced. As a high-order fractal feature, lacunarity has been widely used in many fields, such as remote sensing, infrared, medical and other image analysis, communication, radar and other signal processing [12–15]. This paper intends to introduce it into the characteristic analysis and feature extraction of aircraft echoes from low-resolution radars, and investigates the application of high-order fractal features in aircraft target classification and identification.

2. DEFINITION OF LACUNARITY AND ITS CALCULATION

When studying fractals, it is found that several fractal sets with different surfaces or structures may have the same or similar fractal dimensions. Obviously, it is impossible to effectively distinguish them by using fractal dimension at this time. Given the non-universal or non-uniqueness of fractal dimension, Mandelbrot suggests introducing lacunarity as a complement to fractal dimension. For a fractal set, the fractal dimension describes the degree of irregularity of its surface or structure, while the lacunarity describes how fast its surface or structure fluctuates. The definition of the lacunarity is in [16]

$$\Lambda = E\left\{\left[M/E\left(M\right)-1\right]^2\right\}, \tag{1}$$

where M is the mass of the fractal set, and $E(M)$ is its mathematical expectation. It can be seen from Eq. (1) that the lacunarity is a second-order statistic, so it is called a high-order fractal feature, which reflects the deviation degree between M and its expected value. It is easy to know from the theory of Coastline Length measurement: the mass density distribution of the fractal set is related to the scale r of the measuring instrument used, and there is $M(r) \sim r^{-D}$. Therefore, the lacunarity is a function of r.

For the calculation of the lacunarity value, the probability distribution $P(m, r)$ of the fractal set can be used, where $P(m, r)$ represents the probability that the number of points falling into a box with a border length r in the fractal set is exactly m (the center of the box can be any point in the fractal set), and for any r, there are

$$\sum_{m=1}^{N} P\left(m, r\right) = 1 \tag{2}$$

where N is the maximum number of points in the box with the border length r. Therefore, $P(m, r)$ contains the mass distribution information of the fractal set. If assuming that

$$M\left(r\right) = \sum_{m=1}^{N} m \cdot P\left(m, r\right) \tag{3}$$

and

$$M^2\left(r\right) = \sum_{m=1}^{N} m^2 \cdot P\left(m, r\right), \tag{4}$$

then the lacunarity of the fractal set is [17]

$$\Lambda\left(r\right) = \frac{M^2\left(r\right) - \left[M\left(r\right)\right]^2}{\left[M\left(r\right)\right]^2} \quad . \tag{5}$$

It can be seen from Eq. (5) that the more uneven the mass distribution of the fractal set is, the larger the value of $\Lambda(r)$ is; otherwise, the smaller the value of $\Lambda(r)$ is. As r gradually increases, the points in the fractal set will be more and more evenly distributed in the boxes, so $\Lambda(r)$ will gradually decrease and eventually tend to zero. From the perspective of fractal texture analysis, the speed of $\Lambda(r)$ changing with the scale r reflects the size of the fractal texture primitives, because when the texture primitives are small, the mass distribution in the boxes tends to be uniform rapidly with the increase of r.

3. LACUNARITY CHARACTERISTICS OF TARGET ECHOES IN THE CONTEXT OF ADDITIVE FRACTAL CLUTTER

Many literature studies have shown that radar clutters, such as ground clutter, sea clutter and airspace clutter [9, 17–19] have significant fractal characteristics, and as man-made objects, aircraft echoes do not generally have fractal characteristics, but different types of aircraft targets will have different effects on the fractal characteristics of clutter. Therefore, it is of great significance to study the lacunarity characteristics of target echoes in the context of additive fractal clutter. For this problem, the following theorem can be derived:

Theorem 1: Assuming that x and y are two independent fractal signals, their lacunarity values are $\Lambda_x(r)$ and $\Lambda_y(r)$ respectively under the scale r. If $z = x + y$, then there is

$$\frac{1}{2}H_2 \leq \Lambda_z(r) \leq \max\left[\Lambda_x(r), \Lambda_y(r)\right] \tag{6}$$

where H_2 is the harmonic mean of $\Lambda_x(r)$ and $\Lambda_y(r)$, viz.

$$H_2 = 2\Big/\left[\frac{1}{\Lambda_x(r)} + \frac{1}{\Lambda_y(r)}\right].$$

Proof: Let $M_{1x} = E[M_x(r)]$, $M_{2x} = E\left[M_x^2(r)\right]$, $M_{1y} = E[M_y(r)]$, $M_{2y} = E\left[M_y^2(r)\right]$, then

$$\Lambda_x(r) = E\left\{\left[\frac{M_x(r)}{E(M_x(r))} - 1\right]^2\right\} = \frac{E\left[M_x^2(r)\right] - E^2\left[M_x(r)\right]}{E^2\left[M_x(r)\right]} = \frac{M_{2x} - M_{1x}^2}{M_{1x}^2}.$$

In a similar way, we can get

$$\Lambda_y(r) = \frac{M_{2y} - M_{1y}^2}{M_{1y}^2}.$$

Also due to

$$E\left[M_z(r)\right] = E\left[M_x(r) + M_y(r)\right] = M_{1x} + M_{1y}$$

and

$$E\left[M_z^2(r)\right] = E\left\{\left[M_x(r) + M_y(r)\right]^2\right\} = M_{2x} + M_{2y} + 2M_{1x}M_{1y},$$

thus there is

$$\Lambda_z(r) = \frac{E\left[M_z^2(r)\right] - E^2\left[M_z(r)\right]}{E^2\left[M_z(r)\right]} = \frac{M_{2x} - M_{1x}^2 + M_{2y} - M_{1y}^2}{(M_{1x} + M_{1y})^2} = \frac{M_{1x}^2\Lambda_x(r) + M_{1y}^2\Lambda_y(r)}{(M_{1x} + M_{1y})^2}.$$

If $\lambda = M_{1x}/M_{1y}$, the above equation can be transformed into a quadratic equation for λ as follows

$$\left[\Lambda_z(r) - \Lambda_x(r)\right]\lambda^2 + 2\Lambda_z(r)\lambda + \Lambda_z(r) - \Lambda_y(r) = 0.$$

Because the real solution of the equation is definite, an inequality can be obtained from the relation between the roots and coefficients of quadratic equation as follows

$$\left[2\Lambda_z(r)\right]^2 - 4\left[\Lambda_z(r) - \Lambda_x(r)\right]\left[\Lambda_z(r) - \Lambda_y(r)\right] \geq 0,$$

that is

$$\Lambda_z(r) \geq \frac{\Lambda_x(r) \cdot \Lambda_y(r)}{\Lambda_x(r) + \Lambda_y(r)} = \frac{1}{2}H_2.$$

And because the mss of the fractal set and its expected value are non-negative, another inequality can be obtained by Eq. (7), namely,

$$\Lambda_z(r) \leq \frac{M_{1x}^2 + M_{1y}^2}{(M_{1x} + M_{1y})^2}\max\left[\Lambda_x(r), \Lambda_y(r)\right] \leq \frac{M_{1x}^2 + 2M_{1x}M_{1y} + M_{1y}^2}{(M_{1x} + M_{1y})^2}\max\left[\Lambda_x(r), \Lambda_y(r)\right]$$
$$= \max\left[\Lambda_x(r), \Lambda_y(r)\right]$$

Combining Eqs. (8) and (9), Eq. (6) can be obtained. Q.e.d.

Theorem 1 shows that under the condition of superimposing background clutter, the lacunarity value of the mixed signal is larger than one-half of the harmonic mean of the lacunarity values of the target echo signal and background clutter, and smaller than the larger one between them. Although the target echo signal does not have significant self-similarity, it can still be regarded as a generalized fractal signal. Therefore, the appropriate lacunarity value can also be obtained under the radar actual working condition of "target echo + background clutter".

4. LACUNARITY CHARACTERISTICS OF AIRCRAFT ECHOES

In the previous section, the range of the lacunarity value of a target echo superimposed on the fractal background clutter is theoretically given. In the actual working situation, target echoes are generally superimposed on the background clutter. In this paper, the echo data of multi-type aircraft target recorded on a VHF band air-defense surveillance radar are experimented, and some useful conclusions are obtained through the analysis. It should be noted that in the following analysis, in order to reduce the adverse effects of target distance, flight attitude and other factors on echo characteristics analysis and feature extraction, two kinds of preprocessing have been done: one is attitude partition, and the other is energy normalization. The specific method can be found in [9].

Figures 1(a) and (b) show the fractal dimension statistical histograms of thousands of sets of echo data of a civil aircraft and fighter aircraft flying toward and off the radar station respectively. As can be seen from Fig. 1, it is difficult to distinguish between the fractal dimension values of echo signals of the civil aircraft and the fighter aircraft, especially when they fly off the radar station. In this case, it is not reliable to classify them by means of fractal dimension.

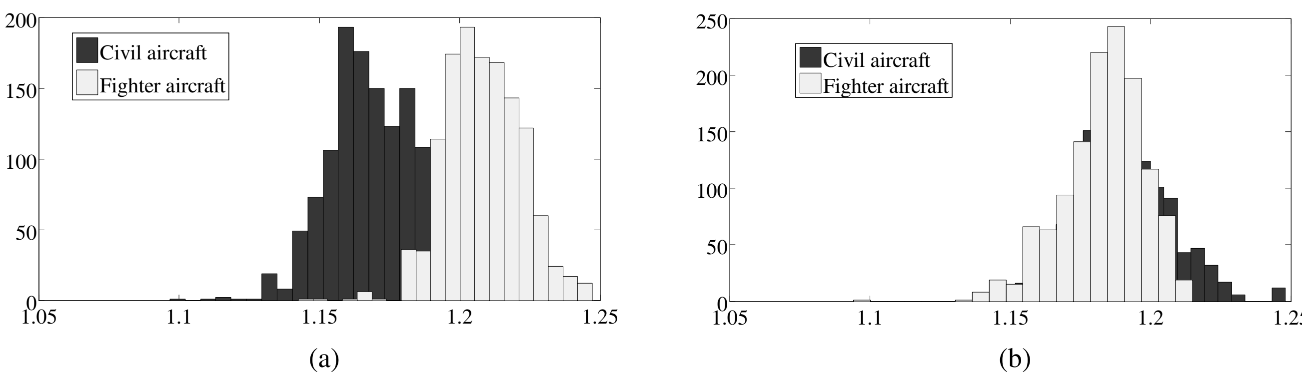

Figure 1. Fractal dimension statistical histograms of radar echoes of different types of aircraft. (a) Flying towards the radar station. (b) Flying off the radar station.

Next, calculate the lacunarity characteristics of target echoes of the two types of aircraft. Figs. 2(a) and (b) show the lacunarity characteristic curves of several sets of echo data of the two types of aircraft targets when they fly towards and off the radar station respectively, and the scale r is from 2^0 to 2^{10}. As a contrast, the lacunarity characteristic curve of the pure clutter has been given at the same time. It should be pointed out that the sample mean and sample standard deviation of the target echo lacunarity values at different scales are given in the figure. As can be seen from Fig. 2, there are many different scales in both flight attitudes, and the echo lacunarity features at these scales can better distinguish these two types of aircraft targets. The variation curves of echo lacunarity features of different types of aircraft targets present a different rate of change. In addition, as can be seen from the figure, in general, the lacunarity values of radar signals containing target echoes are larger than those of pure clutters, so it can be inferred from Theorem 1 that the lacunarity values of target echoes are better than those of pure clutters. It can be further inferred from Theorem 1 that in the case where the target signal is relatively low, the lacunarity values of radar signals containing target echoes may be smaller than those of pure clutters. As shown in Fig. 2(b), at certain scales, the lacunarity values of the civil aircraft echoes are smaller than those of pure clutters.

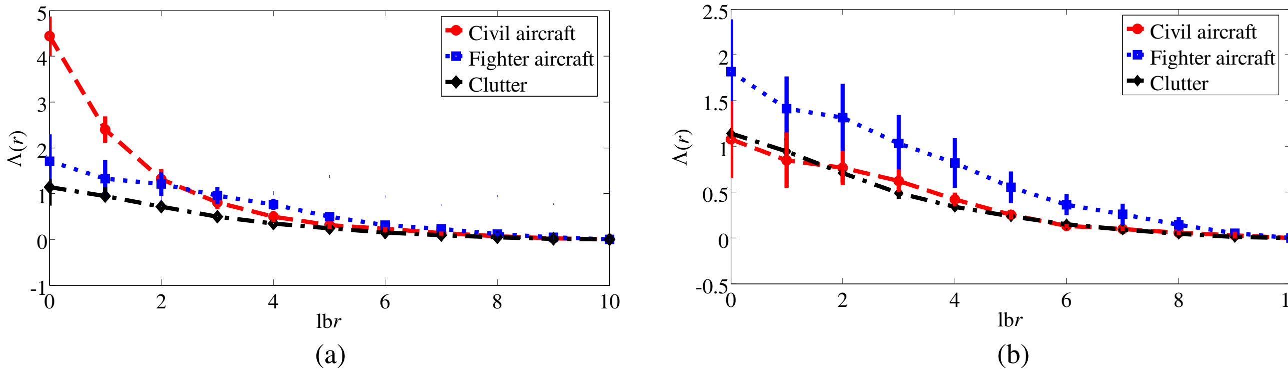

Figure 2. Lacunarity characteristic curves of radar echoes of different types of aircraft. (a) Flying towards the radar station. (b) Flying off the radar station.

In order to better describe the variation law of the lacunarity feature, below we extract a characteristic quantity which is more stable than the lacunarity value itself — the lacunarity scale change rate as the target classification feature, namely:

$$E = \frac{1}{2^N - 1} \sum_{i=1}^{N} \left[\Lambda\left(2^0\right) - \Lambda\left(2^i\right) \right] \tag{7}$$

where 2^N represents the maximum size of the boxes. The fractal dimension reflects the degree of irregularity of the fractal set, while the lacunarity scale change rate reflects the uniformity of mass distribution of the fractal set.

Next, we further consider the ability of the lacunarity scale change rate to describe pure clutters and different types of aircraft echoes. Experiments are carried out by using the same data as in Fig. 2. The results are shown in Table 1.

Table 1. The ability of lacunarity scale change rate to describe clutter and target echoes.

	Flying towards the radar station		Flying off the radar station	
	Mean value	85% confidence interval	Mean value	85% confidence interval
Civil aircraft	0.04×10^{-3}	$\pm 0.68 \times 10^{-19}$	0.01×10^{-3}	$\pm 0.28 \times 10^{-19}$
Fighter aircraft	2.4×10^{-3}	$\pm 0.26 \times 10^{-19}$	-0.83×10^{-3}	$\pm 3.25 \times 10^{-19}$
Pure clutter	0.47×10^{-3}	$\pm 2.71 \times 10^{-19}$	0.47×10^{-3}	$\pm 2.71 \times 10^{-19}$

As can be seen from Table 1, since the lacunarity change rate reflects the high-order fractal features at each analysis scale, it is clear that it has better description ability than the fractal dimension and can better distinguish different types of aircraft targets.

5. AIRCRAFT TARGET CLASSIFICATION BASED ON HIGH-ORDER FRACTAL FEATURES

In this section, we will take the aforementioned echo data from several different types of aircraft as the experimental data. Below we mainly use the echo data recorded when aircraft targets fly towards or off the radar station and classify different types of aircraft targets based on the lacunarity scale change rate features. Compared with other classifiers, support vector machine (SVM) has stronger generalization abilities and a steeper convergence rate [20], so here we will adopt SVM as the classifier. The classifier takes the Gaussian kernel $K(x_i, x_j) = \exp(-||x_i - x_j||^2/\sigma^2)$ as its kernel function. Because there is no prior knowledge about the parameter σ^2, in the following experiments, we will try different parameter values several times without going beyond the calculation burden and take the parameters which can well classify different types of aircraft targets as the kernel function parameters.

Experiment 1: Take SVM as the classifiers to analyze the classification performance of the fractal dimension feature and the lacunarity scale change rate feature contrastively when aircraft targets fly towards the radar station. There are three types of aircraft targets (Type 1 $\sim$ 3) used for the classification experiment, where the total number of samples is 1280 for each type of targets, and for each type of aircraft targets, the feature data extracted from 256 groups of echo data are chosen as training samples. If the correct classification rate (CCR) is defined as the ratio of the number of samples which are classified correctly and the total number of samples, then Table 2 shows the classification results by using the fractal dimension feature and lacunarity scale change rate feature. It can be seen from Table 2 that under the experimental and data conditions of this paper, when aircraft targets fly towards the radar station, whether for the CCR of each type of aircraft targets or the average CCR, the CCR using the lacunarity scale change rate feature reaches 100%, exceeding that using the fractal dimension feature more than 23%. Therefore, the classification effect is very satisfactory.

Table 2. CCRs when aircraft targets fly towards the radar station.

Type \ Feature	Fractal dimension	Lacunarity scale change rate
Type 1	87.32%	100%
Type 2	69.16%	100%
Type 3	73.18%	100%
Average CCR	76.77%	100%

Experiment 2: Take SVM as the classifiers to analyze the classification performance of the fractal dimension feature and the lacunarity scale change rate feature contrastively when aircraft targets fly off the radar station. There are also three types of aircraft targets (Type 4 $\sim$ 6) used for the classification experiment, and the numbers of samples used for classifier training and testing are 256 and 1024, respectively. Table 3 gives the corresponding classification results. As can be seen from Table 3, under the experimental and data conditions of this paper, when aircraft targets fly off the radar station, whether for the CCR of each type of aircraft targets or the average CCR, the classification effect using the lacunarity scale change rate feature is also very ideal, and the CCR also reaches 100%, which is nearly 18% higher than that using the fractal dimension feature.

Finally, we take the target classification when aircraft targets fly towards the radar station as an

Table 3. CCRs when aircraft targets fly off the radar station.

Type \ Feature	Fractal dimension	Lacunarity scale change rate
Type 4	77.51%	100%
Type 5	86.44%	100%
Type 6	81.82%	100%
Average CCR	82.08%	100%

Table 4. Classification results under different SCRs.

CCR \ SCR/dB	2	0	−0.5	−1	−2	−5
Average CCR/%	100	100	99	97	95	90

example, add Gaussian white noise with different intensities to the real recorded echo data, and further investigate the classification performance of the lacunarity scale change rate feature under the condition of low signal-to-clutter ratio (SCR). Table 4 shows the classification results of SVM classifier based on the lacunarity scale change rate features of echo data under different SCRs. It can be seen from the table that the lacunarity scale change rate feature has strong robustness. Even under the condition of SCR $= -5$ dB, the average CCR can still reach more than 90%. So it is a practical and reliable target classification feature.

6. CONCLUSIONS

This paper introduces the theory of high-order fractal analysis into the aircraft echo characteristic analysis and target classification with low-resolution surveillance radars. On basis of introducing the definition of the high-order fractal statistic — lacunarity as well as its calculation method and the lacunarity characteristics of a target echo under the additive fractal clutter background, it analyzes the high-order fractal characteristics of low-resolution radar aircraft echoes and proposes a classification method for aircraft targets based on the lacunarity scale change rate feature. Finally, the effectiveness of the target classification method based on the lacunarity scale change rate feature is verified by the real recorded aircraft echo data. As a contrast, the classification performance based on the fractal dimension feature is analyzed. The experimental results show that high-order fractal features have good classification performance.

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